Mathematics Teaching and Learning with Technology

Part I

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Using Videotaped Components of Lesson Study to Build Communities of Practice for Prospective Mathematics Teachers

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Abstract: Two tiers of pre-service mathematics teachers participated in an exploratory investigation in which components of lesson study were used to develop, teach, refine, revise, re-teach, and evaluate a unit on mathematical relations. Twelve fourth-year prospective mathematics teachers served as the instructional developers, planners, analyzers, and instructors for the unit, and twenty-five, second-year future elementary school mathematics teachers participated as students. Videotapes of the lesson study and the instruction showed that the fourth-year pre-service teachers were modestly successful in developing their second-year peers' understanding of the properties of a mathematical relation, primarily by using arrow diagrams and relations on sets of people to give meaning to the properties. Analysis of the videotapes and written lessons showed that the participants who taught the lessons were prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. The fourth-year pre-service mathematics teachers prepared a final paper reflecting on their experiences with the lesson study. A review of the final papers showed that these participants valued: (1) the opportunity to experience the lesson study process within the setting of an academic classroom, (2) the opportunity to work in a group to develop and improve a lesson, and (3) the experience of observing, and in some cases teaching, a lesson that incorporated the joint efforts of other participants. In retrospect, most of the participants also indicated that they would include more examples in the instructional unit, more examples of equivalence relations, more mathematical examples, and more examples generated and constructed by the students. Increasing class participation was also a concern expressed by most participants.

1. Introduction

Lesson study, a collaborative professional development approach that originated in Japan, is gaining widespread popularity in mathematics education (see [1]). In the lesson study process, a group of teachers meet as a team to set goals and to carefully craft and collaborate on the design of a lesson. Once the lesson is designed one of the teachers in the group teaches the lesson, while the other group members observe the lesson. Later the whole group evaluates and reflects on the lesson discussing ways to reteach it as a means of making it more effective and stimulating for the students. The revised lesson is taught, observed, and reflected upon a second time.

Chokshi and Fernandez [2] believe that lesson study provides a large potential influence on the impact of the professional development of teachers as it creates a professional knowledge among teachers, while offering a connection between educational policy and practice. Hiebert and Stigler [3] concur with this notion and describe lesson study as an institutionalized teaching improvement

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system built on research and the idea that teaching is a complex, cultural activity. Particularly, lesson study enables an activity structure that affords shareable and upscaleable knowledge for teacher professional development.

2. Prior Knowledge and Expectations of the Lesson Study

As part of their senior-level mathematics education seminar, twelve pre-service mathematics teachers reviewed research literature on the lesson study process in preparation for planning and implementing of an actual lesson study. Prior to preparing an instructional unit on mathematical relations, the twelve future teachers prepared discussion papers that outlined their goals, objectives, and expectations for the lesson study. These papers allowed the future teachers to express their personal concerns before undertaking the lesson study.

Within these papers, the pre-service teachers discussed their desire to work well within each study group, to involve students in the instructional activities, to enable students to clearly understand the instructional content, and to enable the future teachers to understand how to evaluate their own and other's teaching. In the discussion papers the future teachers also expressed reservations about only being able to observe the lessons on videotape instead of being present at the lesson and the time-consuming nature of the lesson study process. The future teachers also wrote about their expectations of viewing different types of effective teacher behavior and of their anticipation of using the lesson study process to learn how to adapt and revise a lesson before re-teaching it. In addition, the future teachers who were scheduled to teach the first lesson related their nervousness in teaching a topic that they did not feel they fully grasped, while the future teachers who were scheduled to teach a second (a revision of the first) lesson wrote of their hopes to improve the quality and presentation of the lesson and to use the lesson study experience to make themselves better prepared teachers.

3. Participants

This study involved two tiers of participants. Twelve pre-service mathematics teachers who were enrolled in a senior-level mathematics education seminar planned and implemented the lesson study. Twenty-five students who were enrolled in a second course of mathematics for elementary school teachers were divided into groups and served as the subjects of the study.

4. Process

During their mathematics education seminar, five of the future teachers jointly prepared a lesson plan on relations and equivalent relations that included the lesson's relation to NCTM Standards, teaching procedures, worksheets, and assessment. Two of the pre-service teachers then taught the lesson to a group (Group 1) of prospective elementary school teachers.

The lesson was videotaped and later viewed by all twelve of the pre-service mathematics teachers who wrote reaction papers to the lesson. During a subsequent class the future teachers discussed ways to improve the lesson based on results from the in-class worksheets, assessments, and from debriefings by the two participants who taught lesson.

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Two other seminar participants, neither of whom had written the first lesson plan nor taught the first lesson, prepared a second, revised lesson plan and taught the lesson to another group (Group 2) of prospective elementary school teachers. After the videotaping of the second class, all twelve future teachers viewed the tape and wrote reaction papers. The videotape of the class instruction and the pre-service teachers' reactions were discussed during the next seminar meeting. The prospective teachers were then instructed to write a summary paper of their overall views on lesson study and on their lesson study experiences.

5. Instructional Materials for Lesson One

The instructional materials used in the first lesson included an overhead transparency that described a relation on a set X as "any set of ordered pairs in which the first and second components are from X." Reflexive, symmetric, and transitive properties of a relation were also described for a relation on a set X, and an equivalence relation was described as any relation on X that satisfies these three properties. The lesson also included three tasks on activity sheets for the students to complete.

During the first lesson students were given the definition of a relation and an explanation of its properties. They were then given an example which asked them to indicate whether the "relations on the set of all people are reflexive, symmetric, or transitive" and instructed to complete the following table.

Table 1 Table of Relations and Properties of Relations from the First Activity Sheet

Relation	Reflexive	Symmetric	Transitive
"is an ancestor of"			
"is a different age than"			
"has the same income as"			
"knows a telephone number for"			

The second activity sheet asked the students to determine if the relations given by three arrow diagrams were reflexive, symmetric, or transitive as in the next diagram.

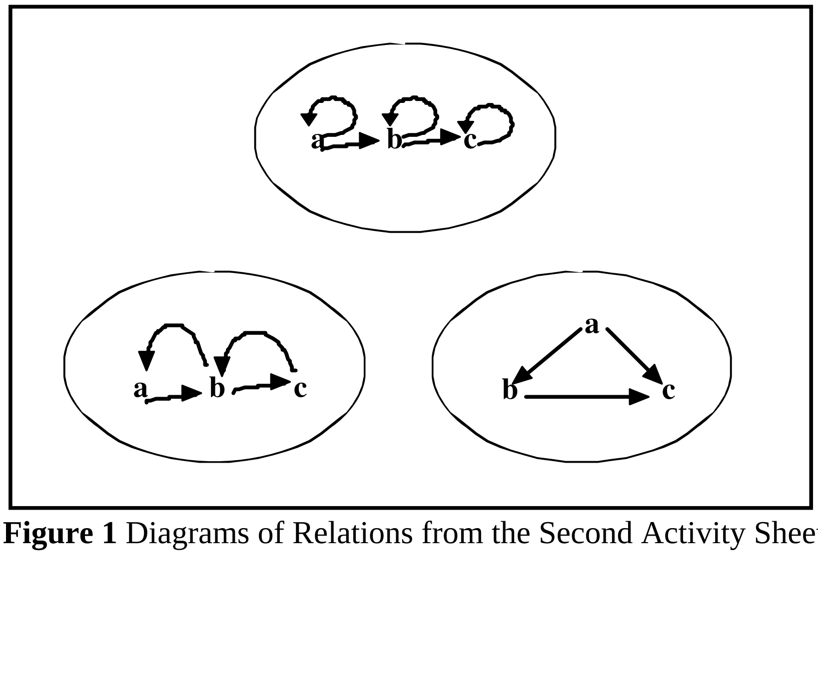

Figure 1 Diagrams of Relations from the Second Activity Sheet

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The third activity sheet asked the students to draw ovals and construct arrow diagrams that showed relations that were: (1) reflexive and symmetric, but not transitive; (2) reflexive and transitive, but not symmetric; and (3) symmetric and transitive, but not reflexive.

6. Instructional Materials for Lesson Two

The refined second lesson plan also included definitions of the reflexive, symmetric, and transitive properties as well as a definition of an equivalence relation. However, a few changes were made based on discussions during the seminar class. The second lesson plan included a paper copy of a transparency sheet containing a definition of a relation along with four examples of relations: “has the same decimal value as”, “is bigger than”, “$x^2 + y^2 = 1$”, and “$x + y < 1$”. For the relation “has the same decimal value as”, an ordered pair member of the relation was presented with the statement “(1 / 2, 2 / 4) is a relation”. For the relation “is bigger than”, the example was the statement “5 is related to 3 but 3 is not related to 5”. For the relation “$x^2 + y^2 = 1$”, the example included the statement that “(1, 0) is a relation, but (2, 2) is not”. For the relation “$x + y < 1$”, the example contained the statement “(-1, 0) is a relation, but (2, 5) is not”. A table similar to the one given to the first group was used except that the relation “is taller than” replaced “is an ancestor of” and the relation “knows a phone number for” replaced “knows a telephone number for”. Also included with the second lesson plan were the same second and third activity sheets that dealt with arrow diagrams.

7. Results

After both Group 1 and Group 2 students had completed the lesson on relations, they completed test items that used arrow illustrations to represent the reflexive, symmetric, and transitive properties. The test items particularly asked students to illustrate situations that were 1) symmetric and transitive, but not reflexive; 2) reflexive and symmetric, but not transitive; and 3) reflexive and transitive, but not symmetric. Results showed that students responded with illustrations that were arrow diagrams.

In Group 1, 60% of the students illustrated a situation that was symmetric and transitive, but not reflexive; while only 3 of the 6 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Also, 30% of Group 1 students illustrated a situation that was reflexive and symmetric, but not transitive while only one Group 1 student illustrated a situation that was reflexive and transitive, but not symmetric.

In Group 2, 30% of the students illustrated a situation that was symmetric and transitive, but not reflexive, and each of the 3 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Half of the Group 2 students correctly illustrated a mathematical situation that satisfied the reflexive and symmetric properties, but not the transitive property. Forty percent of Group 2 students illustrated a situation that was reflexive and transitive, but not symmetric.

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8. Students' Reactions to the Lessons

Students from both Groups 1 and 2 were asked to write their reactions to the lesson on relations, their properties, and equivalence relations. In particular, they were asked to comment on what aspects of the lesson had been most helpful to them and to give an overall assessment of the lesson. When asked which aspect was most helpful, 40% of the participants from Group 1 responded favorably to the diagrams; whole 20% noted the examples and 10% responded each to the explanations and the overhead transparency. Group 2 also responded favorably to the diagrams (40%); followed by 17% for examples and 8% for the explanations. No participants in Group 2 rated the overhead transparency as being helpful. Notably 70% of the students from both Group 1 and Group 2 rated the class presentations as being very good.

9. Seminar Participants' Overall Comments and Discussion of Their Experiences with the Lesson Study

At the conclusion of the viewing of videotape of the second lesson, the pre-service mathematics teachers were asked to prepare a final paper describing their experiences with the lesson study and any overall comments they had about the study. A review of the final papers showed that the participants valued:

(1) Opportunities to experience the lesson study process within the setting of an academic classroom,

(2) Experiences of working in a group to develop and improve a lesson, and

(3) Opportunities to observe, and in some cases teach, a lesson that incorporated the joint efforts of the seminar participants.

The participants also reported that they had added to their repertoire of teaching skills by observing and discussing the videos of the lesson. A majority of the seminar participants also reported that they saw improvements in the lesson implementations in moving from the first to the second lesson.

On the other hand, seminar participants expressed some disappointment that their lesson did not engender more classroom participation and student involvement. They also noted the criticism levied by some participants about the preparation and classroom performance of participants who taught or prepared the plans for the first or second lesson. Nearly half the seminar participants made reference to examples that were presented in the first lesson as being confusing and in need of being reviewed for clarity before the lesson. A major thrust of the suggestions for improvement was for pre-typed overhead transparencies with the definitions, for more explanations and student involvement, and for more examples of relations and their properties.

Overall, most participants indicated that they would include more examples in the lesson, including more examples of equivalence relations, more mathematical examples, and more examples generated and constructed by students in the classroom. Increasing class participation was also a theme of the comments from most of the seminar participants.

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10. Conclusion

According to Lewis, Perry, and Hurd [4], "Lesson study is not just about improving a single lesson. It's about building pathways for ongoing improvement of instruction (p.18)". This was exactly the situation that these pre-service mathematics teachers experienced. They developed working relationships within their groups that allowed them the opportunity to collaborate and reflect on their mathematics lessons. The prospective teachers showed evidence of enhancing their lesson study experiences by not only following a cohesive lesson study cycle that included both mathematical and pedagogical knowledge, but also exhibiting collegial qualities that supported learning.

Another benefit of the pre-service teachers' lesson study activities was the involvement in collaborative tasks that helped them systematically develop their understanding of recent educational reform. The topic of educational reform constituted a sizable component of the seminar's content, but the reality of implementing classroom reform was very concretely encountered by participants through their involvement with the lesson study activities. Analysis of the videotapes and written lessons showed that the participants who taught the lessons were more prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. During the seminar participants' commentaries on the video of the second lesson, it was apparent that most had come to recognize the need to involve students more actively in the construction of relations through experimentation and mathematical reasoning.

The future teachers' implementation of lesson study was not without challenges. In particular, some of the future teachers expressed an anxiety of being videotaped and of making their teaching public as well as finding ways to agree on common ground for joint lesson planning. They also experienced some of the challenges to lesson study that relate to research skills such as posing researchable questions, designing classroom experiments, specifying types of evidence for collection, and interpreting and generalizing results.

Since lesson study is explicit in virtually all areas of the curriculum in Japan and elsewhere, it is certainly feasible to extend the use of the components used in this study to other content in mathematics teacher education. Successful extensions to other content are apt to require less instructional time and effort by selecting topics that are relatively straightforward for prospective teachers to teach as demonstration lessons.

