Using MCL to Discover More Mathematics from Real Life[1]

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Abstract:

The authors of this paper has designed an example using music to support the connection between mathematics and real life practice, with the help of the technology of Mobile Calculator Lab (MCL). The example, which is highly recognized by experts, teachers and student, was carried out in a class by one of the authors. The example showed that, with the help of MCL, students could collect and 'see' the relative data more conveniently, handle it more flexibly and explore the rules relatively easier than conventional practices. It provided more opportunities for students of different levels to make good use of mathematics to organize the exploration of natural phenomenon, and accumulate more mathematics experiences. Most likely, it will deeply influence the future development of science curriculums in China, including integrated science courses.

1. Introduction

1.1 Brief introduction of MCL

MCL（Mobile Calculator Lab）is a portable digital laboratory. It collects data from sensor, and sends data to a computer or a Graphing Calculator (GC), with the Data Streamer software. The MCL combines the functions of collecting outside data, processing data and displaying results, so that the data can be analyzed more freely from different angles and aspects. MCL further develops the functions of the Graphing Calculator[1].

The sensor is used to collect all kinds of information from the surroundings, such as sound, electricity and light. Different sensors will be required for different purposes and different objectives. The GC is used to process the collected data and show the results mathematically.

A GC is a palmtop calculator, which has the functions of calculating, displaying function graphs, programming, data analyzing and so on, especially, according to the characteristics of

science learning, a graphing calculator is designed to improve students' ability of collecting information, analyze problems and solving them [1].

1.2 Introduction of the students

In this paper, the 41 students of ninth-grade joined in the research. They come from Beijing No.22 Middle School, comprising 21 boys and 20 girls. Their scholarly achievements are excellent, even in the entire city of Beijing. Every student is talented in music, dancing, drawing, or sports. They will spend the next three years in this school as senior middle school students. Although they are students of 9th grade, they have completed courses of grade one in senior middle school, and have used GCs for a year. One of the authors of this paper, Li Hong, is their mathematics teacher and the head-teacher of the class. In the teaching process, we try our best to inspire their interests and encourage them to explore and guide them to study knowledge and methods within their interests and passion. Our goal is to teach these students mathematics that they really need in their life time.

1.3 No such research of mathematics teaching in MCL environment has been published in China.

In China, the research using the MCL to teach mainly focuses on physics and chemistry, with their primary attentions on the experiments themselves. No research has been carried out to study the effect of the MCL on mathematics education. Investigation shows that many mathematics teachers believe the usage of MCL in class may weaken students' capacity for calculation. Meanwhile, there are only a few examples in China for the teachers to study and imitate A few teachers are familiar with MCL. In order to solve this problem, we have developed a class example in mathematics teaching using MCL and have implemented the research in the whole process of design, implementation and evaluation. We started with the music, which students are interested in and collected the sound changes in relation to time establishing the connection between music and mathematics. During the lesson, students were guided to collect the data and images by themselves. They guessed a conclusion and tried to validate the guess by analyzing the images and carrying out inferences. The subjects' differences are emphasized in the research, and the key points focus on the mathematics theory itself behind the phenomenon. Excellent teaching results are achieved in this way.

1.4 Research methods

Questionnaire method, individual interviews and LPS video research methods are used to research this paper.

2. Topic selection reasons

Students' interests allow us to conclude that music is a perfect research objective.

2.1 Students are interested in music.

"The application of trigonometric function" is part of the High School Mathematics Textbook of People Education Press in China. There are some topics including the application of the trigonometric function, such as light, electromagnetic, and water waves. Reading materials explaining tides and sound are included in the textbook which give an introduction of the connections between tides, sound and the trigonometric function [2]. The sound and trigonometric connections are added in the new textbook, showing that the application of knowledge is more

emphasized. The individual interviews showed that almost all students are interested in music.

