The Preparation of Secondary Pre- and Inservice Mathematics Teachers on the Integration of Technology in Topics Foundational to Calculus

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Abstract. *The graphing calculator has been on the market for 21 years. It is a fair question to ask whether or not we are taking full advantage of the main capabilities that hand-held graphing technology (HHGT) offers, without Computer Algebra Systems (CAS), to provide precalculus students with the best possible preparation for calculus? To answer this question in the USA, we decided first to establish criteria on how the integration of technology expands the study of families of continuous functions at this particular level. Then, we proceeded to explore the knowledge on the integration of HHGT that secondary pre- and inservice teachers have. A test based on the established criteria was administered to three intact groups consisting of 46 preservice secondary teachers from three universities from the Midwest in the USA. The same test was also given to a group of 74 secondary inservice teachers representing 40 school districts from the same geographical area. The test results were very low. In addition, the preservice and inservice teachers, immediately before taking the test, were asked to answer a survey where they rated their knowledge on the established criteria. The self-evaluation of both groups on the chosen topics ranges from "very little" to "some" knowledge, corroborating their self-awareness on their lack of preparation on these topics.*

1. Introduction

The integration of technology into the teaching and learning of mathematics impacts every aspect of instruction, from course content to teaching methods and assessment. As a result, some of the assumptions made about mathematics curricula prior to the integration of technology in the classroom are no longer valid. Subsequently, the ability to bridge cumbersome calculations via technology allows students at various levels to meaningfully explore concepts and problems only previously proposed to more advanced mathematics students; and to extend the breadth and depth in teaching of these concepts. Therefore topics such as optimization, regression, as well as recursion and other techniques are now accessible to secondary students at different levels prior to calculus instruction. More importantly, the numerical and graphical capabilities of hand-held graphing technology (HHGT) can be used to introduce key concepts foundational to calculus at the secondary level by using different representations, resembling a way that is similar to how these concepts were developed and can be better understood, namely by using approximations.

The National Council of Teachers of Mathematics (NCTM) (1989) *Curriculum and Evaluation Standards for School Mathematics* recommends that:

In grades 9-12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can -.

- Determine maximum and minimum points of a graph and interpret the results in problem situations;
- Investigate limiting processes by examining infinite sequences and series and areas under curves;
- and so that, in addition, college-intending students can-
- Understand the conceptual foundations of limit; the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
- Analyze the graphs of polynomial, rational, radical, and transcendental functions. (p. 180).

These concepts can be developed as natural extensions of topics that students have already encountered (Jockusch & McLoughlin, 1990). Orton (1985) argues that the crucial issue is not when calculus should be taught but how teachers should promote the understanding of calculus and precalculus according to the level of attainment by the pupil. Stroup (2005) comments,

Traditionally, we think of calculus as a culminating course in a secondary mathematics curriculum. It seems backward then, to suggest that calculus can help our younger students make better sense of topics we typically label "the basics". If anything, we assume in our curricula and in our teaching that calculus is a subject to be studied well after the basics are mastered and only after a long series of prerequisite coursework has been taken. As a result, most of our students do not progress as far as calculus; this limits them in terms of their opportunities in post-secondary education. It also limits them in terms of the formal mathematical tools they can bring to situations where rate varies. The world in which we live is dynamic and changing, and all our students should develop powerful ways of talking about change. (p.180)

The NCTM (2000) technology principle for school mathematics states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). They elaborate that "The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it" (NCTM, p.25). Hughes-Hallett (1994) points out that what has become inescapable is the impact computers and calculators are having on what we teach: computers and calculators can now easily compute definite integrals, sketch graphs, solve equations, and find high powers of matrices and are able to do the algebraic manipulations that have been the backbone of high school mathematics for decades. While Ostebee and Zorn (1997), state that whether one views calculus as an introduction to pure mathematics or as a foundation for applications (or both), the conclusion is the same: concepts, not techniques, are truly fundamental to the course.

