Applying GC to Problem Solving: A Case Study of Transfering Handheld Game into Classroom

Chang wenwu
Changwenwu@hotmail.com
Department of Research
Putuo Modern Educational Technology Center
Shanghai, China

Chang wencui
Jsjkajia01@163.com
Department of Computer Science
Jinzhong College
Shanxi, China

Abstract

*This report present a case of high school mathematics project in which graphic calculators (GC) was used as a tool in two ways: one is to use providing small handheld game "Lights Out"; the other way is to use it in solving this puzzle. two classical methods were offered during the solving phase: solving a linear system or applying an algorithm to search the answer. This makes the same game useful in teaching two different parts of mathematics curriculum. This design engages students in team activities and ICT using, which coincides with the spirit of the mathematics reforming movement at present in a full swing in China. In addition, it can be seen as an example of math edutainment or mobile learning. By encourging students to take part in these project activities, the author believes that math course can be made more interesting and fruitful. The pattern of "**C**hoose-**A**nalysis-**D**esign" method is most likely to be a common way in applying game to mathematics projects.*

1. Introduction

Math education reforming in the global world has seen much changes recently. As SIAM and IEEE holding conferences on Edutainments enclosing game into its topics, China also get some new trends in mathematics education. On the one hand, the math teachers are asked to add more activities into their courses in order to forster students' problems solving skills; on the other hand, students and teachers are encouraged to take advantage of modern technology in learning and teaching. However, math teachers even in western world often feel difficulties in choosing appropriate tasks for nurturing student problem-solving strategies and planning appropriate problem-based classroom activities ([7] and [8]). Things are the same in China even when the teachers know some of the knowledge they teach (such as matrix, linear algebra and algorithm of programming) do have close relationship with real world problems.

Hand-held games are making this situation changing. So long as students use Graphic Calculators in math class and homework, they also often play games enbedded in it. Only HP 38g GC has got dozons of games downloadable from the Internet among which are some famous games like Tetris2, Mastermind, Solitary, Hapaman 1.1. And games can also be seen in other hand-held devices such as electronic Dictionary or intellegence mobile phone. These games are often downloaded freely from the internet. Fortunately, some of the games are found useful in teaching mathematics in high school or university.

"Lights Out" is an electronic handheld game, released by Tiger Toys in 1995. In China, it is a handheld game often found in electronic dictionary. The game consists of a board of 5×5 grids of lights; when the game starts, these lights are all switched on. Pressing one of the lights will toggle it and the four lights adjacent to it on and off. The aim of the game is to switch all the lights off by toggling lights on and off. A successful solution is therefore a sequence of presses that toggles all the "on" lights an odd number of times.

Recently, Zhang Yuting and her student [1] have successfully transfered this game into TI 83 Graphical Calculator. The author of this paper have also created a Macro droven program to realize this game in any MS-Office environments. These practices make this game more available for its further use in classroom.

To solve this problem, Margherita Barile [3] gives a method which need to solve a linear system of 25 variables. Someone on Wikipedia give another more clever method based on the two facts: 1. the order in which the lights are pressed does not matter. 2. each light needs to be pressed no more than once. From their analysis we can deduce an algorithm of searching i.e. search from the first raw of five lights.

The two methods mentioned above are closely related to Chinese high school math curriculum. It is rare that a game has such a strong mathematics background and can be understood by high school students. Thus, we would like to contribute this case to those collegues who feel difficult in searching good examples.

2. Mathematics Activities design

In Shanghai, China, matrics, linear equation and algorithm are taught in high school math classes.The following two designs can be used as math lesson plans or both math and computer lesson plans.

2.1 Using linear system as the model

Before the class, we need TI83 or HP39G+ graphic calculators supplied to every student. Each calculator is installed the Lights Out puzzle beforehand. The students will experience 7 procedures during the class.

Procedure 1.Raise the essential question: how to solve the puzzle “Lights Out”? Make the students form small team of exploring, each team contains 3 to 4 people.

Give the student team the task: try to find the answer of the Light Out game.

Procedure 2. Make each team brainstorm the question “what math knowledge can be applied into this problem?”

