Why College or University Students Hate Proofs in Mathematics?

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Abstract: *Empirical studies have shown that students emerge from proof-oriented courses such as high-school geometry, introduction to proof and complex variable are unable to construct anything beyond very trivial proofs. Furthermore, most university students do not know what constitutes a proof and cannot determine whether a purported proof is valid. To motivate students hating proofs and to help mathematics teachers, how a proof can be taught, we investigated in this study the idea of mathematical proofs. To tackle this issue, the modified Moore method and the author method called Z. Mbaïtiga method are introduced follow by two cases of studies on proof of triple integral. Next a survey is conducted on fourth year college students on which of the proposed two cases of studies they understand easily. The result of the survey showed that more than 95% of the students who responded pointed out the proof that is done using details explanations of every theorem used in the proof construction, the case study 2. From the result of this survey, we learned that mathematics teachers have to be very careful about the selection of proofs to include when introducing topics; and filtering out some details which can obscure important ideas and discourage students.*

1. Introduction

When making a comparison between mathematics and others subjects, we can say with certainty that in mathematics things are proved; while in other subjects they are not. This statement needs certain qualifications, but it does express the difference between mathematics and other sciences. In most fields of study knowledge is acquired from observations, by reasoning about the results of observations and by studying the observations, methods and theories of others. Mathematics was once like this too. Ancient Egyptian and Babylonian mathematics consisted of rules for measuring land, computing taxes, predicting eclipses, solving equations. Methods were learnt from the observations and handed down to others. Modern school mathematics is still often practiced in this way. But there were changes in the approach to mathematics. The Ancient Greeks have found that in arithmetic and geometry it is possible to prove that results were true. They found that some truths in mathematics were obvious and that many of the others could be shown to follow logically from obvious ones. Pythagoras' theorem equation (1.1) on right-angle triangle shown in Figure 1.1 for example is not obvious. But a way was found of deducing it from geometrical facts that were apparent. For example: let A and B of Figure1.1a be 5 and 12 in Figure1.1b, find the value of C then prove that equation (1.1) is true.

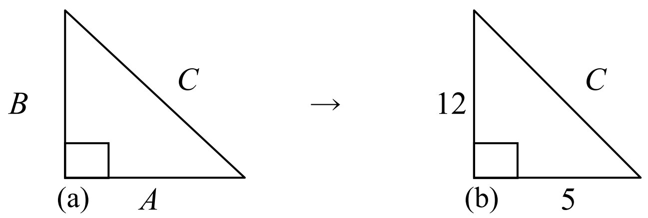

Figure 1.1 Right triangle with legs A and B

Theorem: If a triangle has sides of length *ABC*, with enclosing an angle of 90 °, then:

$$A^2 + B^2 = C^2 \tag{1.1}$$

Finding the value of *C*: Organization of information:

$$\left.\begin{array}{l} A = 5\ ;\ \ A^2 = A \times A = 25 \\ B = 12\ ;\ B^2 = B \times B = 144 \\ A^2 + B^2 = 25 + 144 = 169 \end{array}\right\} \tag{1.2}$$

Replacing the value of $A^2 + B^2$ of equation (1.2) into equation (1.1) and deduces the value of *C*:

$$\begin{array}{l} A^2 + B^2 = C^2 \\ \underbrace{169} = C^2 \end{array} \tag{1.3}$$

$$\sqrt{169} = c \Rightarrow c = 13$$

- Proof of equation(1.1):

$$\begin{array}{l} A^2 + B^2 = C^2 \\ \underbrace{169} = (13)^2 \end{array} \tag{1.4}$$

$$169 = 169$$

- Conclusion: Equation (1.1) is true.

But why *A* is equal to 5 and *B* equal to 12? instead of 3 and 6? If *A* is equal to 3 and *B* equal to 6 really equation (1.1) can be proved? The idea behind these questions is that, mathematics is not about answers, it is about processes to understand why a result is true, hence the importance of proof. At first it was hoped that every subject would become like mathematics, with all the truths following obvious true basic statements. This did not happen, Physics, Biology, Economics and other Sciences discover general truths, but to do so they rely on observations. The theory of relativity is not proved true; it is tested against observations. As a result, mathematic has always been regarded as having a different kind of certainty that obtainable in other sciences. If a scientific theory is accepted because observations have agreed with it, there is always in principle a small doubt that a new observation will not agree with the theory, even if all previous observations have agreed with that theory. If a result is proved correctly, that cannot happen. For more than two thousand years mathematics has attracted those who valued certainty and has served as the supreme example of certain knowledge. For students, what is really difficult in mathematic proof is the concept of proof. The difficulty manifests itself in three principal points:

- Appreciating why proofs are important.
- The relation between verification and understanding.
- The proof construction.