Given then the importance that calculus and, indeed, concepts foundational to calculus have, and the role that technology may play in enhancing and expanding these concepts, we believe that after 21 years of using hand-held graphing technology (HHGT), it is appropriate to ask: *Are we taking full advantage of the main capabilities that this technology offers in order to provide precalculus students with the best possible preparation for calculus?*

To answer this question in the USA we first decided to establish criteria postulating how the integration of technology might expand the study of families of continuous functions at this level. Then, we developed a test containing mostly conceptual questions for evaluating the comprehension of these criteria.

We remark that the term precalculus is used throughout the paper to indicate not a particular course, but rather courses before calculus. As Jockusch and McLoughlin (1990) reminded us "fundamental concepts of calculus can be taught as early as middle school. By middle school, students are ready for concrete experiences with the concept of slope. They can take first steps in exploring the concept of rate of change" (p. 532).

In addition we are not making any reference to HHGT with CAS because, in our view, we should agree on the proper use of HHGT without CAS before we address the new set of curricular and pedagogical questions that symbolic calculators would bring to the schools.

2. How the Integration of Technology Impacts and Expands the Study of Functions in Precalculus

Approaches where, in our view, integrating technology can facilitate the study of families of continuous functions at the precalculus level are:

1. increase the emphasis on conceptual understanding and exploration,
2. introduce relevant concepts and applications now accessible at this level,
3. use the continuous interplay of the graphical, numerical, and analytical representations with every family of continuous functions,
4. use transformations to provide a uniform approach to the study of each family of continuous functions,
5. use the new data types available to provide new approaches to problem solving.

Moreover, major possible changes that HHGT brings to the traditional coverage of functions at the precalculus level are:

1. The learning of basic transformations, such as $f(x)+a, f(x+a), -f(x), a \cdot f(x), f(ax), |f(x)|$, and $f(|x|)$ facilitates the study of families of functions, each with a root or parent function.
2. The simultaneous introduction of the analytical, graphical, and numerical aspects of functions makes it possible to use Demana and Waits (1998) approach, namely, students should learn to: i) support solutions of equations and inequalities graphically and/or numerically analytical, and ii) support graphical and numerical solutions analytically, whenever possible.

In addition to the properties traditionally considered for every family of continuous functions, the integration of HHGT allows one to find irrational zeros as well as local extrema; properties that are not always available analytically at this level. Hence, it is feasible to extend the study of continuous functions at the precalculus level by adding the following topics:

a. finding the range of all continuous functions studied,
b. determining irrational zeros, hence all the real zeros,
c. finding local extrema, hence intervals where the function is increasing or decreasing.

Moreover, HHGT (with or without CAS) provides data types such as tables, lists, sequences, matrices, as well as regression, recursive functions, and even the ability of using recursion from the home screen (Quesada A., 1999). Finally, the latest models of HHGT have incorporated two new environments, a spreadsheet and dynamic geometry software, with the capability of recognizing any user-defined variable in any subsequent environment. These resources further allow expanding the traditional approach to the study of families of continuous functions by including:

a. using sequences to explore the local and end behavior,
b. comparing relative growth of functions from the same or from different families,
c. real world connections by considering relevant examples of data that can be modeled via regression by the family of functions studied,
d. optimization problems.

Since technology enables students to revisit problems from different perspectives based upon the depth of their mathematical knowledge, it is possible to use a spiral approach to some of these concepts, like optimization, through different courses preceding calculus (Quesada and Edwards, 2005). The interested reader is encouraged to look at a selection of examples addressing each of these proposed changes in Quesada (2007).