Hiebert and Stigler [3] concluded their review of lesson study by noting that lesson study's chances of success in the U.S. were closely tied to society's willingness to recalibrate its expectations for change by adopting a long-term improvement strategy that guarantees more effective teaching, not 1 year in the future, but 20 years in the future. The results of this study to use components of lesson study with pre-service teachers bear out this long-range view. The pre-service teachers enrolled in the senior seminar had an advantage in that they did not have some of the ingrained assumptions and solo practices associated with practicing teachers.

Whether or not lesson study will become an important technique for instructional improvement in mathematics education remains to be seen. However, its methodology of working collaboratively to plan, observe, and redesign a lesson certainly warrants innovative implementations.

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[1] Watanabe, T. (2002). “Learning from Japanese Lesson Study.” *Educational Leadership* 59(6): 36-39.

[2] Chokshi, S. and C. Fernandez. (2005). “Reaping the Systemic Benefits of Lesson Study: Insights from the U.S.” *Phi Delta Kappan* 86(9): 674-680.

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Development of the Web resources for Interactive Lessons in Geometry

- Based on the mathematics standards in Iran -

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Abstract: The purpose of this paper is to develop an interactive web resources for the mathematics curriculum concentrated on Geometry. Based on the result of our previous surveys (Behnoodi, M., & Moriyama, J., ATCM 2006) we designed the web resource into eight types of contents categories. A) Lesson planning, B) Dynamic and interactive component, C) Printable worksheets, D) Explanation in details, E) Encourage the students in challenging mathematics learning, F) Projects and presentations, G) Questions and answers, and H) How to use the web resource and math software. By these web resource it is expected that the teachers access virtual sections by printing or using simulations and active functions which can be changed by students. Teachers can see the results of students' progress at the end of each lesson by checking the answers of the questions or the degree of difficulty of each section.. Even more, they can make new simulations by themselves after they read our guide manual for software used in the web resource.

1. Introduction

The National Council of Teachers of Mathematics (NCTM, 1989) recognizes the importance of geometry and spatial sense in its publication "*Curriculum and Evaluation Standards for School Mathematics*." Spatial understanding (3-Dimention imaging) is necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions concerning two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Children who develop a strong sense of spatial relationship and who master the concepts and language of geometry are better prepared to learn number and measurement concepts, as well as other advanced mathematical topics." (p. 48)

The geometry textbooks of the new system of secondary education in Iran differ dramatically from the old ones in regards to their aims, visions, content, approach, and educational purpose. Four hundred and eighty mathematics teachers participated in a nationwide professional development program conducted to facilitate the implementation of the new changes. (Gooya, Z., 2007, p331).

During the last decade, major advances have occurred in the development and use of instructional technologies, such as hypertext-based computer software and multimedia presentation systems.

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When properly used, instructional technology has the potential to produce significant gains in both student achievement and attitude towards geometry.

As technology is now in routine use by students within mathematics classrooms, this technological advancement motivates students to use technologically based educational learning tools to better understand mathematical concepts. Technology transforms what is possible in the teaching and learning of mathematics (Instructional Technology in Mathematics education, IT 1171, The University of Pittsburgh). "The advent of computers and calculators in the classroom facilitates a new approach, one where the focus is on reasoning with a variety of representations and understanding the relationships among those representations" (Dugdale et al., 1995, p. 330). Therefore, it is important to provide the next generation of teachers with opportunities to experience firsthand mathematics learning activities that incorporate technology into their teaching.

The purpose of our research is to develop a web resource for teachers and students based on NCTM standards, mathematics standards in the new geometry textbooks used in Iran and the result of our online survey. In developing their curriculum in mathematics education we attempted to clarify the teachers' needs and *viewpoints* toward web resources of the internet (Behnoodi, M. & Moriyama, J., ATCM 2006). By conducting an online survey, we gathered feedback from mathematics teachers from various countries including Iran and Japan. The teacher's expectations of our mathematics web site were:

A) Lesson planning
B) Dynamic and interactive activities
C) Printable worksheets
D) Detailed explanations of geometrical concepts
E) Challenging geometry problems for advanced students
F) Projects and presentations
G) Questions and answers
H) How to use web resources and math software

2. Concept for Development

The results of the survey suggest that lesson planning with dynamic and interactive activities is the most important need as far as the teachers are concerned. Lesson planning should contain dynamic and interactive components and activities should sometimes be controlled or manipulated by the students who are the active learners in the lesson. The students can answer questions and immediately see the results of their answers. Some extra detailed explanations are necessary for better understanding as well as more difficult questions for those students who are functioning at higher mathematics levels. In this way, the students are engaged in challenging mathematics learning in addition to mastering the different technological software used in the web site. The students become fully engaged and can even create some new parts by themselves and submit them to the website to possibly be linked for usage by other users. Further more, if the teachers have interesting teaching methods in their classrooms they can record their speech and actions then send it for linking. The site also contains a forum for questions and answers.

An additional advantage of this site is the software training provided can be used for making the applets and animations. When software is designed for use across a variety of mathematical topics,

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it can be designed for many different applications and the user can examine the area of mathematics in which the software will be used and develop lessons that promote the type of learning on which they will focus. A dynamic geometry program allows the user to construct, measure, and manipulate what is displayed on the screen, providing immediate feedback as the object changes size or shape.

The main advantages of this kind of web site are, *free and easy access* without asking ID or password, printable worksheets, and unlimited use for any movies, animations or applets in education by teachers and students. Students interactively discover the properties of geometric figures and literally create geometry for themselves. In order to teach students how to use the software, we teach how to produce a variety of activities which integrate the computer skills forming a parallel with the geometry content.

By doing the activities in this web site, the students learn, practice, and apply rotations, translations and reflections and link mathematics to space and form in the world around them. In this way students investigate two-dimensional and three-dimensional space by exploring shape, area, and volume; studying lines, angles, points, and surfaces; and engaging in other visual and concrete experiences.

3. Methods for Development

When we started to make some applets by *Java* we thought mathematics teachers may find programming is too difficult in some cases as most of them are not professional in computer programming; therefore, we searched for an easier way. Cabri Java was the software that answered our needs; it was easier for some of the teachers to find it difficult. Furthermore it can be compatible to transfer into html pages as a Java applet. Some of the animations which were produced by *3D-Studio Max* could be shown as 3-dimensions shapes in Geometry unlike conventional textbooks. By using *Ulead Gif Animator* we could change the AVI files to Gif files and the volume decreased. For capturing a movie like power point slide show to AVI files we used *Camtasia recorder* and then used *Cyber link Power director* to change the AVI files to WMV files. For quiz and examination we used Java script; it can calculate the examination results at the end. Students can compare their answers if they click on each explanation; a new page will open and explain the solution method by animation or simulation. We can save all printable worksheets in PDF file format without any changing the original one in each computer. With well-designed review, and practice software, the role of technology is to reinforce skills through an optimal sequence that assures certain predictable outcomes (Hooper & Hokanson, 2000).

4. Developed Web Resources

The structure of the site map (Figure 4.1) contains five base branches, “about us; online lectures in geometry grade 10; resources; learn software and contact us”. Online lectures contain the interactive lessons in geometry according to the NTCM and Iran standards focused on high school 10^{th} grade text book. This part contains four sessions; Reasoning, Pythagorean Theorem, Similarity and Solid Geometry.

The webpage producer tried to use samples depending on daily life sciences with lesson planning, applets, animations, video streaming, printable worksheets and some tips for teachers to use in their class. Project and presentation that we introduce to students and teachers and the other activities done by the students from all over the world can be joined and collaborated with each

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other. Teachers can share their teaching through movie as a presentation for common users, and links for more usage. In addition the different components of each lesson are arranged separately in special folders in order to allow the user to access the applets they need. If they want to print all the worksheets, they can find all those worksheet in the separate worksheet folder.

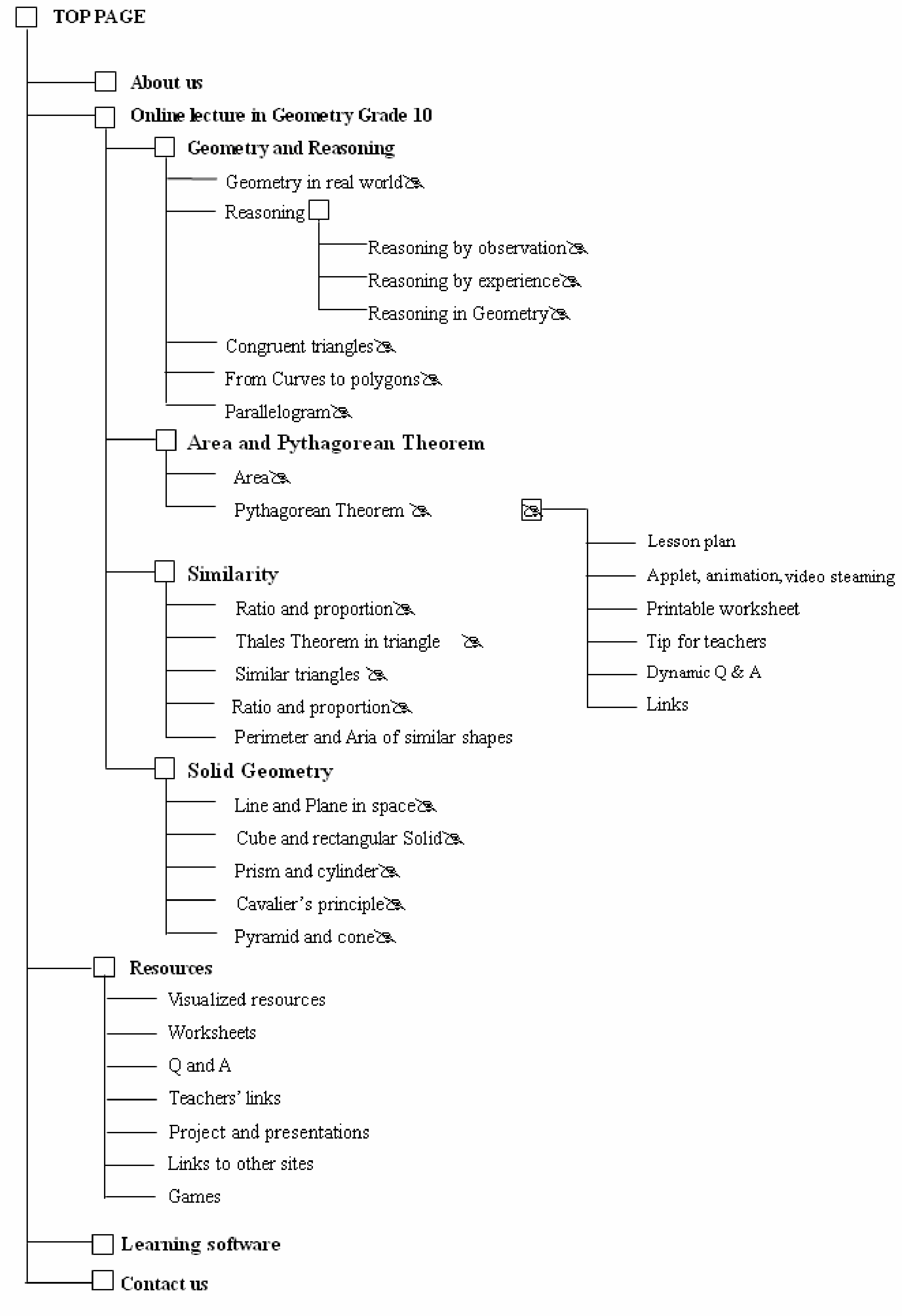

Figure 4.1 The structure of site map

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Samples of developed web resources

Sometimes by observation and practical experiments we can see the proof of theorems.

As shown in (Figure 4.2) the unit squares can be moved from one part to the other part and students can recognize the truth of that theorem. Of course later they will learn Geometrical and Algebraic proof. One of the effective ways in teaching is visualizing the problems for easy understanding. 3D-Max is the software that we used for most of the 3D shapes that can not be recognized on the black board.