The first point describes a spurious or convincing proof. The second point illustrates two important ingredients to develop convincing proof .The third point describes the usage of the theorems.

2. What does proof mean and its role in mathematics?

To this question many mathematics teachers would consider the answers straightforward: A mathematical proof is a formal and logical line of reasoning that begins with a set of axioms and moves through logical steps to conclusion. And the purpose of proving a theorem is to establish its mathematical certainty. A proof confirms truth for a mathematician the way experiment or observations does for the natural scientist (see [1]). Such views are commonly held by mathematics professors and are passed along to students. However, many mathematics professors believe that proofs are much more than this. Davis and Hersh (see [2]) argue that it is probably impossible to define precisely what types of argument will be accepted as a valid proof by the mathematical community. There are some aspects of proof that distinguish it from other types of arguments. As an example, proofs about a concept must use the concept's definition and must proceed deductively, as opposed to examining prototypical a cases or giving an intuitive arguments. And if a result is incorporated in a proof that result must accepted by mathematical community (see [3])

Beyond this, some mathematics professors argue that whether or not an argument is accepted as a proof depends not only on its logical structure but also on how convincing the argument is. At different places in the mathematics educators, a proof has been defined as an argument that convinces an enemy (see [4]), an argument that convince mathematician that knows well the subjects . Other who focuses on the social and contextual nature of proof, offer the following relativist description: We call proof an explanation accepted by a given community at a given time (see [5]). An argument becomes a proof after the social act accepts it as a proof. Many mathematics professors believe that focusing exclusively on the logical nature of proof can be harmful to students' development. But such a narrow view leads students to focus on logical manipulations rather than forming and understanding convincing explanation why a statement is true (see [6]). Here are some alternative purposes of proof proposed by mathematics educators.

- **Explanation**: By examining a proof, a reader can understand why a certain statement is true. Many mathematics educators argue that explanation should be the primary purpose of proof in mathematics classroom (see [7]).
- **Communication**: The language of proof can be used to communicate and debate ideas with other students and mathematicians [8].
- **Justification of a definition**: One can show that a definition is adequate to capture the intuitive essence of a concept by providing that all of the concept's essential properties can be derived from the proposed definition.
- **Discovery of news results**: By exploring the logical consequences of definitions and an axiomatic system, new theories can be developed.
- **Developing intuition**: By examining the logical entailments of a concept's definition, one can sometimes develop a conceptual and intuitive understanding of the concept that one is studying.

3. Why students hate proofs?

There is considerable evidence that students leave school with negative attitudes towards mathematics. Some dislike the subject, others feel inadequate about it, and still others feel it is irrelevant in their lives. Students entering college or university are often very adept at performing algorithms and finding their way through the maze sophisticated calculations or some geometry problems based on calculations. However, they tend to have very little experience with mathematical proofs even though these are central to verifying mathematical facts and buildings corpus of reliable knowledge. It is common for students to say that they like mathematics but hate proofs. For many students proof technique is difficult to overcome and has all of the hallmarks of a threshold concept. The ability to understand and construct proofs is transformative, both in perceiving old ideas and making new and exciting mathematics discoveries. In many cases it appears that negative attitudes toward proofs result from certain teaching practices, the nature of the subject, and the selection of proof problems and inability of professor to explain conceptually difficult concepts in simple terms. When introducing a proof, some professor assumed that students already know, or familiar with the theorems that will be used in the proof construction. Others instead of explaining to students the reason of moving from step A to step B, they content to use the following words:

- Based on the theorem of (Pythagoras for example)
- Using the definition of
- By inserting α into β we have
- After developing ∅ we deduce ∂