3. Method

To get a sense of how well prepared secondary in- and preservice mathematics teachers are on the integration of technology in topics foundational to calculus, we developed a test based on the previous list of major possible changes that HHGT facilitates, in the spring semester of 2008..A sample of the test questions is included in table 1. The test questions were grouped according to the following learning outcomes:

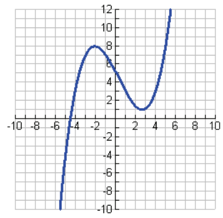
- LO1. Functions attributes: Complete graph, local (at a point) and end (at $\pm\infty$) behavior, one-to-one, inverse behavior, continuity, intercepts, domain, range
- LO2. Solving Equations: From graphs, analytical expressions, etc.
- LO3. Solving Inequalities: From graphs, analytical expressions, etc.
- LO4. Setting up Applications
- LO5. Transformations
- LO6. Modeling Data Via Regression
- LO7. Relative Growth
- LO8. Connections between Representations: Graph to equation or inequality, equation or inequality to graph, abstract to concrete information, etc.
- LO9. Any test problems involving polynomial functions only.
- LO10. Any test problems involving rational functions only.
- LO11. Any test problems involving exponential and logarithmic functions only.
- LO12. Any test problems involving trigonometric functions only.
- LO13. Any test problems involving functions other than those previously listed. These might involve radicals, absolute values, etc.

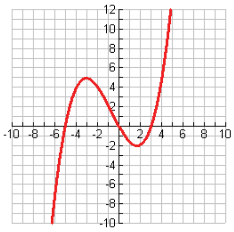
Many of these categories overlap. For example, a question that asked about the shape of a graph of a polynomial function would fall under both the attributes and polynomial categories.

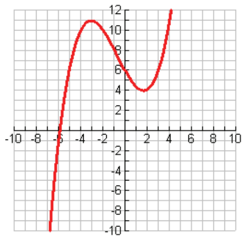
Table 1

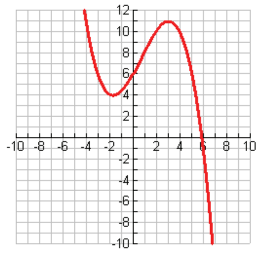
Sample of Test Questions

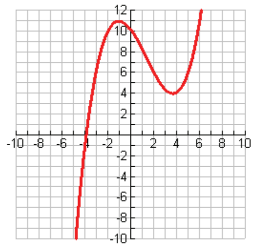
1. The graph to the right shows the curve of $y = f(x)$. Which of the four graphs below shows the curve $y = 3 + f(x-1)$?









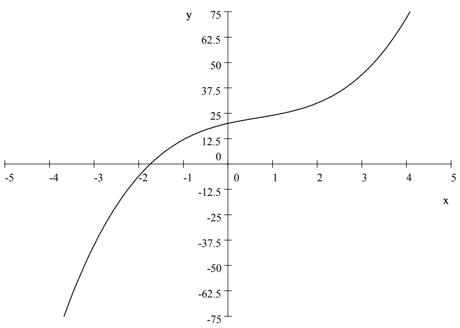


None of these

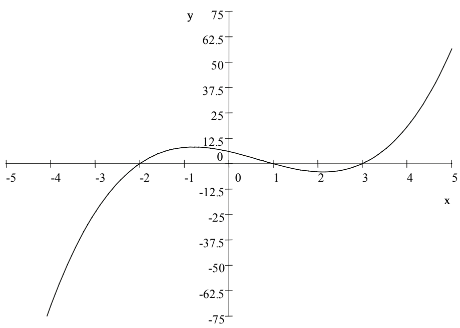
A B C D E

2. Find the domain and range of the function $h(x) = x - \sqrt{x}$.

3. The sum and difference of two functions, $f(x)$ and $g(x)$, are provided below. Determine all values of x in the interval (-5, 5) that satisfy the equation $f(x) = g(x)$.



f(x)+g(x)

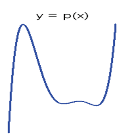


f(x)-g(x)

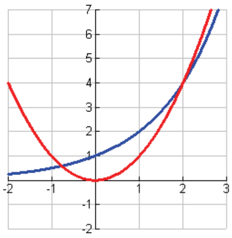
4. Approximate, to the nearest integer, the value of $f(x) = \dfrac{3x^2+1}{1-x^2}$ for $x = 4^{100}$.