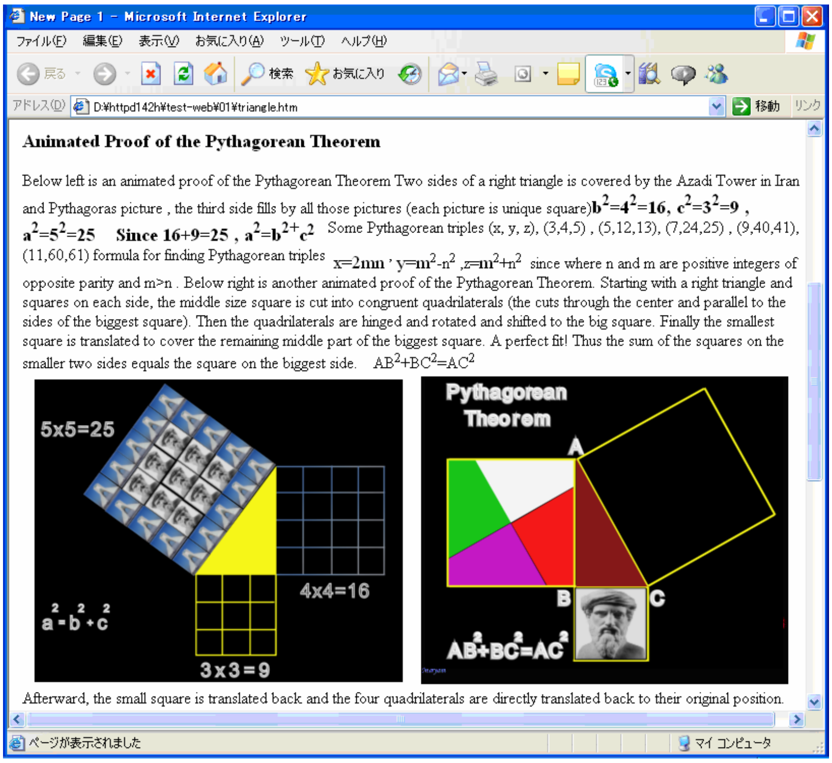
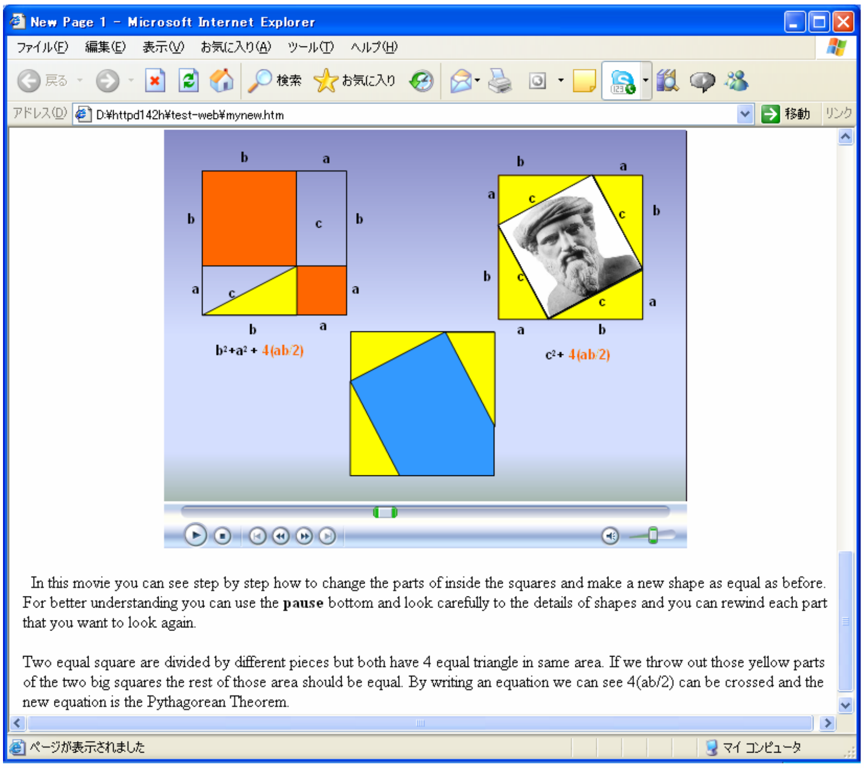


Figure 4.2 A sample of GIF animation created in AVI by 3D-Max



In (Figure 4.3) this video streaming is made by *PowerPoint* presentation and after that we made a movie of the PowerPoint show by the *screen captures* as an *AVI* file. Using of video streaming makes it *Embed* in html file, allows students and teachers to control the display. They can pause and talk about each part that they think need some explanation. Even more the voice and explanation can be recorded on each part.

Figure 4.3 A sample of video streaming from power point show

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Sometimes by observing the movement of a shape, changing the data and comparing those data, the students can see the differences or equalities between the shapes. In this sample (Figure 4.4), It can be seen that four triangles, side by side are equal. So the students, before proving have an imagination of can image the equalities before proving. In many case the students can see the results while they type the data on the webpage. This applet is made by Cabri Java and can be read in html pages. One possible advantage is, Cabri Geomerty II that it doesn't need to make programming. Therefore, a mathematics teacher can easily learn how to work by this software. By Cabri Geometry III software, the teachers can make some 3-dimensional demos that can not be replicated on the traditional blackboard.

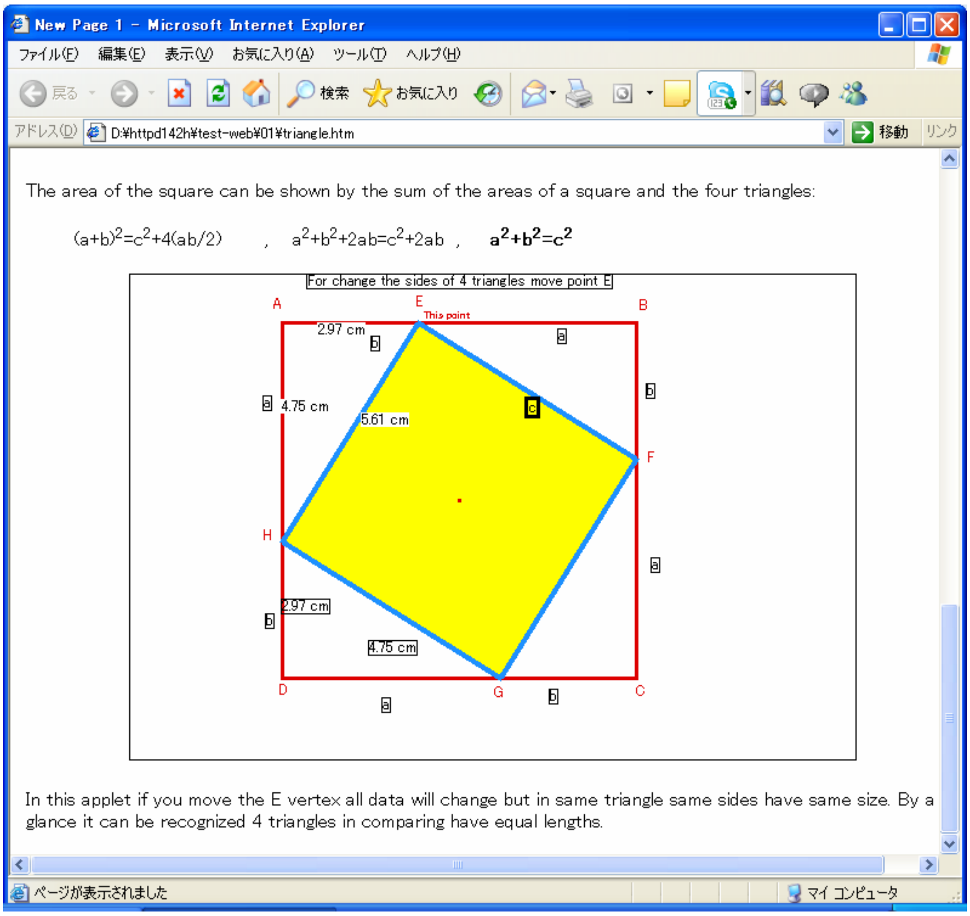

Figure 4.4 A sample of Java applet created by Cabri Geometry II

(Figure 4.5) shows two levels of worksheets. Level A is meant for common students and Level B is meant for discussion in a group training setting. The worksheets include samples related to daily life with colourful photos included. If students have a colour printer available, the questions and their components can be more attractive and aide in clearer understanding for the students. Some computers tend to change fonts or the layout upon printing, so all worksheets are made in PDF format and are printable. The users can copy these pages freely as long as by just acknowledging the name of the site for copyrights.

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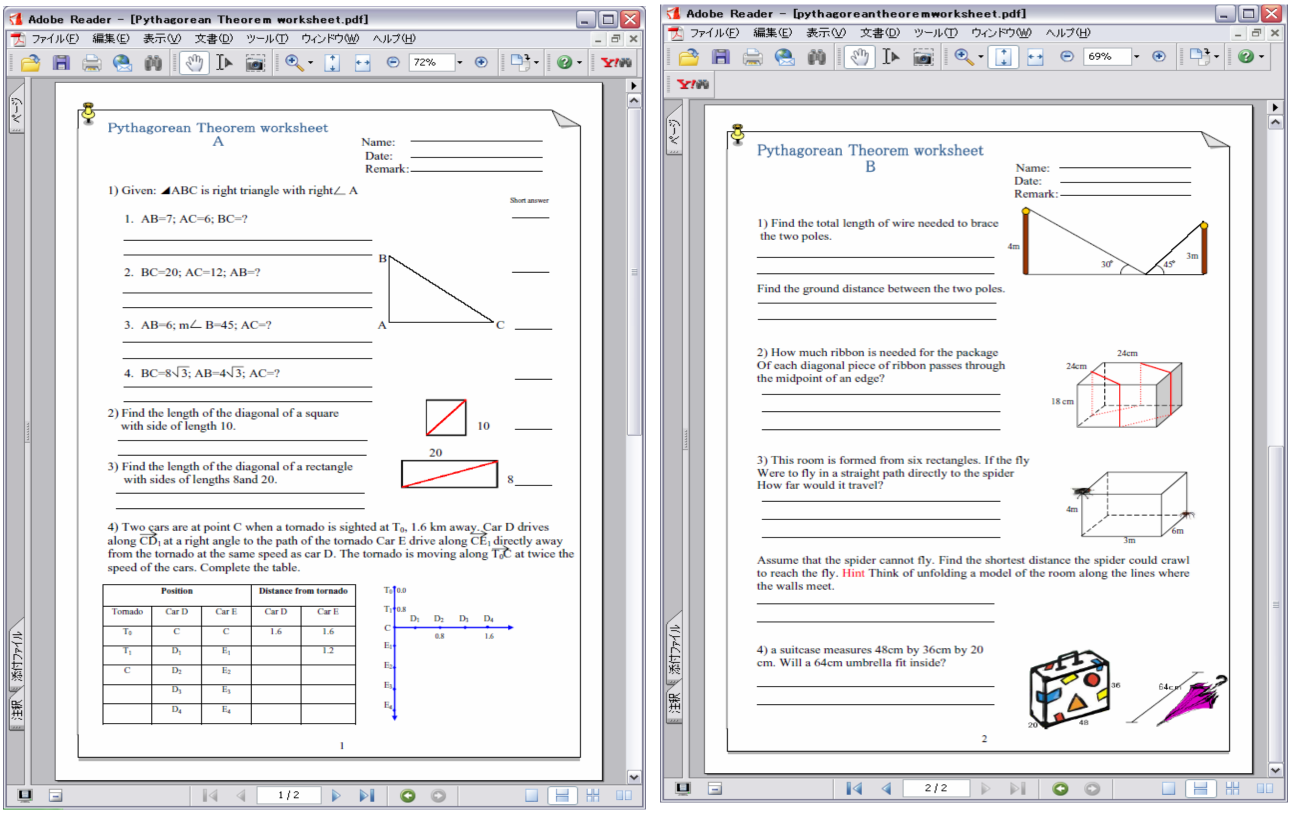

Figure 4.5 Two samples of printable PDF worksheet

As shown in (Figure 4.6), the dynamic question and answer sheets designed by Java Script are presented. There are two frames, for worksheet and answer sheet. In this kind of multiple choice answer sheet the students can answer each question by clicking on A, B, C, or D. After finishing the quiz they can see the results of their examination, and then by clicking on solutions they can compare their answers. In ordinary worksheets the students only observe a paper in black fonts and shapes which are often drawn by hand, however by using the computer generated illustrations the students gain a more accurate sense about questions by looking at the animations. They can come to understand the different situations of each question.

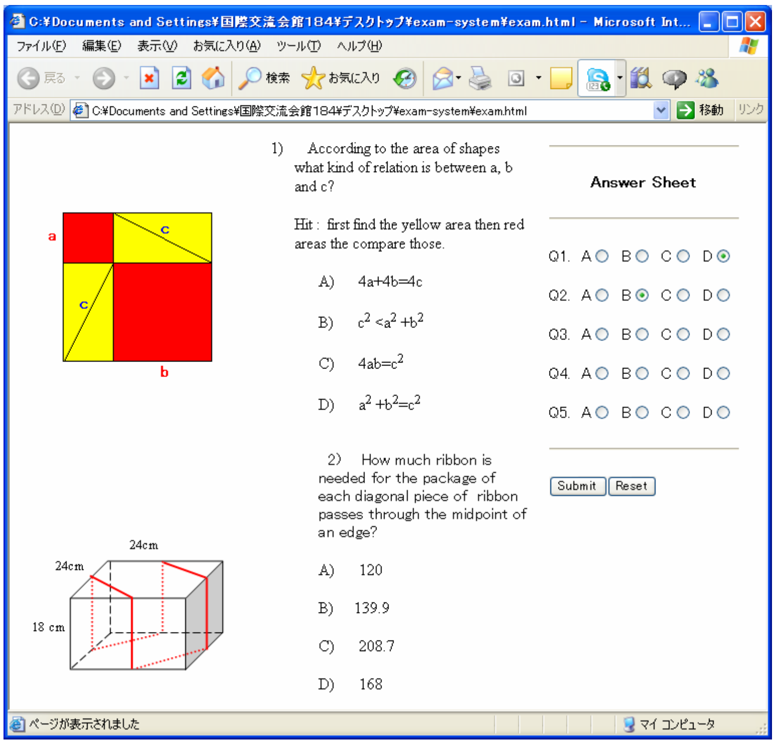

Figure 4.6 Dynamic Q & A by Java Script

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For an extra challenge we made some additional worksheets for students who would like to push further into higher levels of research. (Figure 4.7) If the students solve these problems step by step by the end they can discover some new rules which correspond to the proof that can be seen in their text book. This allows them to look at problems with a different perspective. This part not only contains extra worksheets but also depending on the lesson contains some activities, puzzles and games to guess the answers, depending on their previous knowledge. . Let us try further to introduce some famous unsolved problems for more thinking and finding a solution.