2.2 Students are interested in self-exploration using MCL.

Before the use of MCL, students usually had to search for specific matters in scientific or technical literature to check the results of previous experiments and present a report to the class. This is a common teaching method used in the classroom. This stereotype is unnatural, because students get information from Internet, rather than performing the experiments and recording the results themselves. They are not convinced of the relevance during the teaching phase. We carried out an investigation using questionnaires. The results showed that 38 students out of 41 (which account for 92.7% of the total number of students) were willing to explore by themselves to find the mathematics relationships in music using the tool such as the MCL. Looking for the information on line and exchanging their ideas by reports are no longer what they are interested in.

2.3 Some students in the class have done research on music.

Four students in the class have written a thesis, with the title "Music--The most beautiful curve". They used GC to draw scatter diagram of the rhythm and frequency of the music, and joined the points with smooth curves. They tried to find the common characters between Chinese traditional music and world classic music. Because of the discontinuity of the rhythm, their research needs to be further studied and perfected.

Based on the above three reasons, we have decided to pursue the research, and develop the research of the relationship between music and trigonometry.

3. The display and analysis of the teaching example

3.1 Objectives of teaching

We shall consider learning that takes place in the classroom as a part of the experience of a student's life. Therefore we designed our objectives based on the following:

(a) To find methods to research functions through the study of trigonometric functions.

(b) To do the reasoning by function's graph, and to follow the abstracting process in mathematics.

(c) To understand the relationships between loudness and amplitude, tonality and frequency, separately.

(d) To explain this phenomenon by modeling.

(e) To construct the connections between mathematics and music in daily life, and to strengthen their ability to apply what they've learned and to solve practical problems.

(f) To make them aware of their consciousness to pursuit of beauty.

3.2 Records and analysis in class

(a) Set up the situation

◇ The teacher showed a paper written by some students: "Music--The most beautiful curve". In the paper, four students had chosen to represent Chinese traditional music with "The jasmine" and "You and me", and foreign music with "Edelweiss", as the research objectives. They used GC to draw scatter diagrams of the rhythm and frequency of the music, and found the common traits between native and foreign music. They had found relationships between some important factors, quantified them mathematically, and shown the essence of the problems scientifically. However, some students suspected the thesis of the content about the selection of the songs and the propriety

of choosing the rhythm as variable, resulted in student discussion.

◇ It is indeed a reflection of the function idea through which we guide students to evaluate the existing model and consider how appropriate it is. It shows that it is student based since the problems come from students themselves. Meanwhile, students and teachers stimulate each other. Students promote the instruction of teachers, and on the other hand, the teachers encourage students to learn more.

(b) The simplification of the real world.

◇ Teacher: Research of music is complicated. Do you agree?

Students: Yes, but we can study it from the three factors of tonality, loudness and timbre.

Teacher: Sound has an extended range, but the three factors above are primary. So we can study it with respect to these. Which factor will you choose to start the research?

Student A: I'd like to start with the tonality, because I think it can be easily recognized.

Teacher (nodding): And other students?

Student B: The loudness. It can be easily measured.

Teacher (nodding): Is there anyone considering the timbre?

All students: There is no standard for measurement, so it is not easy for us to study it.

◇ The natural world is complex. With guidance, students propose to start the research from the basic factor of sound, loudness, tonality and timbre. It is an important mathematical concept to grasp the key factor and simplify the real world. On the other hand, we do not require students to answer the question. Instead, they raise questions themselves and explore it, which is effective to their research.

(c) Explorations of the relationship of the loudness, amplitude, tonality and frequency.

Students collected different sounds of music, proposed guesswork, found proof and came to the conclusion that loudness of sound has a connection with the amplitude of the images, while tonality is connected to the frequency of the images.

Exploration 1: Propose guesswork—music from piano.

◇ Collecting time: 2 minutes and 27 seconds.

Activity: One student played the piano in the front row of the classroom, and the others connected the collector, sensor and GCs together to collect the sound signals from the piano. The teacher also connected the collector, sensor and computer together to collect the signals. Sound signals were sent to the computer and GCs, and the signals images were displayed at the same time. The following picture shows one of the images.

After the activity, teacher asked students: "Have you drawn a conclusion?"

Student A: The points at the top and the bottom points have something to do with the loudness.

Student B: If the image is stretched a bit, the spacing of image may be related to the tonality.