These words: based on, using the, by inserting, after and deduce are very confusing for students. As an example before writing this article, deliberately I have used the word using the definition of (X) when proving that 0! = 1! to my students during the mathematics lesson. Surprisingly one of my best students asked me to state the definition again. I responded are you joking? We have learned this definition just two days ago. I am sorry; sir if I am asking you to state it means that I get lost. Get lost mean? I asked him again, I forgot this definition, he replied. This example shows that we cannot tell about students' ability of memorization. Even if teacher is sure students know the theorem or definition that will be used for proof, some students may not remember. So, it is better to always as reminder states the theorems or formulas again so that they can caught what you are presenting or proving. The following comments were made by university students in the Department of Pure Mathematics studying to be high school mathematic teachers. They were asked to reflect on their own experiences of learning proofs in mathematics. They indicate that teacher has a large impact on attitudes toward proofs.

3.1 The Teacher

- The teacher went too fast and did not know how to explain difficult concept to simple terms.
- I had a bad teacher who passed on dislike proofs.
- The teacher did not give a reason that each proof steps is correct.
- Most mathematics lessons were boring and make me sleep.
- The teacher did not state all theorems involving in the proofs.
- The teacher did not convince me about the necessity of the proofs.

The ways teachers teach proofs in mathematics makes difference.

4. How proof should be taught?

The following two methods are some examples of how to teach proofs. The first method is the modified Moore method. The modified Moore is a teaching paradigm that is based on the pedagogical techniques of the mathematician Robert Lee Moore at the University of Texas (see [9]). The second method is the author method called " Z. Mbaïtiga method"

4.1 Modified Moore method

Moore and proponents of this method believe that students will learn little about advanced mathematics by passively writing down the proofs that the professor or instructor presents on the blackboard, and will learn far more about mathematical concepts and proofs if they try to construct the proofs by themselves. Here is a brief description of this influential teaching method. In a typical class using the Moore method, the professor or instructor presents the students with the definitions of mathematical concepts and may be a few motivating examples of those concepts. After this, students are asked to prove or disprove a set of propositions about these concepts. When a student believes that he or she has proved a proposition, that student is invited to present his or her argument on the blackboard. The teacher and the fellow students may critique the student's work or ask the student to clarify his or her argument. If everyone including the professor is convinced by the proof, the class moves to another proposition. If no student is successfully able to prove a theorem, the teacher may ask the students to prove a simpler proposition, put the proposition off to another day, or simply let the proposition go unproved. The teacher may also provide assistance to the students, but the assistance should be minimal amount necessarily for the students to construct the proof. What is critical is that the teacher never provides the students with the actual proof of a proposition. All proofs are generated by the students by themselves.

4.2 Z. Mbaïtiga method

In this method, once the professor presents the problem to be proved on the blackboard asks students to suggest or propose the theorem that can be used to solve the problem and explain how it should be used. After the proposition of the theorem for solving the problem is done and even if the professor knows that the proposed theorem is false, without saying anything uses it and solves the problem, then asks the fellow students their opinion about the proof result. Many arguments will be given by the students and among these arguments the teacher should pick up two propositions: the right proposition and one similar to the one that was proposed if possible. Write them on the other side of the blackboard and asks students again to choose the right formula or theorem. When the theorem or formula is selected, the teacher uses it and solves the problem without erasing the first false result, then asks the class again if they are convinced or not. If everyone is convinced, then the professor compares the two results and explains why the first result is false. But if no student can pointed out what is wrong with the result, the professor assist the students by proposing the theorem or formula to be used and another similar to the right proposition then put the problem to home work for the next day. During the next mathematic class as a Problem-Based Learning the teacher lets students solve the problem by themselves on the blackboard, and can provide only assistance. During the proof class, the teacher should focus only on the proof instead of thinking about moving to the next lesson. Because proof is a scientific language of communication and is a very important tool that can help student to defend themselves when facing a tough problem in other subjects. The teacher should never leave the proof unproved once presented to the students.

In both Robert Lee and Z. Mbaïtiga methods all the efforts are done by the students themselves with professor assistance only. But the difference between the two methods is that in Moore method, if no student is able to prove the theorem the teacher can simply let the proposition go unproved, while in Z. Mbaitiga method the proposition should never be let unproved once presented to the students.