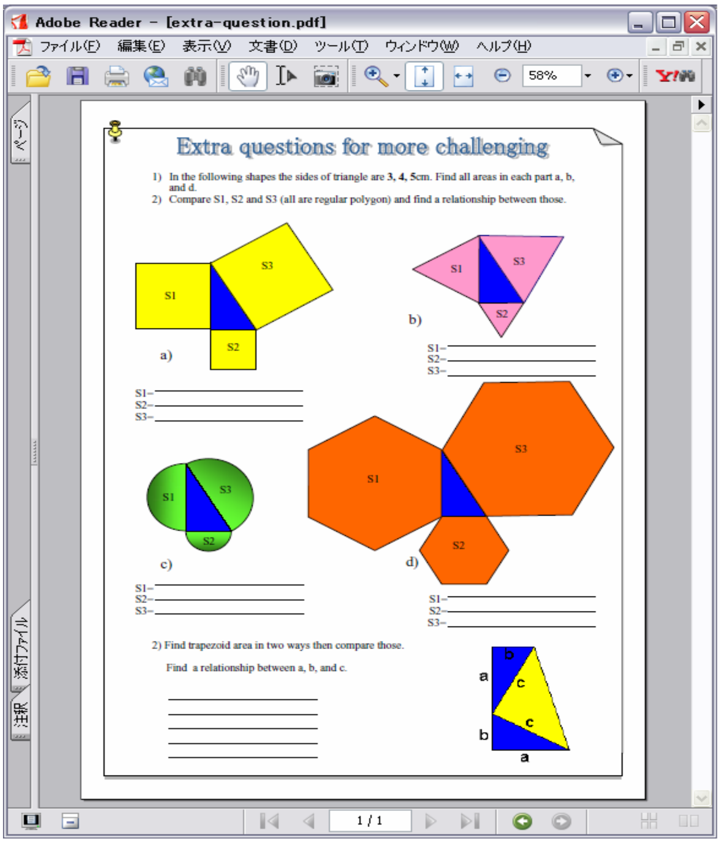
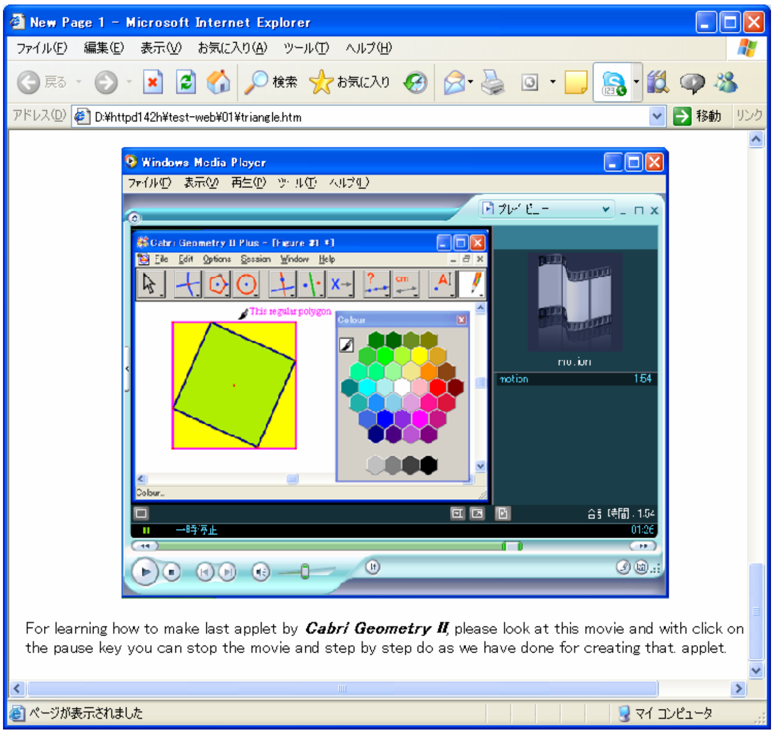


Figure 4.7 A sample of extra sheet for more challenge



The main component of this website which differs from other sites is the learning software. We tried to teach, how to use the software that we used in making interactive parts in each session. **(Figure 4.8)**The users not only can use the interactive parts for learning Geometry but also they can also create new tasks provided their school allows for common usage**.** (Of course some of the software can be downloaded for free on the internet).

We provided movie demonstrations of each part allowing the users to pause as needed and learn step by step.

Figure 4.8 A sample of how to use Cabri software by movie streaming

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5. Conclusion

Multimedia presentations are advantageous because they can developed students levels of understanding in stead of traditional classroom setting. Typically, students do not expect to actively participate in the lecture. They expect that the teacher will come to class and tell them what they need to know; i.e., what material will be on the exam. The students' role is to copy all of this information into their notebooks and memorize each problem without analyzing or synthesizing the material. We should take a different role, such as that of a coach, or a guide, who leads students in the right direction instead of trying to fill their mind with facts. There are a number of potential advantages of using the computer as a tool for instruction in an educational setting. First, technological tools help to support cognitive processes by reducing the memory load of the student and by encouraging awareness of the problem-solving process. Second, tools can share the cognitive load by reducing the time that students spend on computation. Third, the tools allow students to engage in mathematics that would otherwise be out of reach, thereby stretching students' opportunities. Fourth, tools support logical reasoning and hypothesis testing by allowing students to test conjectures easily (Lajoie, 1993). Taylor (1980) described potential computer roles as tutor, tool, or tutee. In this categorization, the student can be tutored by the computer, the student can use the computer as a tool, or the student can tutor the computer through languages or commands. More recently, Handal and Herrington (2003) described categories of computer-based learning in mathematics, including drills, tutorials, games, simulations, hypermedia, and tools (open-ended learning environments).

The tool-based approach has been shown to be an effective means to use technology to enhance student thinking in mathematics (Lederman & Niess, 2000). A tool is defined as a cultural artifact that “predisposes our mind to perceive the world through the ‘lens’ of the capability of that tool,” making it easier or more productive to perform certain activities (Brouwer, 1996-1997, p. 190).

The purpose of this paper was to develop an interactive web resource for the mathematics curriculum concentrated on Geometry. With this web resource it is expected that the teachers access virtual sections by printing or using simulations and active functions which can be controlled by students. They also can see the results of students’ progress at the end of each lesson by checking the answers of the questions or the degree of difficulty of each section. Even more, they can make new simulations by themselves after they read our guide manual about the usage of software used in these resources. These online lectures will continue for other grades. We are going to expand the teacher’s abilities in usage of the computer and software.

For future research we hope to include the following: first to conduct some experiments with students by using this method of teaching in classrooms, followed by a questionnaire which analyses the effects of each part of the lecture to be filled out by students, and second we would like to use the students’ data in order to further develop the Server side system.

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Using MS Word and Field Codes for Teaching Calculus for the Blind

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Abstract: At Winona State University, we have had a few blind students major in or take courses in the mathematical sciences. This talk relates some of my experiences in teaching calculus to a blind student in a classroom situation, including some of the problems we faced, and the strategies, technological and non-technological, that we used to get around these problems. During lectures, I used MS Word for Windows XP, which made typed transcripts of the lectures immediately available for Braille translation. I also used a standard feature of MS Word called Field Codes that are also available for Macs and with which I could create mathematical expressions that could be rendered in Braille. The talk will feature some examples of field code-generated expressions and the creation of shortcut keys for these expressions.

1. Introduction

I got interested in the problem of making mathematics instruction accessible to blind students after Prof. Joan Francioni of the Computer Science Department at Winona State University asked me in June of 2003 about having a blind student in my calculus class for the coming semester. When Prof. Francioni contacted me, she had been working on a project on the problem of mathematics and science instruction for blind and visually impaired students. She had stated that it may be possible that more blind students would choose to major in the mathematical sciences, in particular in computer science, that produce the technological aids that had become more and more pervasive in everyone's lives especially in the lives of those with impaired or without sight [1]. I had heard of blind students majoring and doing well in the mathematical sciences in other places.

I had never had a blind student in any of my classes, but I gave her request serious thought because I believed that the project was important from the standpoint of making education accessible for all. I strongly believed then and still do that education with a solid foundation in mathematics opens doors of opportunity whereas these doors remain tightly shut when we deny someone the chance to learn mathematics because we did not provide him/her the means to achieve.

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2. Discussion of the Logistics and Preparation

My preliminary emails and face-to-face discussions with Prof. Francioni and the student gave me a better idea of the problems that conventional methods of teaching pose for blind students. I also learned some of the possible modifications that I might have to make to the way I taught in order to accommodate the blind student. We discussed the type of aids, technological and otherwise, that were to be available to the student and to myself if and when he was to join my class. During these discussions, everyone agreed that, though I would have to make modifications to my teaching techniques and style, I would have to make them without compromising quality and standards. A few of the suggestions included the following:

- That I should provide the student the outlines of the lectures with the examples, graphs, equations, etc., a day or so prior to each lecture.
- That an electronic transcript of each lecture would have to be prepared either by myself or by somebody else using a font type called the Tiger Font that the Braille machine required (The Braille machine that Winona State currently uses is a Tiger Machine Viewplus Max Embosser.). The student also had a piece of software called JAWS that converted typed text into audio. JAWS, however, did not work with mathematical expressions. This is where the Braille notes played a major supplemental role.
- That I had to be very elaborate in my descriptions during lectures.

I had no problem with the first item since I always had made a point to write up an outline of the core ideas and examples of a lecture prior to presenting that lecture to my class. However, there was going to be very limited help in typing up the transcripts which meant that there could be substantial delays in getting the Braille transcripts to the student. My solution was to lecture using MS Word for Windows XP on my laptop so that the transcript of each lecture was to be ready as soon as I got done presenting that particular lecture. As such, we did not have to use valuable resources that would have been used up for outside help. I just had to work out how I was to create mathematical expressions efficiently in an MS Word document while I lectured.

3. Mathematical Expressions and Field Codes

3.1 Braille Output and Field Codes

While we were discussing how to resolve the problem of transcribing mathematical expressions into Braille, there might have been mention of the Nemeth Code. *Dotsplus*® in conjunction with MathType was already in use in other places [3], but we decided upon what are called "field codes" for three reasons:

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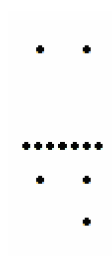
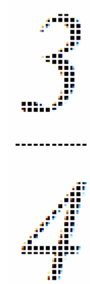
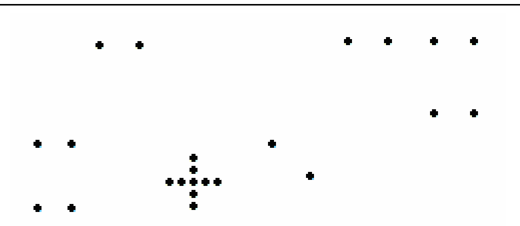
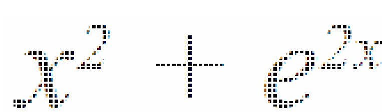
- The package of field codes was as a standard feature of MS Word for Windows XP and for Mac OS.
- I already knew how to use field codes for mathematical expressions.
- Field code-generated mathematical expressions such as " $\frac{x^2 + \sqrt{3e^x}}{2x + sin\ x}$ " or " $\lim_{x \to \infty} f(x)$ " got transcribed with *Braille-type* characters as opposed to *embossed* characters when run through the Braille machine (See Table 1). This was very important for the student who was to supplement his notes with the lecture transcripts. Note that all characters <u>including symbols should be in plain type and not in italics</u>.

Table 1. Sample Braille Machine Output Using the Equation Editor and Field Codes

Word (Field Codes)	**Field Codes Transcribed by the Braille Machine**	**Equation Editor Transcribed by the Braille Machine**
$\frac{3}{4}$		
$x^2 + e^{2x}$		

Someone had suggested using the Equation Editor, but the expressions from the Equation Editor did not translate well into Braille because the characters in mathematical expressions turn out as embossed rather than in Braille.

Note that symbols such as the horizontal bar between the numerator and the denominator of fractions and "+" as shown in Table 1 end up as embossed symbols even for field codes. But the student was able to figure these out quickly enough, especially when these symbols were put in a specific context.

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3.2. Creating Mathematical Expressions with Field Codes

We now take a look at the steps and keystrokes in creating mathematical expressions using field codes in an MS Word document using Windows XP. (I should point out that MS Word for the Mac can create and read the mathematical expressions generated on Windows XP. In fact, I learned about creating equations using field codes on a Mac back in 1992.)

- Within a new MS Word document start by pressing CTRL+F9 (Command+F9 on a Mac). Two brackets "{ }" should appear with the cursor between the brackets. The two brackets/braces are not the same as the usual brackets that one gets by using the bracket keys on the keyboard.

Table 2. Examples of Mathematical Expressions with Corresponding Field Code Lines

Mathematical Expression	**Field Code**
Fraction: $\frac{x}{y}$	{ eq \f(*x, y*)}
Square Root: $\sqrt{x}$	{eq \r(*x*)}
The n^{th} Root of x: $\sqrt[n]{x}$	{eq \r(*n,x*)}
Exponent: x^n	x{eq \s\up6(*n*)}
Subscript: x_n	x{eq \s\do6(*n*)}
Subscript and Superscript: x_k^j	x{eq \s(*j,k*)}
Summation: $\sum_{j=1}^{n} f(j)$	{eq \i\su(*j=1,n,f(j)*)}
Integral: $\int_a^b f(x)\,dx$	{eq \i(*a,b,f(x) dx*)}

- With the cursor in between the two braces, type in "eq \" to get the following: { eq \ }. The symbol "eq" calls for the generation of "equation" or mathematical expression field codes.
- All we need now is to type in the proper command line for each of the different mathematical expressions. Table 2 provides examples of some of these field code-generated expressions.

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- Once the full command line has been typed in, press ALT+F9 to view the expression (Option+F9 on a Mac). One can toggle between the field code and the math format by pressing ALT+F9 (Option+F9 on a Mac).

3.3 Shortcuts and Macros

One can create shortcut keys to speed up the creation of mathematical expressions, especially those that are used most often in class such as the ones given in Table 2. This can be done by going to the item "Macro" under the "Tools" menu in MS Word.

Table 3. A Sample Lecture Using Field Codes

Ex. $y' = (1/2)*[\frac{2x+3}{x^2+3x} - \frac{4}{4x+1}] *y$

Since $y = \sqrt{\frac{x^2+3x}{4x+1}}$,

$y' = (1/2)*[\frac{2x+3}{x^2+3x} - \frac{4}{4x+1}] * \sqrt{\frac{x^2+3x}{4x+1}}$

3.4 Other Considerations

After deciding to use field codes for mathematical expressions, we had to consider a few other things that affected the spacing of the Braille dots and characters on the Braille transcripts:

- I was to use a fairly large font size of 36.
- I had to put generous spaces between terms. For example, I should write the fraction "x^2+2x+1" as "$x^2\ +\ 2x\ +\ 1$" where there is clear spacing between terms. One gets exactly the same spacing in the Braille transcript as in the original.
- Exponents and subscripts, respectively, had to be raised or lowered a bit more than usual so that they were easily identifiable as exponents or subscripts in the Braille transcripts. Hence, " x^2 " might appear as " $x^{\;2}$ ".