Procedure 3. If need, teacher can give some hints like “when will be a light on or off?” “does the game implied some linear system in it?” “what is the variables?” With all this help, students are expected to obtain following equations:

$$\begin{cases} x_1 + x_2 + x_6 = 1 \\ x_1 + x_2 + x_3 + x_7 = 1 \\ \cdots \\ x_{20} + x_{24} + x_{25} = 1 \end{cases} \tag{1}$$

As there are all together 25 variables, how do deal with such a huge linear system becomes a new problem.

Procedure 4.Tell the students that their graphic calculator (say HP 39G+) has a math command RREF (Reduced Row Echelon Form), which will work dealing with this . Now it is suggested that the students consult the reference book or use syntex key to learn how to use this command to solve the equation.

Still, some questions can be asked here to support the students’ work of exploring: 1. How to use RREF? 2. Can you try to use “helpwith” to learn? 3. where and how to input matrix ? 4. How to save the result into another matrix?

Now the equations can to converted into corresponding matrix expression:

$$\left(\begin{array}{ccccc|c} P & I & & & & 1 \\ I & P & I & & & 1 \\ & I & P & I & & \vdots \\ & & I & P & I & \vdots \\ & & & I & P & 1 \end{array}\right)_{25\times 26}, \quad \text{where “I” is the identity matrix of rank 5, and}$$

$$P = \begin{pmatrix} 1 & 1 & & & \\ 1 & 1 & 1 & & \\ & 1 & 1 & 1 & \\ & & 1 & 1 & 1 \\ & & & 1 & 1 \end{pmatrix}.$$

But how to input such a big matrix without errors is the problem.

Procedure 5. make the students test the command bellow. Have them think Why can this simple line of command yield the wanted matrix?

```
MAKEMAT(|I-J|<2 OR |I-J|==5 OR J==26,25,26)
```

figure 1 The Simple way of Yielding Matrix in Eqation (1)

(By typing this command will yield within a HP 39G a matrix of 25×26 which is nearly coinsides with the target matrix. Students only need to correct the 8 entries handly.)

Procedure 6. Students use RREF command to find the solution to the linear system. But students are bound to meet a strange result by execute the command RREF(M1). Most numbers in answer column are recurring decimals of period 2.

M2	23	24	25	26
1	0	0	1	.727273
2	0	1	0	.636364
3	0	-1	-1	-.45455
4	0	-1	0	0
5	0	2	1	1.36364

figure 2

Figure 2 shows the result metrics of the previous command. This time the discussion topic focus on "why we got a strange answer on column 26?" "Why it seems to have same periods?" "What if we multiply the origional constant column by the factor 99?"

Procedure 7. By now we have got an answer to the equations. But we have to explain why there are two zeros at the bottom of the reduced raw echlon form? Will this game have infinityly many solutions? If we eliminate those have the same remainder with respect to module 2, how many solutions left?

Figure 3 shows the 4 answers that the students will find:

$$(1\ \ 1\ \ 0\ \ 0\ \ 0) \quad (1\ \ 0\ \ 1\ \ 1\ \ 0) \quad (0\ \ 0\ \ 0\ \ 1\ \ 1) \quad (0\ \ 1\ \ 1\ \ 0\ \ 1)$$

figure 3

2.2 Using Searching Algorithm as the Model

This design is more suitable for programming class, though in math class we also teach algorithms.

TI83 or HP39G+ needed and installed Lights Outs puzzle. LED projector, PPT and EXCEL environments.

Five procedures are needed to do this exploration.

Procedure 1.Raise the essential question: How to solve the puzzle game Lights Out? Each student is supplied a HP 39G+ or TI 83 which has the Lights out game installed. They may work together with a team. Remember to think about how to use algorithm to solve it.

Students are expected to find certain searching algorithm as their method.

Procedure 2.Teacher explain how huge 2^{25} would be if the searching algorithm not be optimized. Then a discussion be held upon how to reduce the searching space and search the 25 lamps raw by raw recursively is found to be the best way. Teacher shows how the previous clicking be inherited to present raw and how present clicking manner be decided by last raw's final state.