5. The minimum possible degree of the polynomial $p(x)$ depicted to the right is

A. 6 B. 5 C. 4 D. 3 E. Can not be determined from the graph



6. Below, incomplete graphs of the functions $y = 2^x$ and $y = x^2$ are provided. How many solutions are there to $2^x = x^2$?



The test was administered to three intact groups of 13, 15, and 18 preservice secondary mathematic teachers from three universities in the Midwest of the USA. In two of the universities participants were registered in a course. The third university was in between terms, so participants were found by advertising the test among secondary mathematics majors and raffling a graphing calculator among those taking the test. Participants were juniors and seniors, and had therefore completed the calculus sequence.

It is important to remark that the participants came from three recognized and well-respected universities, two public and one private. Moreover, the mean grade point average (GPA) of all participants was 3.6 out of 4.0, while their mean GPA in mathematics courses was 3.5; grades that would seem to indicate a well-prepared group of students capable of coping with a set of questions at the precalculus level.

The same test was also given in the spring semester, during a workshop, to a group of 74 secondary and middle school inservice teachers from 43 high schools and 6 middle schools. These teachers represented 40 school districts from the same geographical area. The average number of years of experience was 12. In addition, 56 of the teachers had completed master degrees in education, 4 of the teachers had master's degrees in mathematics, and another 2 had master's degree in business.

Fifty-three percent of the preservice teachers and 58% of the inservice teachers were female. Descriptive statistics with the test results for in- and preservice teachers are included in table 2.

The mean scores on the tests for the inservice and preservice teachers were 54% and 46%, respectively. These low averages indicate a need for improvement on the topics that were on this test. In fact, none of the thirteen categories scored a mean over 60%. However, some categories fared worse than others. The categories that need the most improvement are Regression (mean scores ranged from 10% – 30%), Solving Equations (mean scores ranged from 30% – 41%), and Relative Growth (mean scores ranged from 34% – 40%). These are followed by Applications (mean scores ranged from 31% – 52%), Other Types of Functions (mean scores ranged from 38% – 50%), and Connections (mean scores ranged from 41% – 48%). The subcategories that need the most improvement are Attribute questions involving Inverses (mean scores ranged from 17% – 34%), Attribute questions involving Continuity and Intercepts (27% for both groups), Solving Equations from Equations (mean scores ranged from 24% – 36%), Solving Equations from Graphs (mean scores ranged from 36% – 47%), and Connections from Abstract Information to Concrete Information (mean scores ranged from 39% – 47%). To reiterate, all of the categories need improvement, some more than others.

Table 2

Descriptive Statistics for Inservice and Preservice Teachers

		N	Mean	Percent	Std. Deviation	Std. Error Mean
Test	In	74	0.543	54.3%	0.239	0.028
	Pre	46	0.463	46.3%	0.185	0.027
Attribute	In	74	0.594	59.4%	0.224	0.026
	Pre	46	0.511	51.1%	0.183	0.027
Complete	In	74	0.653	65.3%	0.260	0.030
	Pre	46	0.548	54.8%	0.244	0.036
Local/End	In	74	0.655	65.5%	0.267	0.031
	Pre	46	0.606	60.6%	0.231	0.034
One-to-One	In	74	0.676	67.6%	0.471	0.055
	Pre	46	0.739	73.9%	0.444	0.065
Inverse	In	74	0.338	33.8%	0.476	0.055
	Pre	46	0.174	17.4%	0.383	0.057
Cont and Interc	In	74	0.270	27.0%	0.447	0.052
	Pre	46	0.272	27.2%	0.431	0.064
DR	In	74	0.588	58.8%	0.288	0.033
	Pre	46	0.557	55.7%	0.217	0.032
Eqtn	In	74	0.410	41.0%	0.327	0.038
	Pre	46	0.300	30.0%	0.293	0.043
EqtnG	In	74	0.466	46.6%	0.382	0.044
	Pre	46	0.359	35.9%	0.351	0.052
EqtnEq	In	74	0.355	35.5%	0.375	0.044
	Pre	46	0.241	24.1%	0.312	0.046
Ineq	In	74	0.544	54.4%	0.351	0.041
	Pre	46	0.400	40.0%	0.295	0.043
IneqEq	In	74	0.552	55.2%	0.408	0.047
	Pre	46	0.391	39.1%	0.346	0.051
Application	In	74	0.521	52.1%	0.324	0.038
	Pre	46	0.307	30.7%	0.283	0.042
Transf	In	74	0.556	55.6%	0.292	0.034
	Pre	46	0.422	42.2%	0.245	0.036
Regression	In	74	0.297	29.7%	0.429	0.050
	Pre	46	0.098	9.8%	0.291	0.043