5. Case study on proof of triple integral

So, which kinds of proof method are the most appropriate for a lecture or class presentation? The short answer is those which lead to deep explanation of the formulas that the teacher uses for proof construction, or those which lead as quickly as possible to deep conceptual understanding. Here an example is given from author's recent teaching of triple integral to fourth year college students who have only a high school background. It uses two cases of studies. After each case study students were asked about the case study that is easy for them to understand.

Problem to be proved: By using the spherical coordinates prove that,

$$\iiint_R (x^2 + y^2 + z^2)\ \mathrm{dxdydz} \ = \frac{56\pi}{15} \tag{2.1}$$

5.1 Case study 1:

In this case study, the author assumed that students already have learned or understand well how to find the spherical coordinates and can easily manipulate them to solve equation (2.1). The author also assumed that students have no problem at all on trigonometric formulas conversion.

5.1.1 Proposition 1

Let R be a space area of x, y, z and R′ the set of $(\rho, \theta, \varphi) \in [0, \infty) \times [0, 2\pi] \times [0, 2\pi]$ such as $(\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \sin\varphi) \in R$. The area corresponding to the space *ρ, θ, φ* shown in Figure 5.1 has a real function *f: R→R* such as the triple integral,

$$\iiint_R (x^2 + y^2 + z^2)\ \mathrm{dxdydz} \quad \text{exist so that}$$

$$\iiint_{R'} f(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\theta)\rho^2 \sin\varphi\, \mathrm{d}\rho\, \mathrm{d}\theta\, \mathrm{d}\varphi \tag{2.2}$$

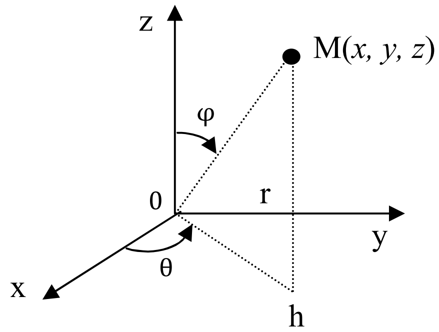

Figure 5.1 Spherical coordinates without details information.

5.1.2 Proposition 2

The area R′corresponding to R in spherical coordinates is:

$$R' = \begin{cases} \{(\rho, \theta, \varphi) \mid 0 \le \ \rho \le 1/\sin\varphi, \\ \qquad 0 \le \ \theta \le 2\pi, \\ \qquad \varphi_{\min} \le \varphi \le \pi - \varphi_{\min} \ \} \end{cases} \tag{2.3}$$

It is easy to verify that:

$$\begin{cases} \varphi_{\min} = 1/2 \\ \sin\varphi_{\min} = 1/\sqrt{5} \\ \cos\varphi_{\min} = 1/\sqrt{5} \end{cases}, \quad \begin{cases} \sin(\pi - \varphi_{\min}) = 1/\sqrt{5} \\ \cos(\pi - \varphi_{\min}) = -2/\sqrt{5} \end{cases}$$

We used the fact that $\rho \sin\varphi = 1$ for the point on the vertical cylindrical edge of our area.

5.1.3 Proof 1

From proposition 1 and proposition 2, equation (2.1) becomes:

$$\iiint_R (x^2+y^2+z^2)\,\mathrm{dxdydz} = \iiint_R \rho^2\cdot\rho^2\sin\varphi\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\varphi = \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}}\left(\int_0^{1/\sin\varphi}\rho^4\sin\varphi\,\mathrm{d}\rho\right)\mathrm{d}\varphi\right)\mathrm{d}\theta$$

$$= \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}}\left(\frac{\rho^5}{5}\right]_{\rho=0}^{\rho=1/\sin\varphi}\sin\varphi\,\mathrm{d}\varphi\right)\mathrm{d}\theta = \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}}\frac{1}{5\sin^4\varphi}d\varphi\right)\mathrm{d}\theta = \frac{28}{15}\int_0^{2\pi}\mathrm{d}\theta = \frac{28}{15}(\,\theta\,]_{\theta=0}^{\theta=2\pi} = \frac{56\pi}{15} \quad \blacksquare \tag{2.4}$$

5.2 Case study 2

In this case study the author assumed that, students have learned the spherical coordinates but did not understand how to use them. Also they have some difficulties on trigonometric formulas conversions and have limited or no experience with proof construction. Therefore more details are required.