Procedure 3. The top down order of searching is decided, and students realize that only the first raw of lights are free varialbes. That is to say, once the state of the first raw decided, following raw's clicking way is decided too. Some cleverer students may further reduce the searching process to just following the 16 initials in table 1:

table 1 Initial values of first raw of lights

00000	00001	00010	00011
00100	00101	00110	00111
01000	01001	01010	01011
01100	01101	01110	01111

Procedure 4. Flow chart of the searching algorithm is worked out (see appendix 2) and programming in GC can thus be done.

Some explainations about the flowchart may needed for the students to understand it.

1. L_1 is the clicking list on present line defined by counting on the above table (when j=1) or $1-L_3$. It's also the effect list to the next line.

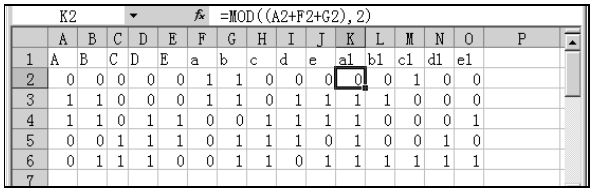
2. L_2 is the present line state list.

3. L_3 is the effect state list of the present line defined by

$$L_3 = L_1(i-1) + L_1(i) + L_1(i+1) + L_2(i), i = 1,2,\ldots,5.$$

(item with index out of range be treated as zero.)

4. The flow chart can be verified in Excel environment (see figure 4)with formula input and copy mathod to those student who feel difficult in programming.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	A	B	C	D	E	a	b	c	d	e	a1	b1	c1	d1	e1	
2	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	
3	1	1	0	0	0	1	1	0	1	1	1	1	0	0	0	
4	1	1	0	1	1	0	0	1	1	1	1	0	0	0	1	
5	0	0	1	1	1	0	1	1	1	0	1	0	0	1	0	
6	0	1	1	1	0	0	1	1	0	1	1	1	1	1	1	
7																

figure 4

Procedure 5. Coding on the GC and testify the results.

3. Conclusion

We have designed two unit plans based on the same game “Light out”. We showed that technology and mathematics can be combined together into an interesting activities natually. The duel roles of the mobile device of GC are to provide an enviroment of play and to support solving. The mathematics knowledge in this case is still textbooks related. But students must be more engaged in since they will surely be motivated by the fun of the game. Thus, students will deduce that math is usful even in playing games. Solving one problem in two different way teaches students good lesson on how to use creative thinking.

From this case we can see that choosing the suitable puzzle game is especially important. But once the game is chosen, analysis on it become essential. And finally the teacher need plan the teaching procedures so that the course is developed step by step. We call this whole scheme **C**hoose-**A**nalysis-**D**esign scheme.

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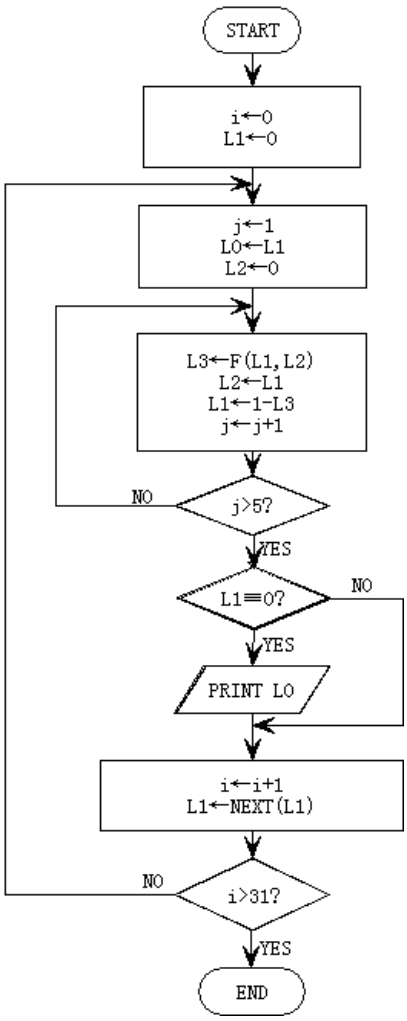
Appendix

Flow char of the searching algorithm.


START
i←0
L1←0
j←1
L0←L1
L2←0
L3←F(L1,L2)
L2←L1
L1←1-L3
j←j+1
NO
j>5?
YES
L1≡0?
NO
YES
PRINT L0
i←i+1
L1←NEXT(L1)
NO
i>31?
YES
END