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4. What Was the Student to Do

The student's difficulties when writing mathematical expressions were more complex compared to those that he encountered while reading them. While figuring out how to get around this problem, I felt that the most important thing was that he first be able to get his ideas across regardless of the format. Format is important in mathematics, but I felt that the student should be able to focus more on the mathematics. I did not want to have him have to learn field codes if it was going to take time away and detract him from learning calculus. I decided that he could write mathematical expressions using a format that was similar to what one might see in computer programming languages since, well, the student was a computer science major. For example, he could write "*x^3 + 4*e^(x^2)*" for the expression "$x^3 + 4e^{x^2}$", of course with the proper pairs of brackets or parentheses. For the more complicated expressions such as $\frac{x^3 + 4e^{x^2}}{\sqrt{2x + 1}}$, I suggested breaking the whole into pieces such as writing the numerator and the denominator separately when describing the fraction as "The numerator is *x^3 + 4*e^(x^2)* and the denominator is the square root of *(2x + 1)*." (See Table 4).

Table 4. Examples of Student Output from the Student

Mathematical Notation	**What He Wrote**
$\lim_{x \to 5} f(x)$	the limit of *f(x)* as *x* approaches *5*
$\sqrt{x - 3}$	The square root of *(x - 3)*
$\frac{x^2 - 5x + 7e^x}{\sqrt{2x^2 + 7}}$	Fraction Numerator: *x^2 - 5*x + 7*(e^x)* Denominator: Square root of *(2*(x^2) + 7)*

In the course of the two semesters that he took Calculus I and Calculus II, I asked and received constant feedback from the student to make sure that he kept apace with the class, to find out what worked and what did not, and to make quick changes when things did not work out. He did ask questions more often than everyone else during lectures, but these were primarily about notation. A sample situation could be when I say, "the fraction with the numerator $x^7 + 3x + 4$ and with denominator $\sqrt{x + 2}$," he might ask about the numerator and if the exponent of x was 7 or $7 + 3x + 4$.

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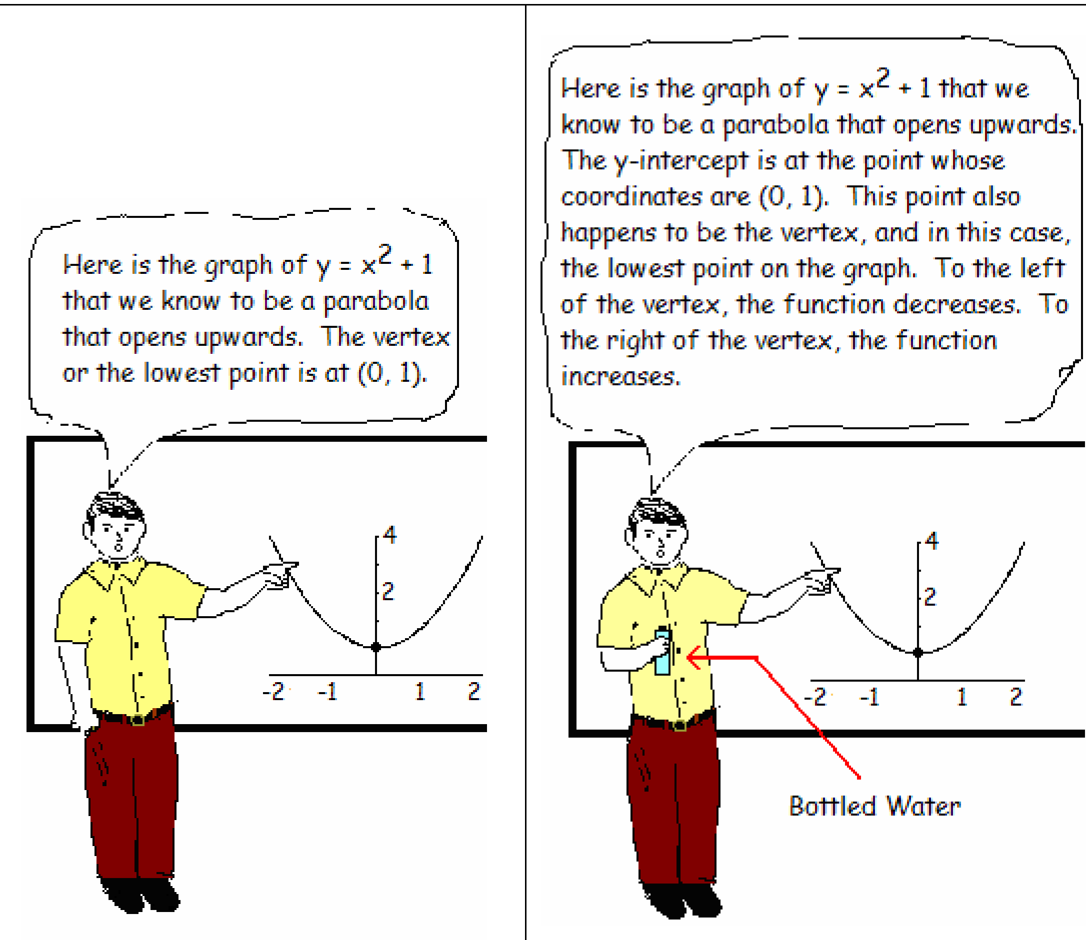
5. Graphs and Diagrams

The issue of graphing and the sketching of diagrams is the one problem that I think I had not addressed sufficiently, and it was not for lack of trying. This is one of the reasons that motivated me to write this paper and give a talk on this topic, that I feel that I had not completely "enabled" the student. Maybe this is where the reader or audience of this paper can provide suggestions and ideas as to how this problem can be worked out.

From my end, lecturing with graphs and diagrams was not a problem for the student. I just had to be more descriptive in class when working with graphs and figures as illustrated by Figure 1.



a. Without a Blind Student   b. With a Blind Student

Figure 1. Lecturing In Class

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The other students found this as a source of amusement, but they also thought that it was a good thing since, according to them, I often pointed out things that they usually would not have paid much attention to or noticed.

Reading graphs presented only minor problems for the student since he was able to do tactile explorations of the graphs that were rendered as embossed by the Braille machine. The problem when he had to draw graphs and diagrams was more difficult to work around. He had a piece of software that worked with graphs and such that the computer generated varying tones depending upon the positions of points on the graph -- the higher the points on the graph, the higher the pitch. Still, while his results for very simple curves such as parabolas and lines were more than satisfactory, we had mixed results for the more complicated graphs. I am not sure if this is a problem of becoming more adept at matching and identifying subtle changes in sound and pitch to subtle changes in graph properties such as monotonicity and curvature. If it is, I am not sure if this is an ability that can be acquired with repeated use of the software and feedback from an instructor. Even if this had been so, this might have required time that we did not have so that I suggested that he include detailed descriptions of the behavior of the curve in support of his work. Again, the important thing was to have him get his ideas across clearly. What happened was that I often received detailed descriptions of the graphs, descriptions that were more meticulously "drawn" out compared to most of the graphs and descriptions that I got from the other students (See Table 5.).

Table 5: An Example of the Student's Supporting Description of a Curve

In sketching the graph of $f(x) = \dfrac{x}{x^2 + 1}$ for the interval $[0, \infty)$, he wrote the following:

> *f increases from x = 0 to x = 1 where f has a critical point. f is concave downwards on this interval. f(0) = 0 and f(1) = 1/2*
> *f decreases from x = 1 to x = square root of 3. f is still concave downwards on this interval. f(square root of 3) = square root of 3 all over 4 which is approximately 0.43.*
> *f decreases but is concave upwards for x > square root of 3. Hence, f has a point of inflection at x = square root of 3.*
> *f tends to 0 from the right when x approaches infinity.*

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6. Final Thoughts

I discovered that teaching a blind student, in many respects, is fundamentally the same as teaching a traditional student. It is true that I had to make changes in my teaching style and techniques, but this is something that I often do when teaching a class of traditional students. The only difference is that the changes I have had to make this time are of a different nature. More importantly, however, an important objective is still to make mathematics accessible even to the most unique of individuals without sacrificing standards by finding tools and ways through which these individuals can participate in the learning experience on an even footing with everybody else.

I believe that having a blind student in class enhanced my teaching because it forced me to search wider afield for a common ground on which the student and I could meet without watering down the level of the course or alienating the other students. On the contrary, I feel that this inadvertently made mathematics in some ways more comprehensible for the traditional students.

Using the laptop allowed for more elaborate and more organized lecture notes that all my students could access. I noticed a similar change in many of the works of the other students who became more thorough in their own descriptions and explanations apart from becoming more organized in their final works. Some even started to use field codes although I did not require it nor did they get extra credit for using field codes.

The set-up I had for teaching calculus to blind students was adequate with room for improvement, especially on the matter of the student having to sketch graphs and diagrams. As such, I feel that I had not thoroughly helped the student. This will probably be not the last time that this will be an issue at Winona State. This past spring alone at Winona State, one of our faculty members encountered this same problem when a visually impaired student took her statistics class. I am not sure how this problem can be resolved. But I will be on a constant lookout for ways of providing a bridge to help blind students achieve as others would.

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I wish to thank Ms. Nancy Dumke of the Disability Resource Center at Winona State for her help in providing resources for this paper. I wish to thank Prof. Joan Francioni of the Computer Science Department at Winona State for giving me the opportunity to teach a blind student. And to my student (whose name I can't mention at this point because of privacy issues) for helping me grow as an instructor.

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Graphic Tool for Communication with Visually Impaired Persons

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Abstract: Graphic content is very useful and convenient for effective communication, including that of mathematical concepts. However, this proves to be a barrier for visually impaired people. Our software is a tool for communicating such graphic content between sighted and visually impaired persons. The targets of our system are two-dimensional graphic documents, and the objects used are applied in Japanese high school mathematics. A visually impaired person generally does not understand complicated graphics and the cross relationships of graphical contents. Thus, our main objective is to simplify the graphic content and the information they convey. Our system, which makes use of a tactile display, is primarily designed for communication between sighted and blind persons. However, this system may provide the answer to the question, "How can we create or convey graphic content without visual information?" thereby making it useful for all visually impaired persons.

1. Introduction

We often use electronic information on a daily basis, and these communications are very convenient for visually impaired persons if they are text-based. For example, if we send an e-mail with no image files attached, special provisions are not required for visually impaired persons. However, communication of mathematical content consists of a few graphics, which create barriers particularly for a blind person. By designing communication software for communication that includes some mathematical graphic content, we attempt to remove these barriers.

Recently, we have seen many software and hardware breakthroughs aimed at assisting visually impaired persons, and there has been a great deal of researches improving these technologies ([1], [2], [3]). M. Kobayashi created the MIMIZU System (see [4] for example) comprising a tactile display and a commercially available ultra-sonic pen. The user draws some lines on the tactile pin display with the ultra-sonic pen. This tactile display is also commercialized at this time. Then, anyone can use that system with ease. A complete description of how to design the MIMIZU System is on Kobayashi's web page (http://www.cs.k.tsukuba-tech.ac.jp/labo/koba/research/dv-1.php, last access 2007.9.10).

Our system runs on the same hardware as the MIMIZU system. The previous version of our system ([5]) provided a graphic editor for visually impaired persons (primarily for the blind persons). With the help of simple graphics, the user can select graphic object that appear on the tactile display. Verbal explanations are available for the graphic contents. Using this system, a

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blind user can draw mathematical graphic objects. However, on practical use of this system, we encountered a problem. The system was not convenient for real-time communication. For example, when a user began working with using our system, it was necessary to explain how to use it. In the case of a blind user, occasionally, we were unable to give clear instructions, making it necessary to take his hand and lead his fingers to the graphic contents on the tactile display. It is clear that the absence of an efficient user interface to express the graphic contents is the reason for this obstacle.
In trying to express the graphic contents verbally, we faced the same difficulties that arise in text-based communication. Therefore, for a person to understand how to use this system, some communication functions are necessary at the initial stage itself.

Our new system consists of two sub-systems; one has the tactile display, and the other uses only a standard monitor display. In this system, we have several communication functions. We cannot display some graphic contents as visible images because the tactile display is very coarse, the tactile information is local, and the user cannot understand the cross relationships. Then the two sub-systems have common graphic content parameters, which are used to draw same shapes in an x-y plane. On the tactile display of our system, there are usually small number (not more than 3) of objects, and the user selects one of them as a main object. When there are several graphic objects (more than 3), the user selects some of them to be displayed on the tactile display. The sub-systems also share information on what objects the user selects as main and display. This allows two users to understand each other's intentions, using some additional communication functions.

2. Outline of the System

This system consists of two sub-systems. One is a graphic editor for visually impaired persons (mainly for the blind), and the second is for sighted persons. With the system for visually impaired persons, a user can edit graphic objects using the tactile display (Dot View, commercialized by KGS Corp), and an ultra sonic pen (Pegasus Technologies) (Figure 1). The other system does not require special hardware, except for a standard personal computer. These sub-systems exchange information using a local area network.