5.2.1 Proposition 3

Spherical coordinates consist of the following three quantities:

Radius: $\rho = \overrightarrow{OM}_p$

Azimuth: $\theta = (\overrightarrow{U}_x, \overrightarrow{OH})$

Colatitude: $\varphi = (\overrightarrow{U}_x, \overrightarrow{U}_p)$ = 90°- δ (δ = latitude)

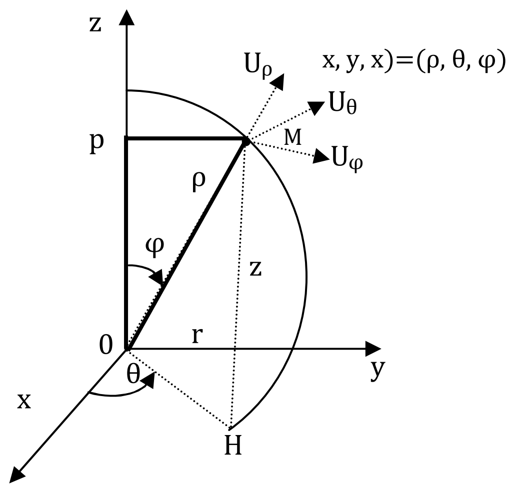

Figure 5.2 Spherical coordinates with details information.

ρ = Distance from the origin to the point M with $\rho \geq 0$.

θ = The same angle we see in polar-cylindrical coordinates. It is the angle between the positive x-axis and the line above denoted by r which is also the same r as in polar-cylindrical coordinates shown in Figure 5.2. There is no restriction on θ. That is, $0 \leq \theta \leq 2\pi$.

φ = Angle between the positive z-axis and the line from the origin to the point M with $0 \leq \varphi \leq \pi$.

In summary, ρ is the distance from the origin to the point M, φ is the angle that we need to rotate down from the positive z-axis to get the point M and θ is how much we need to rotate around the z-axis to get to the point M. Now we should first derive some conversion formulas.

Let's first start with a point in spherical coordinates and ask what the cylindrical coordinates of the point are. So, we know ρ, θ, φ and what to find r, θ, z and of course we really only need to find r and z since θ is the same angle in both coordinates systems. We will be able to do all of our work by looking at the right angle shown in Figure 5.2.With little geometry using the triangle represented by *OPM* we see that the angle between z and ρ is φ and we can see that:

$$z = \rho \cos\varphi \qquad (2.5)$$
$$r = \rho \sin \varphi$$

And there are exactly the formulas we were looking for. So given a point in spherical coordinates the cylindrical coordinates of the point will be:

$$r = \rho \sin \varphi$$
$$\Theta = \theta \qquad (2.6)$$
$$z = \rho \cos\varphi$$

Next, let's find the Cartesian coordinates of the same point. To do this we will start with the cylindrical conversion formulas, Figure5.3. The conversions for x and y are the same conversions that we used back in when we were looking at polar coordinates. So if we have a point in cylindrical coordinates the Cartesian coordinates can be found by using the following conversions:

$$x = r\cos\theta$$
$$y = r\sin\theta \qquad (2.7)$$
$$z = z$$

The third equation is just an acknowledgement that the z-coordinate of a point in Cartesian and polar coordinates is the same. Now all that we need to do is to use equation (2.6) for r and z to get

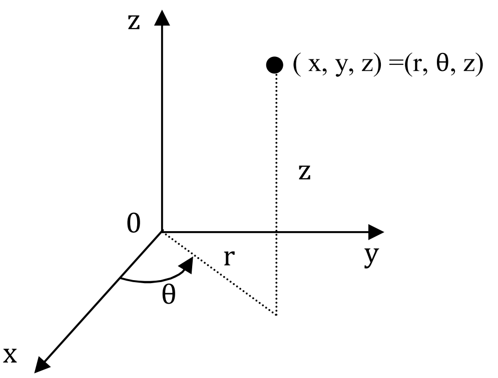

Figure 5.3 Cylindrical coordinates

$$\begin{cases} x = \rho \sin\varphi \cos\theta \\ y = \rho \sin\varphi \sin\theta \\ z = \rho \cos\theta \end{cases} \qquad (2.8)$$

also note that since we know that: $r^2 = x^2 + y^2$.