Teacher: What secret is hidden in it? Student B suggested stretching the image a little. And I choose part of the image and stretched it. You can see that the image becomes larger continuously. It seems that the changes may have a connection to the tonality, correct? And another assumption shows the loudness has something to do with the top and the bottom points. How do we describe it in mathematics language?

All students: Amplitude.

Teacher: We conclude that the loudness has a relationship with the amplitude. Let's continue.

What does the transverse axis mean?

All students: Time.

Teacher: Let me ask the author of the thesis "Music--The most beautiful curve" a question. What does the transverse axis mean in your article?

Author: Rhythm.

Teacher: We have some similarities of choosing transverse axis. But our axis here is continuous.

◇ Students are led to analyze the function image in a mathematical way. The conclusion is drawn that the loudness and amplitude are connected while the tonality is related to the frequency. The everyday language and the language of mathematics are the same in this exploration. The images generated by MCL are compared with the images in students' thesis, which allow students to make comparisons. Rhythm is not continuous, while the time, which is represented as the transverse axis in the MCL image, is a variable that changes continuously. Therefore the original model is modified. We do as much as we can in class. The students are the focus of the activity.
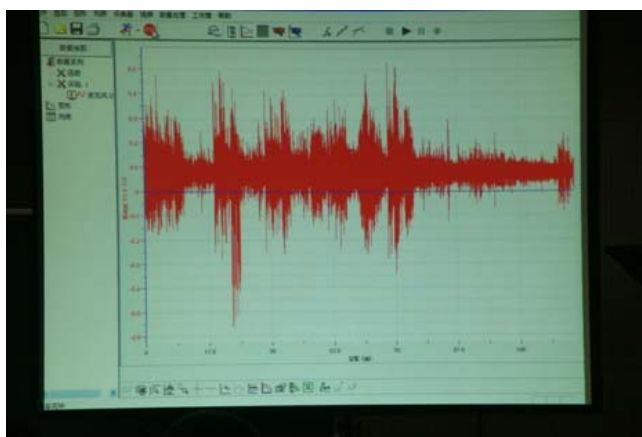

Figure 1   The real-time image displayed by MCL

Exploration 2: Verify guesswork – Music from flute.

◇ Collecting time: 40 seconds.

Activity ended.

Teacher: We collect the flute sound to verify the former conclusion.

Students: With the help of the MCL, sound signals from a flute are collected.

Students accepted the relationship between loudness and amplitude.

◇ Students collect the data by themselves and verify the relationship of loudness and amplitude during the experiment. Loudness is greater while amplitude is large, and loudness is less while amplitude is small.

Exploration 3: Verify guesswork – The music from piano.

◇ Collecting time: 14 seconds

Activity:

Teacher invited a girl student to play the piano. The teacher then asked the students to watch the images on screen as she was playing. Students should watch carefully to see whether the changes of loudness and the changes of the amplitude were corresponding to each other?

Students nodded, watching the changes of amplitude in the image with the elegant music

playing.

◇ Students watch the screen and see the change of amplitude as the loudness of the sound changes. The relationship between the loudness of sound and the amplitude in images is further verified. Students practice and observe that the image change by themselves. In this way, the students can experience the progress of verification and get a taste of the excitement of correct conclusions.

Exploration 4: Verify guesswork – The sound from tuning fork.

◇ Collecting time: 2 minutes and 26 seconds.

Activity:

The teacher pointed out that the purpose of the experiment was then different. The relationship between tonality and frequency was to be studied. The teacher then distributed tuning forks to each group, and students completed the exploration in the form of group discussion. The results of their exploration showed that different tuning forks display different waveform.

Activity ended.

Students were invited to go to the platform and sound their tuning forks. The images of sound were displayed on the screen in real time.

Teacher asked students: Have you noticed the mark of "440Hz" in the bottom of the tuning fork?

A student was invited to observe such a mark on the fork, and it was marked with 440Hz. So the frequency of the tuning fork was confirmed.

Teacher and students reached the conclusion: The pitch of tonality is related to the cycle of the image. Since the cycle is the reciprocal of the frequency, the tonality pitch is related with the frequency.