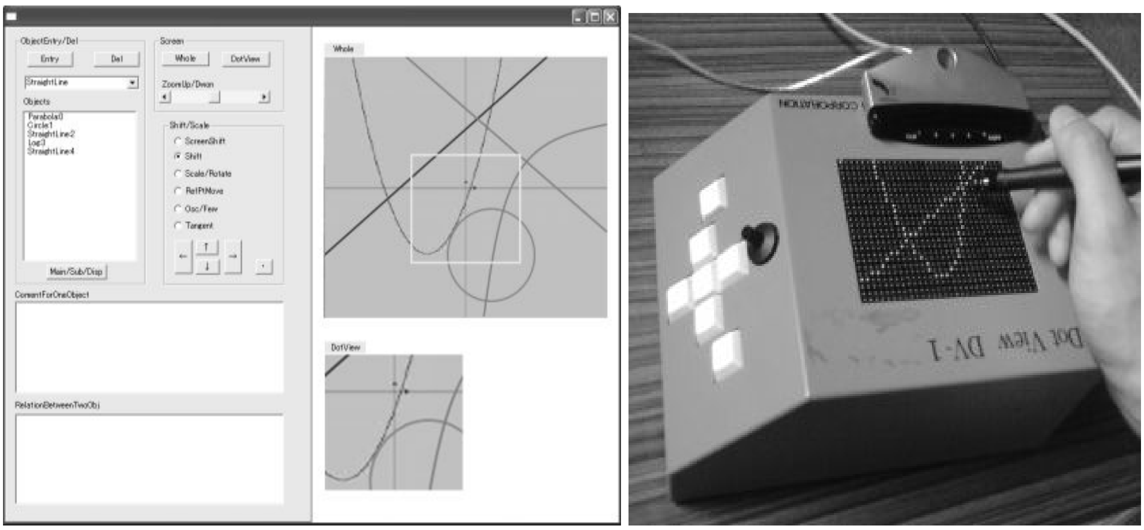

Figure 1 Two Subsystems

2.1 Target Objects

Our system is a graphic communication tool used in high school mathematics. In this model, we restrict the targets to objects in a two-dimensional coordinate system. The types of target objects

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are straight lines, parabola curves, trigonometric functions (sin, cos and tan), exponential and logarithmic functions (exp and log), hyperbolic functions (sinh and cosh), and basic figures (triangles, squares, circles, and ellipses). For each object type, we form sufficient parameters to determine each object uniquely, and express these in an XML format. A detailed explanation will appear in section 3.

2.2 The System with a Tactile Display and an Ultrasonic Pen

For this subsystem, we use the following hardware:

(1) A tactile pin display (Dot View by KGS).
  24(vertical) x 32(horizontal) =768 pins.
  Pitch between two pins = 3mm.
  One joystick and seven keys.
  RS232C connection with PC.

(2) An ultra sonic pen (Pegasus Technology).
  Area size = A4 paper size.
  Resolution = 100 dpi.
  USB connection with PC.

Each sub-system is connected to a common PC, and the user uses a standard keyboard along with the hardware (1) and (2). Our main functions of these sub-systems are as follows:

(a) Edit graphic contents.
(b) Save and Load.
(c) Communicate with the user who uses the other system.

We assume that a user of this sub-system is visually impaired (may be blind), therefore, the system must be controllable without visual information. A blind user can edit graphic objects as follows: First, the user inputs a graphic object on the tactile display by selecting an object type and pushing the corresponding key, causing an object with default parameters to appear on the tactile display. The user can then change its shape or position using a joystick and keys attached to the tactile display. During the edit, verbal explanations for one graphic object and for the relationship between two objects are available.

The display area is very coarse, and cannot accommodate many objects. Therefore, for simplicity, the user selects just a few objects for display. Other objects are not for display; however, they remain in the graphic document. The user also selects a main object that is the target for edit functions (changing its shape and position) and explanations. The user can also obtain explanations on the relationships between the main object and the other object, which the user selects from among the objects for display. There are two categories of graphic objects, function graph and basic figure. Table 1 shows the list of edit functions for each category. In the beginning, an object has its default figure, and the user changes its shape and position using these functions.

Function Graph			Basic Figure		
Edit Type	Parameters	Details	Edit Type	Parameters	Details
Shift	(X,Y) (a vector)	f(x) => f(x-X)+Y	Shift	(X,Y) (a vector)	(x,y) =>(x+X,y+Y)
Oscilation scale up/down	A (a real num.)	f(x) =>Af(x)	Scale up/down	A (a real num.)	(x,y) => (A x, A y)
Cycle scale up/down	A (a real num.)	f(x) => f(A x)	Rotate	B (angle)	(x,y)=>(x cos B- y sin B, x sin B+ y cos B)

Table 1 Edit Functions

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In the details column, under the basic figure category of the table, (x, y) represents the relative coordinates of any point, with respect to some reference points for each figure.

In a mathematical graphic document, a graphic object is sometimes tangent to some other object. Despite the importance in mathematical properties, it is almost impossible to create it using a joystick without certain special functions. The function touch move (touch shift, and touch scale up and down) changes the shape or position of some target object (called the touch target) with respect to one curve touching to another curve (called the touch base). The following table shows the list of candidates for touch target or touch base.

	Object Type
Touch Target	Circle, StLine(straight line), ParaboraCrv
Touch Base	Polynomial, Trigonometric, Exponential, Hyperbolic curve

Table 2 Touch Target and Touch Base

Only touch shift works for a straight line while the function touch scale up and down cannot change the parameter of the straight line.

2.3 The System for a Sighted User

One may be skeptic about whether a blind person needs graphic information. The answer may be “no” if they are not communicating with sighted person. However, sighted persons often use made mathematical concepts and properties, and some of these are closely related to graphic concepts. Moreover, many sighted persons understand these concepts with respect to their graphic contents. Thus, for communication with sighted persons, a blind person must use some graphic objects, even if he (or she) can understand all mathematical concepts and properties without any graphic concepts. Thus, we can say that all persons need graphic objects; however, they are convenient only for sighted people.

From this point of view, there are two barriers of communication with graphic content. Graphic content is not easy for visually impaired persons to understand, and it is difficult for sighted persons, to convey the important information in the graphic content, as the information is subconsciously apparent. To remove these barriers, we added the following functions beside the basic functions (a), (b), and (c) in this sub-system.

(c) Check and Control the area of the tactile pin display.

(d) Check and Control the selected objects (main and display objects).

The area of the tactile pin display is coarse (24 x 32 pins); therefore, when a user checks for details in an object, the area is often very small. When he (or she) checks another part or makes a global check, the size and position of the display area often change accordingly. As a result, there are two display areas in the system for sighted persons, one provides global view of graphic contents, and the other replicates the images on the tactile display of the other system.

3. XML Formats

The system document, including graphic content, is stored as an XML file. In this document, there is sufficient information to construct the shapes and positions of the objects. For communication between the two sub-systems, we use an XML format to express the graphic document. As mentioned in section 2, the information on the state of a tactile display is shared information between both the sub-systems. This is also represented in the XML-document.

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3.1 XML Format for Graphic Document

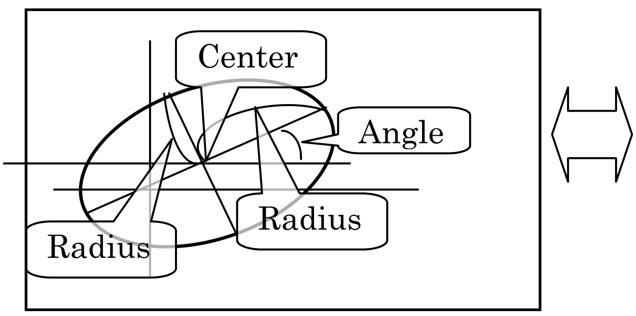
All graphic contents of a document are represented by curves determined by certain parameters. Table 3 provides a list of the elements and the names attributed to them. Each attribute has its value type [(i), (r), or (v)]. (i) stands for integer values, (r) for real values, and (v) indicates two-dimensional vectors.

For a graphic representation of a polynomial function (a straight line and a parabola curve), the general expression of the function is $f(x) = c_0 + c_1 x + c_2 x^2$, and the attributes Coeff0, Coeff1 and Coeff2 represent c_0, c_1, c_2. For trigonometric, exponential and hyperbolic functions, we consider a generalized function $s\,g(c_0 + c_1 x) + t$. Then, for each base function $g(x)$, the attributes Shift, Scale, Coeff0 and Coeff1 define the function. According to the expression rule, we can express the function as $s\,g(c_0 + c_1 x + c_2 x^2) + t$. We can display this function if the XML source file includes this document. However, we only have functions, (x,y)-shift, oscillation scale up and down and cycle scale up and down. Therefore, this system cannot change the default function $g(x)$ to $g(c_0 + c_1 x + c_2 x^2)$.

Each element always has an attribute Id, which is a serial number for the graphic objects. The system identifies an object using this attribute. A circle is defined by its Center and Radius. An ellipse is defined by its Center, Radius, Radius2 and Angle (see Figure 2). The vertices of the corresponding number define a polygon. Every attribute has its default value, that is, every object has its default shape. We can define their default values using an XML file. Figure 2 is an example of a XML source code for a graphic document and its figure.

Curve	*Element Names*	*Attribution name (value type)*
Common	*****	Id (i)
Polynomial	StLine, Parabora	Coeff0 (r), Coeff1 (r), Coeff2 (r)
Trigonometric	SinCurve,CosCurve TanCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Hyperbolic	SinhCurve,CoshCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Exponential	ExpCurve,LogCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Circle	Circle, Ellipse	Center (v), Radius (r), Radius2 (r),Angle (r)
Polygon	Triangle, Square	Pt1 (v), Pt2(v),Pt3(v), Pt4(v)

Table 3 Element Names and Attribution Names




```
<Ellipse Center=”(3,1)”
         Radius=”7.0”
         Radius2=”3.5”
         Angle=”0.523” />
```

Figure 2 Ellipse and its Xml Expression

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An XML sub-element Relation may belong to any XML element corresponding to a graphic object. This sub-element has an attribute SubId and text document. The text document explains some relationship between the graphic object and the other graphic object corresponding to the id number SubId. This sub-element is important to understand the document, and we explain their details in section 4.

3.3 XML Expression for Tactile Display State

For the functions (c) and (d) in subsection 2.2 we express the following information by XML text document.

(1) Area of tactile display.
(2) The main object and the objects for display.
(3) How to edit the document.

Table 4 shows a list of XML elements and their attributes that express this information. The tactile display has a pin display area (24 x 32), and this corresponds to rectangle area in a coordinate plane and two vector attributions stand for two corners ((left, bottom) and (right, top)) of this rectangle. Two elements, SelectedObj and EditState, are sub-elements of DvArea, and DisplayObj is a sub-element of SelectedObj. The attributes MainId, SubId, and Id are the ID numbers of the corresponding graphic objects.

An object type (n) implies that the value is a string, selected among prepared strings. The value of Target is selected from among {Object, Screen, and Touch}, and that of EditType from {Shift, Rot_Scl, Osc_Frq, TchShift, and TchScale}.

	Element Name	Attribution	Sub Element
(1)	DvArea	LeftBottom (v) RightTop (v)	SelectedObj
(2)	SelectedOb j	MainId, SubId (i)	DisplayObj
	DisplayObj	Id (i)	
(3)	EditState	Target (n) EditType (n)	

Table 4 Xml Expression for State

```
<DvArea LeftBottom=”(0,0)”
                RightTop=(1.0,1.0)>
    <SelectObj mainId=”1” >
          <DisplayObj id=”2”/>
    </SelectObj>
   <EditState Target=”Object”
              EditType=”Shift” />
</DvArea>
```

Figure 3 Sample State for Display

4. Information Sharing

There are two barriers in the graphic communication between sighted and visually impaired persons. One barrier is difficulty for a visually impaired person to get adequate graphic information from graphic content, and the other is, for sighted person, difficulty to be conscious on essential tips of information. Two sub-systems exchange information of curve shapes and positions, cross relationships and states of the tactile display. The system displays the shapes and positions in the tactile display and several explanation functions are available, these will remove the first barrier. It is also very important how the visually impaired user use these functions. Thus, we add the function to exchange the information of the states of the tactile display to remove the second barrier for sighted persons when they communicate with visually impaired persons. In this section, we will explain the information for removing the second barrier.

The XML element Relation is a sub-element of an XML element corresponding to some graphic object. A content of this element is string: an explanation about relations between the main and the

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other (user selected) object. Voice explanations are available in the sub-system with a tactile display using this information. Some of these contents, “a positional relation of reference points of both objects” and “the number of cross points and tangent points”, are given by the system automatically”. We can also input any text document as a content of “Relation” in both sub systems. The Figure 4 is a part of a sample XML document. In the “Circle” element, there are two “Relation” elements. The first one is standard one, which the system makes automatically. The second one is specialized for the situation.

The information of the state of a tactile display consists of the area rectangle, the object selection and the edit state. The attributes Target and EditType in the Xml Element EditState explain how the user controls the system. Table 4 shows the candidates for these attributes.

```
< Circle  id=”1”   Center=”(0.0,0.0)” Radius=”2.0” >
           <Relation refId=”2” >
                Two Cross Points.
          </Relation>
           <Relation refId=”2”>
             Straight line through the center.
             Another straight line is tangent to this circle,
               And two straight lines cross at right angle
               on tangent point.
          </Relation>
  </Circle>
```

Figure 4 Sample Xml Expression for “Relation”

Target		*Edit Type*	
Object	Edit the main object.	Shift	(x,y)-shift
Screen	Change the area rectangle.	Rot_Scl	Rotate or Scale up/down
Touch	Edit the main object touching to other object.	Ocs_Frq	Scale up or down for Oscillation or Frequency
		TchShift	Shift touching to other object
		TchScale	Scale up/down under touching

Table 4’ Move Targets and Edit Types

5. Extension of Object Type

The target of our system is graphic content in Japanese high school mathematics. This does not cover all situations even in the high school mathematical education. For example, we cannot create a graph of $\sin(x)+e^{x}$ with this system. Thus, in the near future, we will have to extend the system for new object types. For the extension, we have only to prepare the following functions (1) ~ (5) according to a new object type, if the object is a curve parameterized by one real number.

(1) Get a beginning parameter and an ending parameter.
(2) Get a curve point corresponding to a parameter.
(3) Get a tangent vector corresponding to a parameter.
(4) Get the parameter of the nearest curve point from given point.
(5) Get main reference point and sub reference points and their total number.

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In the case where the curve is a function graph, (2) ~ (4) are replaced by
(6) Get a function value corresponding to a parameter.
(7) Get a differential coefficient corresponding to a parameter.
The system calls (1) ~ (5) in many situations. We list below some of them
(a) Draw a curve on displays of two subsystems.
(b) Find a nearest curve from a point.
(c) Touch move (for a function graph as a "Touch Base").
The function "Touch move" (see Table 1), for example, a starting tangent point is set using (4) then the position or shape of "Touch Target" are determined using (2) and (3). Then, next position and shape are calculated using (2) ~ (4) according to the type of "Touch Move". Thus, general graph of a function can be "Touch Base".