RelGrow	In	74	0.399	39.9%	0.351	0.041
	Pre	46	0.337	33.7%	0.366	0.054
Conn	In	74	0.484	48.4%	0.266	0.031
	Pre	46	0.414	41.4%	0.214	0.032
ConnGEq	In	74	0.521	52.1%	0.283	0.033
	Pre	46	0.467	46.7%	0.218	0.032
ConnEqG	In	74	0.544	54.4%	0.369	0.043
	Pre	46	0.486	48.6%	0.394	0.058
ConnAbstr	In	74	0.466	46.6%	0.337	0.039
	Pre	46	0.391	39.1%	0.311	0.046
Polynomial	In	74	0.538	53.8%	0.281	0.033
	Pre	46	0.433	43.3%	0.224	0.033
Rational	In	74	0.562	56.2%	0.252	0.029
	Pre	46	0.442	44.2%	0.202	0.030
Exponential or Log	In	74	0.561	56.1%	0.372	0.043
	Pre	46	0.527	52.7%	0.322	0.047
Trigonometric	In	74	0.534	53.4%	0.373	0.043
	Pre	46	0.418	41.8%	0.361	0.053
Other Functions	In	74	0.502	50.2%	0.381	0.044
	Pre	46	0.384	38.4%	0.349	0.052

Independent sample t tests were performed on the test results. The outcomes are in table 3. Levene's test for equality of variances was used to determine whether a pooled- or separate-variance t test should be employed. Equality of variances was assumed if the F statistic had a significance level of at least .05. In this case the pooled-variance t test was used. The mean score on the test for the inservice teachers ($M = 54\%$, $SD = 0.24$) was significantly higher than the mean score on the test for the preservice teachers ($M = 46\%$, $SD = 0.19$), $t(112) = 2.05$, $p = .04$ (two-tailed). In seven of the thirteen categories, the t tests showed that the inservice teachers had significantly higher means (Attributes, Inequalities, Applications, Transformations, Regression, Polynomial functions and Rational functions). There was not a significant difference in the other six categories.