Figure 2   The scene of students' exploration

◇ The relationship between tonality and frequency is not as obvious as the relationship between loudness and amplitude. If students are told the answer directly, they will miss a good opportunity to experience the live experience of data collection. Tuning forks gives out a pure tone. We use this characteristic to let students carry out the experiment to find the frequency of the image, using a simple model. They then conclude a relationship between tonality and frequency. Simplification of the model is a key idea in functions.

Exploration 5: Verify guesswork – The sound from piano.

◇ Collecting time: 5 seconds.

Activity:

The teacher told a student to play the high-pitched octave and the low-pitched octave with the same strength continuously. The images from the two sounds were displayed together. The teacher stretched the images, and found that the frequencies of the high-pitched and low-pitched octaves were 520Hz and 260Hz. The high frequency was exactly twice as many as the low one. The students were excited to see this.
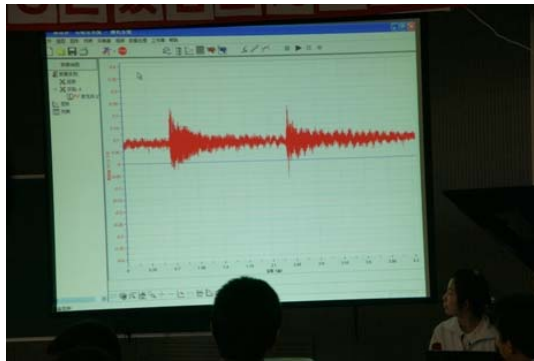

Figure 3   The function image of the high-pitched and low-pitched octave of the piano
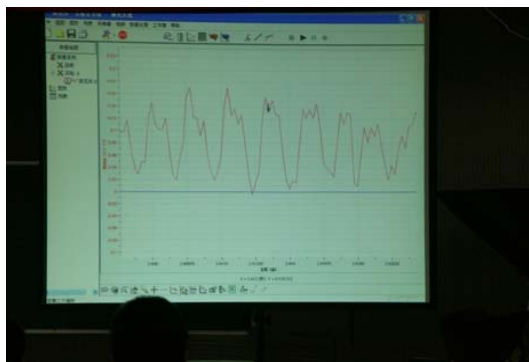

Figure 4   The stretched function image

◇ The high-pitched octave and low-pitched octave of piano are displayed on the computer and the screen in real time. Students can observe the relationship between tonality and frequency directly and easily. The tonality is high when the frequency is large, while the tonality is low when the frequency is small.

(d) Exploration of the Curve fitting by MCL equipment

◇ Exploration time: 3 minutes and 22 seconds.

Teacher asked: We have verified our original conclusion. What should we research further?

Students: We want to fit the curve.

Because of the periodicity of the sound image, the trigonometric function $y = A\sin(\omega x + \varphi)$, which also has periodicity, can be used to fit the curve. The teacher asked the students in each group to do the curve-fitting of the images using the MCL equipment in their hands.

The teacher observed students doing this and gave them some real-time instructions according

to their needs.

The activity ended.

Each group assigned a representative to report what they had concluded.

Student A: Our group chose the tuning fork to research. The result from the fork's sound is an image of a standard trigonometric function, without any superimposition. The information is transferred to the GC and the trigonometric function is fitted obtaining an analytic expression. A sinusoid expression $y = A\sin(\omega x + \varphi)$, is obtained, which is the fitting result.

Student B: We chose three cycles to fit the curve. The collecting time interval is adjusted to a shorter one, which makes the fitting more exact. Some of the data that are regular are inputted for statistic calculating and the scatter diagram is drawn. We try to find the regularity and complete the curve-fitting. The waveform is similar to the trigonometric functions. The expression is checked and the frequency is calculated, which tallies with the frequency of the tuning fork.

◇ Both the students can simplify the model and fit the curve of the function image from the simple waveform of the tuning fork. Student B does a wonderful job. He knows to shorten the time interval to reduce errors. He is also able to use the statistics function of GC to draw the scatter diagram. By observing the image, he chooses an appropriate expression to fit the curve, and verify the frequency of the tuning fork from the data from the curve-fitting. The students' performance is far beyond what we expected.