6. Evaluations

Voice explanations are main functions for removing barriers in communication with graphic contents. Especially, we expect that the explanations for the cross relationships are useful for visually impaired person to understand graphic objects. Then our evaluation test is checking their abilities. There are twelve test subjects and all of them are sighted (Table 5).

	age	sex	Comment
1	57	male	Math. Teacher
2	38	female	Math. Teacher
3	23	male	Univ. Student(M2)
4	22	male	Univ.Student(M1)
5	21	male	Univ.Student(B4)
6	21	male	Univ.Student(B4)
7	21	male	Univ.Student(B4)
8	23	male	Univ.Student(M2)
9	24	male	Univ.Student(M1)
10	58	male	Office worker
11	58	female	House wife
12	28	female	Office worker

Table 5 Test subjects

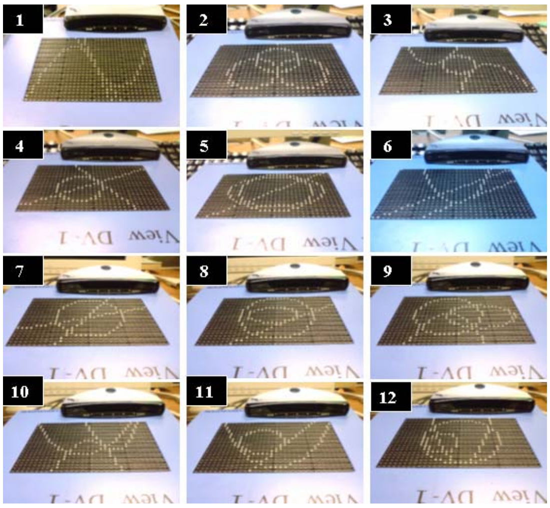

Figure 5 Graphic Dcuments

There are twelve graphic documents (Figure 5) and we use one document for one test. The tests are divided into 3 categories according to the explanation attached to the document. For the document of first category, there is no relational explanation, only the names of curves are given. For the second, positional relation and number of cross points are attached. For the third, we made an adequate explanation for each pair of objects (Table 6). One test subject takes six tests (two tests for each category) .

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Object Pair	Explanation
Parabola 1 and Straight Line	A parabola curve with convex upward is tangent to a straight line. The tangent point is in the right area of the peak point.
Parabola 2 and Straight Line	A parabola curve with convex downward is tangent to a straight line. The tangent point is in the left area of the peak point.
Two Parabola Curves	A parabola curve tangent to the other parabola curve. The tangent point is a tangent point to a straight line. Two peak points are point symmetry with respect to the tangent point.

Table 6 Explanation for Document 1

We gave a test according to the following steps.

1. Wear an eye mask and touch the document in a tactile display with above explanation.
2. Remove the objects from the display, and take off an eye mask. After that, the subject draws a picture of the graphic document.
3. We store the total time, and the object evaluate his (or her) picture as a point (1 ~ 5), comparing with a correct figure.

Figures 6 and 7 show the results of the evaluation test. These are graphs of average values and standard deviation of test times and self-evaluation points for each test category. The standard deviation of the third category is very small.

Let $V_T(k)$ and $V_P(k)$ $(k = 1,2,3)$ denote the unbiased variance of the test time and self-evaluation point for k'th category. The statistic lows of the fractions $\frac{V_T(j)}{V_T(k)}$ and $\frac{V_P(j)}{V_P(k)}$ are F-distribution with 23 and 23 degrees of freedom if we assume that the population variances of two categories are same and that these values are stochastically independent. In this experience we obtain $\frac{V_T(1)}{V_T(3)} = 2.86$, $\frac{V_T(2)}{V_T(3)} = 3.33$, $\frac{V_P(1)}{V_p(3)} = 4.07$, then by the F-test with the risk rate 0.01, we obtain statistic judgments: the population variance of the category 3 is less than that of the categories 1 and 2 for the data of test time, is less than that of category 1 for the data of self-evaluation point. This implies stable and comfortable use of the system.

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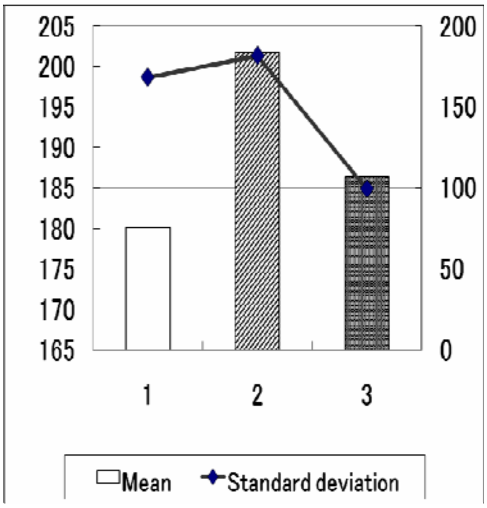

Figure 6 Result(time)

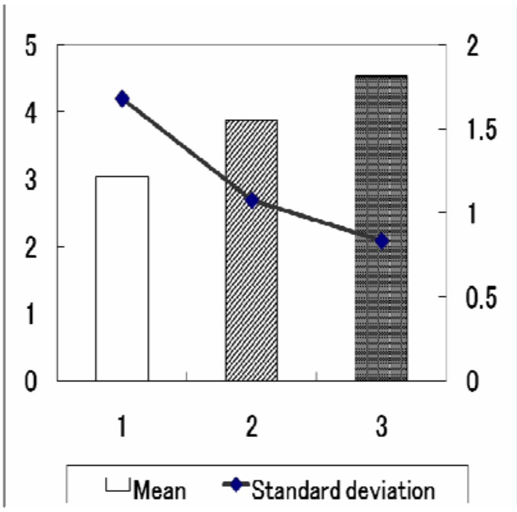

Figure 7 Result(self evaluation)

7. Conclusions

We have designed a communication system for a graphic object. This system enables us mutual communication with graphic objects between sighted and visually impaired persons. Concerning about the LAN connection between two sub systems, it is not difficult to increase the number of subsystems. Then, this system will be useful for e-learning including some visually impaired persons, if we can solve the following problems

1. Improve the user interface and the expression rule for graphic objects.
2. Automatic creation of voice explanations.
3. Connection with other documents or databases.

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Beyond the Web-Based Cognitive and Interactive Metacognition Interface within Teaching Similar Triangles

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Abstract: Most of good performances of teaching and learning outcomes are based on the suitable information management and communication during the teaching and learning processes. Every educational communication is not a simple thing which usually involves encoding and decoding information from teaching and learning interface. The interface maintains information which includes different views in teaching, learning, developing, and administering to transmit and receive criteria information and knowledge during teaching and learning processes. However the transmitters and receivers are not always skillful in expression and comprehension or not always energetic enough to send or receive the information exactly. Consequently, the losing information or misconception happened commonly during each information communication. Similarly, while instructor and learner communicate the concepts via the web pages in the web-based educational system, the event of losing information occurs in the same time. In other words, the communication interface may not always maintain a good or suitable expression in each concept or knowledge and the learners may not always skillful enough to comprehend and receive the expressions via the interface. Then the web-based teaching and learning communicated interface are not always suitable for supporting, diagnosing and monitoring in teaching and learning misconceptions. This paper presents a **Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)** to support learners and instructors to percept, monitor and communicate the critical information and metacognition within teaching similar triangles in web-based educational system. Teaching scenario and empirical evaluation (N= 105, Grade 8, 14~15 years old, from three classrooms) have been done to express multi-stage supporting and multi-level detections of learner's misconception to detect and guide the metacognition of learners to show the WBCIMI is available.

1. Introduction

Metacognition is both as the abilities of individuals' knowledge which can monitor, control,

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reflect, and understand their own learning processes sensibly and as the resources which can be used and applied to enhance the learning outcomes. Flavell (1976) proposed the concept of metacognition of learners' cognitive processes and regulation of learning processes. The metacognitive knowledge interplay with individual characteristics, task properties and available strategies (Flavell, 1979) in learning situations and the metacognitive skills focus on learners' self-regulatory activities. Schraw & Dennison (1994) defined the metacognition is the abilities of learners which reflect, understand, and control their own learning. However the assessment of metacognitive skills through self-reports was problematic because it appeared that learners had poor insight into their own behaviors (Nisbett &Wilson, 1977; Prins, Busato, Elshout, & Hamaker, 1998; Veenman, Prins, Verheij, 2003, & Veenman, Prins, & Elshout, 2006). Accordingly, mental image became a black box that is not the real subjects can be insight and exploration, whereas a durable improvement of more specific cognitive abilities through training and supporting are an essential issue during teaching and learning processes. Owing to the metacognitive skills are teachable and supportable (Brown & Palincsar, 1989; Schraw, 1999; Veenman, Elshout, & Busato, 1994) then instructors and learners need the efficient ways to communicate their main ideas more transparently and directly. In this paper we present an interface, named **Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)** to implement teaching and learning interface which integrates metacognition knowledge detecting and metacognition strategies supporting. The precision behaviors of **WBCIMI** are identified by well defined teaching programs, real teaching/learning actions and the detectable/reachable interactively information processes between instructors and learners.While learning is happen, the **WBCIMI** tries to maintain and transform the learning processes into a visible and manipulated objects for detecting, reasoning, training, and supporting learning processes. Furthermore, the WBCIMI build a specific knowledge framework to characterize metacognition knowledge, strategies to detect and map these cognition behavior, and then to support the guidance for learners to learn more fluently and correctly.

2. Knowledge Framework

Cornoldi and Vianello (1992) suggested that metacognition knowledge can be characterized with reference to a series of aspects such as its level of specificity, it include the range of application, the ease degree of access, whether it can be verbalized, visualized, and then manipulated. In WBCIMI, the knowledge of similar triangle is a framework that represents a set of concepts within similar triangle and a set of linkages between those concepts and solutions of problems. The concept framework refers to the objectives, and subordinate objectives to the learning objects and the relationships between concepts from one to another. And the represented framework of concepts

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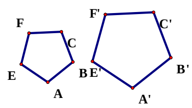
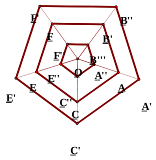
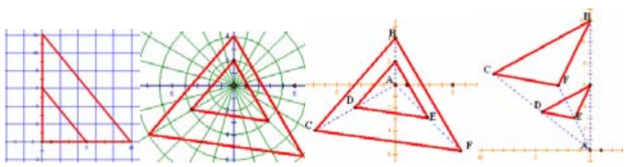
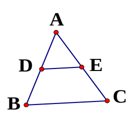
defines what can be expressed in the objectives represented, how it will be constructed and comprehended by learners. Table 1 illustrates the concept framework of similar triangles.

Table 1: Concepts framework of similar triangle

Teaching Stages	Learning concepts
Prior knowledge stage	To describe, identify, and comprehend the rates, ratios, and proportionality. To confirm, rebuild or recover the concepts of corresponding angle, vertices and side. The sum of all three angles of a triangle is 180∘. Equality of Cross Products: For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b}=\frac{c}{d}$ if and only if ad = bc. To recall similar polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are proportional. Similarity means that: (i) The two figures have congruent angles (ii) The sides are in proportion.  
Parallel lines intersect proportional segments	If two lines are parallel and they are intersected by another two lines then two sides of the two intersected lines separates these sides into segments of proportional lengths. CorollaryⅠ: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. CorollaryⅡ: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.
Define similarity	The mathematical definition for similar triangles is that they both have corresponding angles that are equal, while the lengths of the corresponding sides are in proportion. Similar triangles have the same shape, but the size may be different. "~" is "similar to". Examples ΔABC ~ ΔDEF (i.e. ΔABC is similar to ΔDEF)
Plot the similar triangle	Plot the similar triangles in different ways. An enlargement must have a center of enlargement and an enlargement factor. 
Parallel Lines interested the proportional segments of the triangle	To explain, proof, illustrate, and apply the follows: 

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	$If\ \ \overline{DE}\,//\,\overline{BC} \Rightarrow \dfrac{\overline{AD}}{\overline{DB}} = \dfrac{\overline{AE}}{\overline{EC}}\ \ (up\ \ vs.\ \ down)$ $\Rightarrow \dfrac{\overline{AD}}{\overline{AB}} = \dfrac{\overline{DE}}{\overline{BC}} = \dfrac{\overline{AE}}{\overline{AC}}\ \ (up\ \ vs.\ \ all)$ $\Rightarrow \dfrac{\overline{BD}}{\overline{AD}} = \dfrac{\overline{EC}}{\overline{AC}}\ \ (down\ \ vs.\ \ all)$
Similarity prosperities	(i) AA (angle-angle similarity) or AAA (3 angles): If two corresponding angles of one triangle are congruent then the two triangles are similar. The sum of all three angles of a triangle is 180◦. Therefore if two angles of two triangles are congruent, the third is automatically congruent. Therefore the sufficient condition requires only two angles to be congruent. In other words, if you know two of the angles of two triangles are congruent, then the third angles of the two triangles are congruent, and then two triangles are similar. (ii) SAS (side-included angle-side similarity): This one says that if you have two sides that are in proportion (that is, they reduce to the same fraction) and the corresponding included angles are congruent, then the triangles are similar. (iii) SSS (side-side-side similarity): This one says that if the three sides are in proportion (they reduce to the same fraction), then the triangles are similar.
Proportional related properties	If two triangles are similar then the proportions of corresponding sides will equal the proportions of corresponding perimeters, corresponding angle bisectors, corresponding medians, and corresponding heights.
Parallel Lines and Proportional Parts of Triangle	Triangle Proportionality: If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths. Theorem A: If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side. Theorem B: A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is one-half the length of the third side.