Table 3

Independent Sample t Tests

	Equal var. assumed (EVA) Equal var. not assumed (EVNA)	Levene's Test for Equality of Variances		t test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Differ.	Std. Error Differ.	95% Confidence Interval of the Difference	
									Lower	Upper
Test	EVNA	5.274	0.023	2.054	112	0.042	0.080	0.039	0.003	0.157
Attribute	EVA	1.931	0.167	2.122	118	0.036	0.084	0.039	0.006	0.161
Complete	EVA	0.123	0.727	2.193	118	0.030	0.105	0.048	0.010	0.199
LocalEnd	EVA	0.518	0.473	1.037	118	0.302	0.049	0.048	-0.045	0.144
One-to-One	EVA	2.301	0.132	-0.733	118	0.465	-0.063	0.087	-0.235	0.108
Inverse	EVNA	18.841	0.000	2.072	110	0.041	0.164	0.079	0.007	0.321
Cont and Interc	EVA	0.185	0.668	-0.018	118	0.986	-0.001	0.083	-0.165	0.162
DR	EVNA	5.776	0.018	0.665	114	0.508	0.031	0.046	-0.061	0.123
Eqtn	EVA	1.173	0.281	1.872	118	0.064	0.111	0.059	-0.006	0.228
EqtnG	EVA	0.108	0.743	1.546	118	0.125	0.108	0.070	-0.030	0.245
EqtnEq	EVNA	4.387	0.038	1.796	108	0.075	0.114	0.063	-0.012	0.239
Ineq	EVNA	4.883	0.029	2.418	108	0.017	0.144	0.060	0.026	0.262
IneqEq	EVNA	5.937	0.016	2.302	107	0.023	0.160	0.070	0.022	0.299
Application	EVA	3.789	0.054	3.699	118	0.000	0.214	0.058	0.100	0.329
Transf	EVNA	2.618	0.108	2.710	108	0.008	0.134	0.050	0.036	0.233
Regression	EVNA	30.732	0.000	3.031	117	0.003	0.199	0.066	0.069	0.330
RelGrow	EVA	0.830	0.364	0.921	118	0.359	0.062	0.067	-0.071	0.194
Conn	EVA	3.807	0.053	1.521	118	0.131	0.071	0.046	-0.021	0.163
ConnGEq	EVNA	5.643	0.019	1.165	113	0.246	0.054	0.046	-0.038	0.145
ConnEqG	EVA	0.417	0.520	0.822	118	0.413	0.058	0.071	-0.082	0.199
ConnAbstr	EVA	0.298	0.586	1.218	118	0.225	0.075	0.061	-0.047	0.197
Polynomial	EVNA	4.531	0.035	2.263	111	0.026	0.105	0.046	0.013	0.197
Rational	EVA	3.308	0.071	2.714	118	0.008	0.119	0.044	0.032	0.206
Exp or Log	EVA	3.878	0.051	0.507	118	0.613	0.034	0.066	-0.098	0.165
Trig	EVA	0.446	0.505	1.674	118	0.097	0.116	0.069	-0.021	0.253
Other Functions	EVA	2.027	0.157	1.706	118	0.091	0.118	0.069	-0.019	0.255

Even though the inservice teachers performed better on the test, their question scores were positively correlated (.898) with the pre-service scores, $p = .00$ (two-tailed). This indicates that as the preservice scores increased on a question, so too did the inservice scores. These results are in table 4.

Table 4

Correlations on pre- and inservice questions' averages

		Pre-service Average on Questions	In-service Average on Questions
Pre-service Average on Questions	Pearson Correlation	1	.898(**)
	Sig. (2-tailed)		.000
	N	30	30

** Correlation is significant at the 0.01 level (2-tailed).

Table 5

Self-Ratings on Technology Used in Teaching or Learning Content

	The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously Pre-service: Rate your knowledge integrating technology into each of the following areas. In-service: How often (if at all) do you teach each topic via technology?	Pre-Service Averages	In-service Averages
1	The use of nontraditional tools such as lists, sequences, recursion to solve different problems?	2.69	2.38
2	The consistent interplay of these 3 representations	3.20	2.85
3	Calculating the range of functions using extrema?	2.81	2.30
4	Calculating intervals where a function is increasing or decreasing?	3.39	2.59
5	The local behavior of functions via approximations?	3.02	2.33
6	The global behavior of functions via approximations?	2.74	2.20
7	Optimization problems for each family of functions besides quadratics?	2.54	2.10
8	Solving transcendental equations graphically?	2.48	2.09
9	Solving transcendental equations numerically?	2.62	2.11
10	Solving transcendental inequalities?	2.52	1.90
11	Questions such as "when will the answer be at least (at most) some number?" rather than just asking "when will the answer be some number."	3.17	2.32
12	Family of functions as coming from a root, via transformations?	2.66	2.32
13	Modeling real data using nonlinear regression for each family of continuous functions?	2.42	2.09
14	Matrix applications (Networks, Markov, Transf…)	2.64	1.86
15	Recursion	2.50	1.73
Average Score		2.76	2.21