(e) Superimpose simple trigonometric functions' images into image of music.
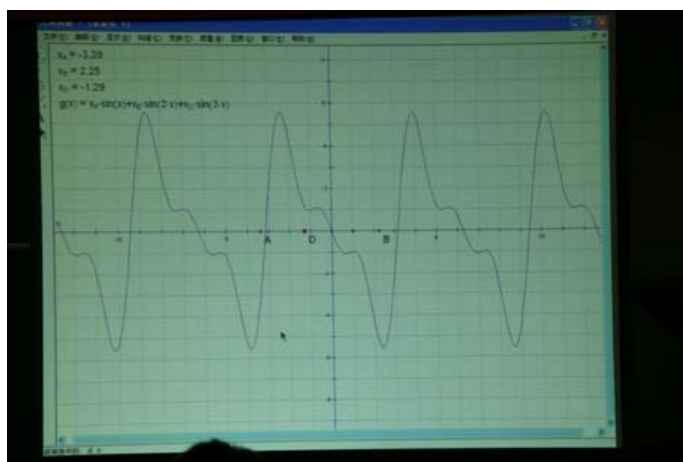

Figure 5   Complex function image from the superposition of simple trigonometric functions

◇ Teacher: We have just collected the sound signals found the function images from computers or GCs. Now, we will give some artificial expression of simple functions, such as $y = \sin x$, $y = \sin mx$, $y = \cos x$, $y = \cos nx$. Let's see whether or not the superimposition is similar to the collected data.

The teacher showed the superimposition on computer with the help of geometric drawing board software. Students and teacher would find that the two kinds of images are quite similar.

◇ In the former teaching in class, the superimposition of trigonometric functions is shown only by geometric drawing board and is very difficult for students to understand. But today, they accept it.

(f) Comprehensive analysis

The time of this lesson is 50 minutes, in which 9 minutes and 14 seconds is used for exploration by students, amounting to 18.47% of the total class time. The proportion is higher than normally in China (less than 5%)[3]. We give students more time to practice by themselves and join the exploration. They can see the relationship between mathematics and the real world while practicing.

4. Teaching assessment

The teaching assessments include the evaluations from students, teachers as class attendees and experts in mathematics and mathematics education. The assessments from students and the attendees are collected by questionnaires. Expert evaluations are carried out by group discussion. The attendees and experts evaluate the teaching just ten minutes after the class, while the students do their assessments two months later. The questionnaire investigation includes two aspects. Firstly, the content of the class is assessed by students and attendees' mastery degree of the lesson. So the teaching effect in MCL environment is properly evaluated. The second part of the questionnaire includes some simple questions. The people who complete the questionnaires can quickly give their opinions including negatives.

4.1 Students' assessment

(a) The investigation result shows that 37 out of 41 students (90.2% of total students) want to use MCL equipments frequently. More lessons like this one are expected by the students.

(b) Students can basically finish the explorations in class. Almost all the students try the exploration and draw conclusions. However, most of them, about 60%, do not get the specific results, because of the time limit. It shows that the time for exploration is too short. It is suggested that more time be allowed for their exploration in class.

The Table 1 below shows the specific conditions in class.

Because Explorations of the relationship of the loudness, amplitude, tonality and frequency are the foundation of the Exploration of the Curve fitting and superimpose simple trigonometric functions' images into image of music, we classify the first one as exploration of ordinary difficulty and the second as high difficulty.

Table 1　Students' completion of explorations of ordinary difficulty and high difficulty

Completion situation	A	B	C	D
Student numbers（ordinary difficulty）	17	24	0	0
Student numbers（high difficulty）	15	25	1	0

Notes:

A: Understands and gets the ideas quickly, and finishes independently in time.

B: Try to gets the ideas, but doesn't get the result in time limits.

C: Has no ideas, and understands the problem after teacher's explanation.

D: Has no ideas, and cannot understand even after teacher's explanation.