We get: $\rho^2 = x^2 + y^2 + z^2$ and $dx\ dy\ dz = \rho^2 \sin\varphi\ d\rho\ d\theta\ d\varphi$

5.2.2 Proposition 4

Area R′ corresponding to R in spherical coordinates is:

$$R' = \begin{cases} \{(\rho, \theta, \varphi) \mid 0 \le \rho \le 1/\sin\varphi, \\ \qquad 0 \le \theta \le 2\pi, \\ \qquad \varphi_{\min} \le \varphi \le \pi - \varphi_{\min}\ \} \end{cases} \tag{2.9}$$

The author has used the trigonometric circle to explain how the values of equation (2.10) have been obtained to students by considering the fact that $\rho\ \sin\varphi = 1$ for the point on the vertical cylindrical edge of our area. But for the sake of brevity the author refrains from giving the details in the article.

$$\begin{cases} \varphi_{\min} = 1/2 \\ \sin\varphi_{\min} = 1/\sqrt{5} \\ \cos\varphi_{\min} = 1/\sqrt{5} \end{cases}, \quad \begin{cases} \sin(\pi - \varphi_{\min}) = 1/\sqrt{5} \\ \cos(\pi - \varphi_{\min}) = -2/\sqrt{5} \end{cases} \tag{2.10}$$

5.2.3 Proof 2:

From proposition 3 and 4 equation (2.1) becomes:

$$\begin{aligned} &\iiint_R (x^2+y^2+z^2)\, dxdydz = \iiint_R \rho^2 \cdot \rho^2 \sin\varphi d\rho\, d\theta\, d\varphi = \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}}\left(\int_0^{1/\sin\varphi} \rho^4 \sin\varphi\, d\rho\right) d\varphi\right) d\theta \\ &= \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}}\left(\frac{\rho^5}{5}\right]_{\rho=0}^{\rho=1/\sin\varphi} \sin\varphi\, d\varphi\right) d\theta = \int_0^{2\pi}\left(\int_{\varphi_{\min}}^{\pi-\varphi_{\min}} \frac{1}{5\sin^4\varphi}\, d\varphi\right) d\theta \end{aligned} \tag{2.11}$$

Now let's find the primitive of: $\int \frac{1}{5\sin^4\varphi} d\varphi$. We know that: $\text{cotan}\varphi = \frac{\cos\varphi}{\sin\varphi}$ and its derivation is:

$$\frac{d}{d\varphi}\text{cotan}\varphi = \frac{d}{d\varphi}\left(\frac{\cos\varphi}{\sin\varphi}\right) = \frac{-1}{\sin^2\varphi} \tag{2.12}$$

Then:

$$\begin{aligned} &\int \frac{1}{\sin^2\varphi} d\varphi \int \frac{1}{\sin^2\varphi} \frac{d}{d\varphi}\left(\frac{-\cos\varphi}{\sin\varphi}\right) d\varphi = \frac{1}{\sin^2\varphi}\left(\frac{-\cos\varphi}{\sin\varphi}\right) - \int \left(\frac{-\cos\varphi}{\sin\varphi}\right) - \frac{(-2)\cos\varphi}{\sin^3\varphi} d\varphi \\ &= \left(\frac{-\cos\varphi}{\sin^3\varphi}\right) - 2\int \frac{\cos^2\varphi}{\sin^4\varphi} d\varphi \end{aligned} \tag{2.13}$$

Replacing $cos^2\varphi$ by $1\text{-}sin^2\varphi$ into the second term of equation (2.13), since we know that $cos^2\varphi + sin^2\varphi = 1$, we have:

$$= \left(\frac{-\cos\varphi}{\sin^3\varphi}\right) - 2\int \frac{(1-\sin^2\varphi)}{\sin^4\varphi} d\varphi = \left(\frac{-\cos\varphi}{\sin^3\varphi}\right) - 2\int \frac{1}{\sin^4\varphi} d\varphi + 2\int \frac{\sin^2\varphi}{\sin^4\varphi} d\varphi = \left(\frac{-\cos\varphi}{\sin^3\varphi}\right) - 2\int \frac{1}{\sin^4\varphi} d\varphi + 2\int \frac{1}{\sin^2\varphi} d\varphi \tag{2.14}$$