The preservice and inservice teachers responded to survey questions that related integrating technology with a variety of content. The focus for the preservice teachers was their knowledge of how to do this and the focus for the inservice teachers was how often they taught this way. The results are in table 5**.** The average ratings mainly occurred inside the "very little" and "some" categories. These low ratings are consistent with the low test scores achieved by the respondents. These low ratings also indicate that the respondents were aware of their weaknesses in these areas. It is interesting to note that the preservice teachers gave themselves higher ratings than the inservice teachers on every question. Furthermore, table 6 shows that the correlation between the average question scores for the pre-service teachers and in-service teachers is positive (0.786) with a .00 significance level (two-tailed). This suggests that a deficiency in knowledge of integrating technology with content before becoming a teacher leads to an even higher deficiency in using technology when teaching content.

Table 6

Correlations between average survey questions' scores

		In-service teachers' use of technology in teaching content
Pre-service teachers' encounters with technology being used to teach content in classes they are taking	Pearson Correlation	.786(**)
	Sig. (2-tailed)	.001
	N	15

** Correlation is significant at the 0.01 level (2-tailed).

The pre-service and in-service teachers also responded to survey questions where they rated the usage of different types of technology in the classroom. The focus for the pre-service teachers was usage in the classes that they had taken and the focus for the inservice teachers was usage in the classes that they had taught. The results are in table 7. The inservice teachers gave themselves the highest ratings in calculator usage. The average ratings for calculator usage occurred between the "some" and "often" categories. Combining these responses with the test results and previous self-ratings, it seems likely that even though the inservice teachers are using calculators regularly in their classrooms, they are using them with a limited scope. The ratings from the preservice teachers were lower on average and had a smaller range that mainly occurred between the "very little" and "some" categories. They gave their highest ratings for graphing calculator and dynamic geometry software usage. These ratings support the test results and previous self-ratings.

Table 7

Self-Ratings on Types of Technology used in Classroom

The following Likert scale was used on these questions: 1=Not at all 2=very little 3=some 4=often 5=Continuously		Preservice	Inservice
I	Preservice: Do the classes you have taken integrate the use of technology? Inservice: Do you integrate the use of technology in the teaching/learning of mathematics?	2.92	3.38
II	Rank how often those classes integrated each technology.		
	a) Scientific Calculators	1.95	3.53
	b) Graphing calculators	3.07	3.47
	c) Math Software, which one(s)?	2.00	1.99
	d) Dynamic Geometry Software (Cabri, GSKPD, Other:	2.91	1.67
	e) Other kind of technology, which one(s):	1.83	2.38
Average Score		2.45	2.73

4. Conclusions

The performance of preservice and inservice teachers on the test questions and learning outcomes seem to indicate a lack of exposure to these topics in their preparation to become teachers. This is corroborated by the self-evaluation of both groups on the chosen topics. Judging by these results the answer to our initial question seems to be that we are not taking full advantage of the range of capabilities that HHGT offers in order to provide precalculus students with the best possible preparation for calculus. It seems that neither the mathematics courses nor the methods courses that these participants had taken prepared them to answer the representative set of questions using HHGT. It may be the case that some of the topics we chose are not as relevant as we would like to think. Even if this is the case, we need to ask why our inservice and preservice teachers are not prepared to deal with the questions addressing the remaining topics. Is the mathematics educators and mathematics faculty who are teaching these students knowledgeable about the new approaches and concepts that HHGT makes possible? Is the information about the educational possibilities of HHGT and the research results, that point to the positive impact of this technology, reaching this faculty? The regular use of HHGT by inservice teachers in their classrooms does not seem to have contributed to an improvement in the scope of what they teach. Therefore, new and more advanced technologies will probably not help either, unless we improve the preparation of preservice teachers on the proper integration of technology.

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