We can find from the table that students perform similarly in the situations of low and high difficulties. From the student interviews, we know that the experience of exploration in the low difficulty will be helpful when they deal with high difficulty exploration. So they also do well in the exploration of high difficulty.

(c) Students learn a lot in this class. Emotionally, they find mathematics problems in music, get perceptual knowledge of mathematics, and feel the harmony of mathematics and music in a new perspective. In the aspect of knowledge, they understand and manage the MCL's function and usage, which gives them a foundation for their future research with MCL. The students find that mathematics problems in music allow then to draw mathematical conclusions and verify them. Also, they establish relationships between physics, music and mathematics. In the aspect of ability, they experience the exploration by themselves and in the group. They find that mathematics doesn't mean always doing homework, since there are so many unknown things for them to discover. This class enhances their ability to research and experiment by using new technology.

(d) Although two months has passed, the experience impresses the students deeply. 21 students, 51.2% of the total number, believe that they did get most profound impressions by self-explorations. The proportion is the largest in the class, compared to other situations. The students also have deep impressions from the following activities, such as collecting sound wave by MCL equipment and establishing relationships between physics, music and mathematics, verifying the frequencies of piano and tuning fork, and fitting the curve by GC. Some student pointed out that the smile of the teacher's face really impressed them.

4.2 Attendees' assessment

a) All attendees present are interested in the class. The conclusions are as follows.

The problem studied in this class comes from practical application. We can see mathematics is useful.

The class shows the beauty of mathematics. When music is played and real-time image is shown, music and mathematics are joined together organically.

In this class, the students are guided to find a problem and solve it themselves. The ability of analysis and problem-solving and the consciousness of application are strengthened.

The teacher and students interact sufficiently. The teacher is experienced and she respects the students and appreciates the beauty from the students.

b) Present teachers think the class wouldn't be so successful without the help of MCL. The conclusions are as follows.

MCL equipment is a fantastic tool for collecting and analyzing data. It transforms the collected signals into images. And students can see the changes of the images in real time when the sound changes. The relationship between mathematics and practices can be established in their mind.

With MCL equipment, students can experience the process of discovering and exploring the world, which is helpful to the students understanding of mathematics and their exploratory learning. And it creates a new area of the research-based learning.

The MCL is an effective attempt to improve present teaching mode.

c) Several teachers have doubts about the usage of MCL. They worry about the students computing capability may be weakened and their college entrance examination achievements may be influenced. We will make further research to address this concern.

4.3 Experts' assessment

Experts' assessment shows that there are beauties in this class. This students realizes the subject integration of mathematics, physics and music and learn knowledge, emotion, attitude and values.

a) There are beauties in this class. According to the aesthetic standards, the beauty has characteristics of simplicity, symmetry, singularity and harmoniousness. In this class, the model based on the original student thesis is improved. The optimization of the model shows the beauty of simplicity. The periodicity of the function images reflects the symmetry. And there are many examples of the beauty of singularity, such as the analyzing music mathematically, collecting data from different instruments using the MCL, proposing and verifying guesswork and curve-fitting. However, there are also the unifications in the singularities. For example, music from different instruments have similarity. Besides, the teaching environment is harmonious, where teacher and students and different groups cooperate in harmony.

b) This class realizes the subject integration of mathematics, physics and music. The mathematics language, such as cycle and extremum, the physics language, such as frequency and amplitude, and the music language, such as tonality, loudness and timbre, are also integrated. Besides, the integration of technology and subject is also realized here.

c) Students learn the method to solve realistic problems which are more important than the knowledge itself. Not only knowledge, but also emotion, attitude and values should be involved in a class. This class can be full of vigor in this way. What students learn is not only the method about the mathematics knowledge itself, but also the method to establish connections between mathematics and practice. With teacher's guidance, students learn to analyze the real world and learn to find out what factors can be quantified and what can't. For example, the timbre can't be researched as a variable. This process is important to students especially. In class, the teacher should teach students to analyze problems like this. If students are interested, they can carry out their research in a similar way.

d) Without the techniques support, this class may be directed indistinctly. It can't be studied like this. For this lesson, the objective of knowledge itself is not an important one.

5. Acknowledgements

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