Hence:

$$3\int \frac{1}{\sin^4\varphi} d\varphi = \frac{-\cos\varphi}{\sin^3\varphi} + 2\int \frac{1}{\sin^4\varphi} d\varphi \Rightarrow \int \frac{1}{\sin^4\varphi} d\varphi = \frac{-\cos\varphi}{3\sin^3\varphi} - 2\int \frac{\cos\varphi}{3\sin\varphi} + c \tag{2.15}$$

Using equation (2.15) our equation(2.11) becomes:

$$\frac{1}{5}\int_0^{2\pi}\left[\frac{-\cos\varphi}{3\sin^3\varphi}-2\frac{\cos\varphi}{3\sin\varphi}\right]_{\varphi=\varphi_{\min}}^{\varphi=\pi-\varphi_{\min}}\mathrm{d}\theta=\frac{1}{5}\int_0^{2\pi}\left(\frac{-1(-2/\sqrt{5})}{3(1/\sqrt{5})^3}-\frac{-2(-2/\sqrt{5})}{3(1/\sqrt{5})}\right)-\left(\frac{-1(2/\sqrt{5})}{3(1/\sqrt{5})^3}-\frac{2(2/\sqrt{5})}{3(1/\sqrt{5})}\right)\mathrm{d}\theta$$

$$=\frac{28}{15}\int_0^{2\pi}\mathrm{d}\theta=\frac{28}{15}\left[\theta\right]_{\theta=0}^{\theta=2\pi}=\frac{56\pi}{15}$$ Hence the proof of equation (2.1) is completed. ■

6. Results and Discussion

Very few teachers would cite assurance of truth as the sole reason for teaching proof in the classroom, of course. Proof is also a method of communicating results to others in a clear and fairly conventional form. This purpose relatively straightforward; a good proof show in details the problem to be proved follows from other already-known facts by a chain of good reasoning. The way teacher teaches proofs makes difference and can have an impact on students. Let's see the results of the survey conducted on four year college students shown from Figure 6.1 to Figure 6.3 on case study 1 and 2. The class was divided in to 3 groups as follows: High level students which consist of 12 students, average level students which consist of 20 students and low level students which consist of 13 students respectively. Each group was asked on which of the four propositions and two proofs they have no problem, some problems understanding it or simply cannot understand.

6.1 High level students: Case study 1, 2:

In Figure 6.1a on 12 students who responded. 4, 10 and 4 students said that they have no problem at all understanding the proposition 1, 2 and proof 1. 6, 2 and 7 have some problems, while in Figure 6.1b all 12 students who responded said with interesting comments that they have no problem at all understating the proposition 3,4 and proof 2.

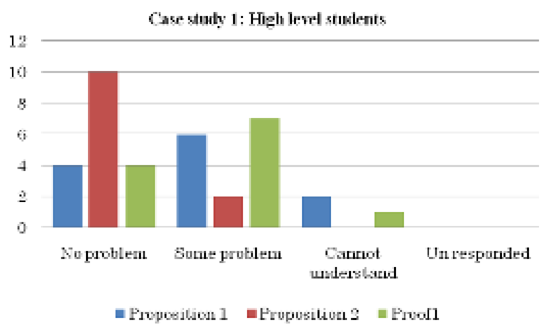

Figure 6.1a: Survey result of case study 1.

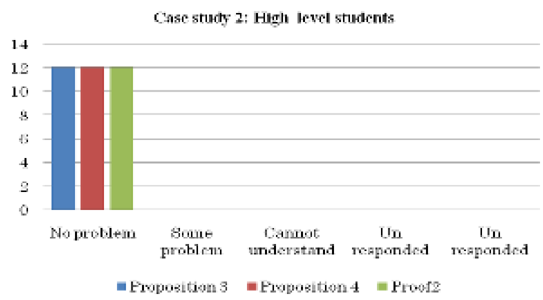

Figure 6.1b: Survey result of case study 2.

6.2 Average level students: Case study 1, 2

In Figure 6.2, on 19 students who responded, Figure 6a: 2, 4 and 1 students said that they have no problem at all understanding the proposition 1, 2 and proof 1.10, 8 and 2 have some problems, 7, 8 and 17 cannot simply understand. There was one student who did not respond for proposition1. While in Figure 6.2b, 18, 19 and 18 have no problem for proposition 3, 4 and proof 2 and only 2, 1 and 2 have some problems.

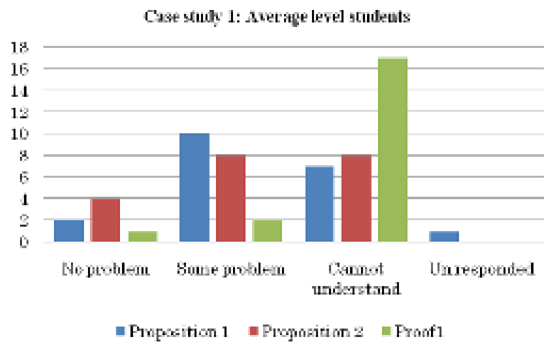

Figure 6.2a: Survey result of case study 1.

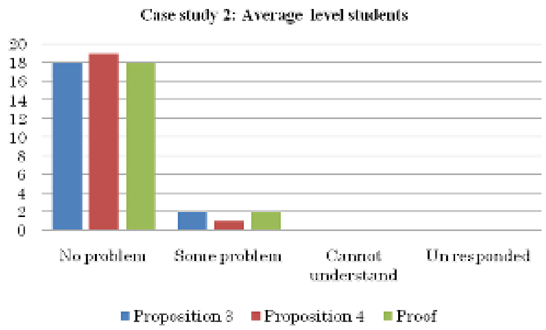

Figure 6.2b: Survey result of case study 2.

6.3 Low level students: Case study 1, 2

Regarding the low level results shown in Figure 6.3a and Figure 6.3b, on 13 students who responded no student have been found to have understood the proposition1, 2 and proof 1. 1, 4 and 1 have some problems and 12, 10 and 11 cannot simply understand; as there was one student who did not responded for proof 1. While in Figure 6.3b the result shows that 9, 11 and 10 have no problem understanding the proposition 3, 4 and proof 2 and only 3, 2 and 3 have some problems. The result of this survey shows that most students have no problem understanding the proof as well as theorems involving in the proof construction with more details.

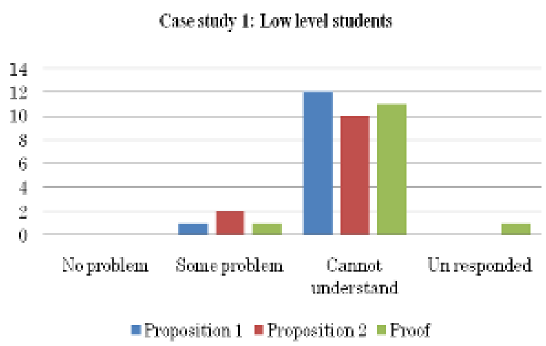

Fig. 6.3a: Survey result of case study 1.

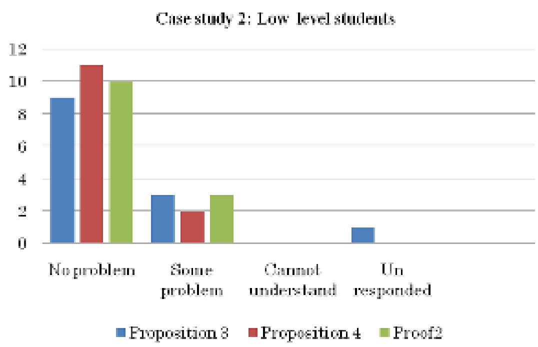

Fig. 6.3b: Survey result of case study 2.

7. Conclusion

Mathematics education in college and university aims at providing a certain level of understanding of mathematic and mathematical methods, but most of students will not continue their studies of mathematics, but they will have to apply their knowledge of mathematics in such fields as sciences and business and so no. Therefore mathematics teachers have to be very careful about the selection of proofs to include when introducing topics and filtering out certain which can obscure important ideas. Indeed the word proof is often equated with obfuscation. A poorly presented proof even if meticulously prepared, can be frustrating and wasteful in terms of time and effort in concentration and it is common for students to get lost. In many cases it appears that negative attitudes toward proofs result from certain teaching practices, the nature of the subject and the selection of proof problems and inability of teachers to explain conceptually difficult concepts in simple terms. The objective of this study is not to take mathematics professors or educators responsible of the dislike of proofs by students, but instead to let them know that students rely on them so they have to help them to erase the mystery behind the proof.

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