3. Cognitive pool

Owing to learning behavior is influenced by learners' viewpoints and believes, by the way in which learners interpret learning concepts and objectives by learners themselves. Accordingly, the WBCIMI tried to maintain and integrate the knowledge, misconception and metacognitive knowledge/strategies as the cognitive pool of similar triangles in order to set the related concepts and as knowledge base to detect the conceptions or misconception from learners themselves transparently and interactively. In other words, WBCIMI tried to figure out what learner thought about a specific problem via the selected terminology by learner themselves. After the terminology

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was selected by learner, then the system followed the factors to see the knowledge, metacognition or misconceptions that learners might have, and to direct learner to essential learning objects or hints which are the necessary concepts of similarities for learners to go further more confidently. The cognitive pool includes five components that are terminology, knowledge/concept hierarchy, problem spaces, common misconception, and linkages. The five components of cognitive pool illustrate in the following: 1) Terminology is a set of the properties, concepts, and postulates of similar triangles. 2) Knowledge/concept hierarchy is the overall knowledge and concepts in teaching/learning objectives. 3) Problem spaces are based in part on the concepts of similar triangle that is the fundamental process for attainment the solutions for difficult problems. Problem spaces were based on the three-level abstract/separate/deliberate control structure. And the problem spaces were separated by different subordinate objectives of similar triangles. 4) Common misconception is the faults, difficulties, or misconception from learners that was found by experience instructors. 5) Linkages are the relationships among the selected terminology, meant concepts and suitable directions to solve these misconception or problems. In other word, the linkages are the relationships among the terminology, knowledge/concept hierarchy, specific problem, common misconception, and guidance of the solutions

4. Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)

Initial metacognition detection: The purposes of web-based cognitive and interactive metacognition interface are transplanting problems with the related concepts and terminology to the web sites to enhance the accessibility, usability, comprehension, and application of similar triangles. However, the misconception and metacongition knowledge are not visible features for the system to acquire and enhance learning/teaching efficiency during learning/teaching processes. Furthermore, learning is a complex task; several different aspects must be taken into consideration. The WBCIMI built ways to visualize and manipulate misconceptions and metacognition knowledge during learning similar triangles. Figure 1 illustrates the structure of the initial metacognition detection with terminology selection.

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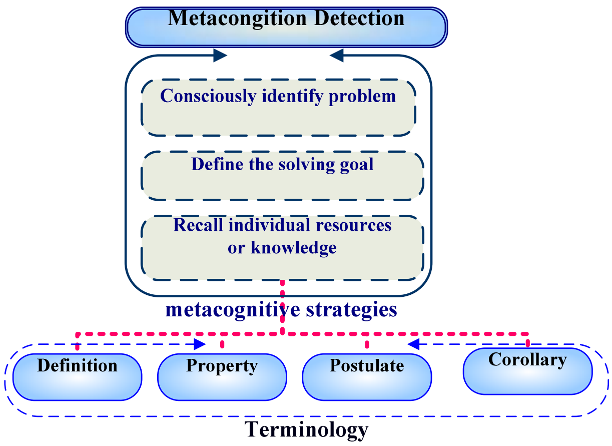

Figure 1: The initial metacognitive detection within terminology selection

WBCIMI showed the specific problems for learners to select the essential terminology, then to figure out the viewpoints, beliefs or intuitions of learner in the specific problems which were the metacognition knowledge or self-awareness via the individual terminology selections. By the way, WBCIMI could detect what ideas about pre-solving stage of the specific problem do learner bring to his work, and how does learner shape the concepts to do the problem. Substantially, right senses and concepts of what learner know or wrong ideas and concepts of what learner keep will be the bases to make sure the metal image or metacongition knowledge of learner. In deeply, learner's approach to a problem or understands of how to solve that problem are affected by the knowledge of learner which can realistically assess.

Training and supporting stage: After the initial metacognition detection, WBCIMI wants to know "what are the implications of the results?", "what are the assisted points to learners in this interface?", and "what are the learning objects which will adapt to train and support the learning situations?". In WBCIMI, the problems were sketched in a three-level abstract/separate/deliberate control structure. Each problem was abstracted the most important issue and essential concepts which related the problem and knowledge of similar triangles. These abstracted items were set to be the terminology pool. This is the fundamental level which might help to concentrate the focus of the specific problem. Next level is to separate the complex problem into many individual one for system and learner to make sure and commit the individual issue clearly. This is the second level for system and learner to identify the abilities of learners in each part problem solving. The final level, deliberate level, is to integrate the over all issues, guidance, hints, and related concepts of the original problem for learners to find the answer. In practice, learners can compare the selected terminologies which were chosen by learns themselves with the hints on the interface of WBCIMI system. While learners compared the same or different main terminology on the interface, they were forced to think over the related concepts and knowledge of similar triangles. In solving problems,

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learners will be trained and supported to solve the problem via the issue by issue individually.

Metacognitive Strategies of WBCIMI : The purposes of WBCIMI are to detect, commit, explore, apply and enhance the metacognitive capabilities of all learners via the solving processes of the similar triangles problems. After detecting metacognition of the specific problems, it refers to higher order thinking, learner involves thinking processes during learning similar triangles. Learning will be identified in the understanding, monitoring, comprehending, and evaluating progress toward the completion of the problem solving with metacognition. Because metacognition plays a critical role in the initial problem solving processes, the WBCIMI tried to connect the knowledge between cognitive aspects of similar triangles and the skill of problem solving to detect, train, and support the problem solving processes. Mayer (1998) proposed the skill (cognitive strategies), metaskill (metacognitive strategies), that will be adapted in different problem solving processes. At this level, WBCIMI identifies three strategies which are creating a clear mental representation of solving intentions, linking solving intention to a series of actions and finally monitoring learner's solving intentions in the face of obstacle and confusion solving intentions. They involve problem solving and decisions that help 1) to allocate resources, metacognition and misconceptions, to the current problem solving, 2) to determine the individual concepts and separate the solving steps to be taken to reach the problem solving, and 3) to set the intension or acknowledge the metacognition at which one should solve step by step.

WBCIMI Experiment

The participants of this study were 105 students who were second year junior high school students of ZCJH. One week prior to this study, the prior knowledge test of 105 was assessed by a series of ability tests, representing 20 questions of 8 primary objective of similar triangles (Prior knowledge stage, parallel lines intersect proportional segments, define similarity, plot the similar triangle, parallel Lines interested the proportional segments of the triangle, similarity prosperities, proportional related properties, parallel lines and proportional parts of triangle). The participants came from three classes that were taught by the same teacher. This study was conducted the coordinate planes teaching programs at the ZCJH, a junior high school. The mission statements of the final and subordinated objectives of similar triangles and well-defined learning actions are identified in teaching programs and WBCIMI refined and separated the teaching objectives into eight stages. The stages are the follows: 1) Prior knowledge stage; 2) parallel lines intersect proportional segments; 3) define similarity; 4) plot the similar triangle; 5) parallel Lines interested

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the proportional segments of the triangle; 6) similarity prosperities; 7) proportional related properties; 8) parallel lines and proportional parts of triangle; The experiment spanned 2 weeks, with one pretest, one posttest (fourth a week, 45 min each). One week before the formal experiment began; we gave the pretest to the learners in three classes (40 min) to test their abilities. In this quantitative study, 105 junior high students attended the WBCIMI experiment. The teaching program lasted two weeks from the pre-knowledge, content objects, and interactive active problem solving. Accordingly, the WBCIMI maintains the ways that learners may be more confident on the learning phenomena and the potential for meaningful review after the action itself during the interim steps. The means and standard deviations of learners' scores on the pretest, posttest-1 (not WBCIMI base), and posttest-2(WBCIMI base) are the follows: The independent sample t-test for pretest (mean value: 38.6) and post-test (mean value: 61.1) implies significant difference in pretest (0.00<0.05) and significant difference in post-test (mean value: 61.1) and posttest with WBCIMI (mean value: 68.9) (0.000<0.05). The results expressly show the model is available and effective in improving learning performance.

5. Conclusion

In this study, WBCIMI sought to propose a cognitive, interactive, and metacognition interface which identifies and describes the components of cognitive pool in learning similar triangles. Although WBCIMI involves general cognitive actions of mathematic comprehension and reasoning, the proposed interface provides an integral approach to solve similar triangles problems. WBCIMI attempted to (a) develop an initial framework for describing, predicting and monitoring how learner think in inductive similar triangles via cognitive pool, (b) refine and validate the framework using a test of inductive mathematics reasoning involving problems that require students to apply inductive processes. Thus, the focus of the study of WBCIMI was on the integration of various objectives similar triangles problem into classification scheme. Activated scheme is applied and detected to incoming information in an implicit manner, the WBCIMI will map the information to cognitive pool for more abilities to guide and assist learners to go further confidently. WBCIMI achieved the cognition by solving in problem spaces, and architecturally supports this by a flexible, three R-level recall-recognize-react control structure. In the interface design, this kind of terminology context was also particularly beneficial for those learners who displayed a more transformative orientation towards knowledge construction (i.e., who exhibited a strong drive to modify and construct knowledge of similar triangle in terminology). It follows that knowledge construction was enhanced by the framework of engagement (i.e., the study questions and hints via terminology), where participants worked to select the related terminology of the specific question towards the

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instrumental goal of solving specific problems in their practice. Finally, the functions of WBCIMI are 1) Reduce learners' cognitive load; 2) Identify and separate the problem solving in three levels: abstract, separate, and deliberate; 3) Making explicit the navigational structure in problem solving stages; 4) Providing learners with a navigational map to solve and cognize the problems; 5) Decease the level of disorientation learner's experience; 6) Decompose a problem to several subtask routines; 7) To detect and maintain a metacognition knowledge and strategies to assist learning. At the design of WBCIMI, learners may engage in further metacognitive monitoring and assisting about the utility of tactics and strategies to solve the similar triangles problems confidently.

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Exploring Ellipses by Shuttling Between the 2D & 3D Worlds

-Integrated Learning Function and Geometry to Foster "Function Sense"-

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Abstract: The ellipse often appears in astronomy and we can see a lot of ellipses in our daily lives. It has been studied for a long time by researchers, and has deep roots in the history of science. Although an ellipse is familiar to us, there are very few mathematics teaching materials about them nor are ellipses usually included in Japanese school curricula. It may be one of the reasons why Japanese teachers and students have not been able to draw and explore figures of ellipses and other conic sections easily, especially spatial ones. The development of dynamic geometry software (DGS), for example Cabri Geometry II Plus and Cabri3D[1], has helped the spatial study of ellipses. In our research, we developed many teaching materials for ellipses and we show them in an e-book (Kakihana, 2007). By using these materials, students are able to explore an ellipse spatially and foster a "function sense" (Kyoko Kakihana, Chieko Fukuda, & Katsuhiko Shimizu (2002); Fukuda, and Kakihana (2005), etc.) in an integrated learning environment for functions and geometry.

1. Background

An ellipse often appears in astronomy and it has been investigated by researchers for a long time and has deep roots in the history of science (Toyama, 1985). We can observe a lot of ellipses in daily life and these ellipses are very interesting subjects for exploration both in geometry and in functions. Although an ellipse is familiar to us, there are very few mathematics teaching materials about ellipses in school curricula. In Japanese curricula, an ellipse only appears in "expressions and curves" in the high school Mathematics Level C (Sugaku C) course. It is an optional subject therefore most students don't learn about ellipses. In this course, students learn the definition of an ellipse and how to express it algebraically. It is not so exciting. There are interesting machines to draw ellipses in science classes (Isoda, 2005) but these machines are not available for all students to use in exploring ellipses. Now, DGS Cabri Geometry II Plus and Cabri 3D help students draw ellipses while providing many chances to observe and explore ellipses spatially.

[1] Developed by CABRILOG

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The object of this research is to show the process of exploring ellipses by using DGS. At first, we take some ellipses from daily life and then simulate some heavenly body phenomena. Then, we verify whether these curves are real ellipses while also the determining the appropriate drawing method from each situation in which an ellipse appears. Finally, we will show some examples for further exploration.

2. The process of exploring ellipses

2.1 Step 1: Let's find some examples of ellipses from daily life

(1) When you shine a flash right on the ground, you can see an ellipse (Fig.1).
(2) When a plane cuts through a cone without going through the top, you can see an ellipse (Fig.2).
(3) When you cut a sausage, a Chinese radish (daikon), etc., you can see an ellipse (Fig.3).
(4) When you fill an ice cream cone with water and then tilt it, you can see an ellipse (Fig.4).

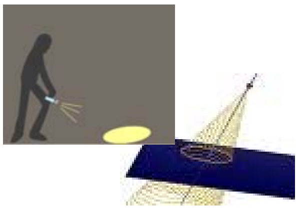
Fig.1

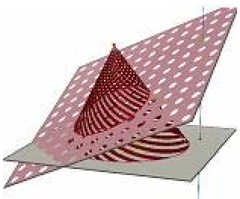
Fig.2

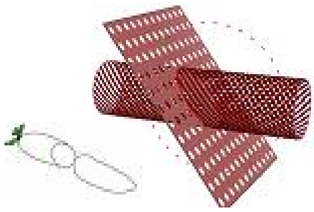
Fig.3

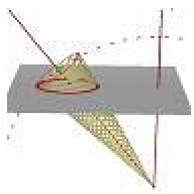
Fig.4

(5) When you fold a round paper to align a point Q (on the circumference) on the fixed point P in the circle and then move point Q along the circumference, you can see an ellipse (Fig. 5).
(6) When you compress a circular cylinder, you can see an ellipse (Fig.6).
(7) Push 2 pins into a flat surface. Tie the ends of a piece of string together and put it around the 2 pins. Pull the string tight with a pencil and you can draw an ellipse (Fig. 7).

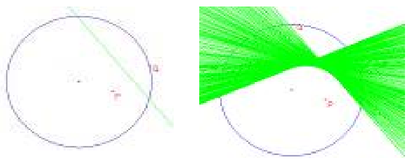
Fig.5


Fig.6

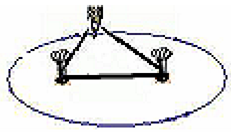
Fig.7

(8) The observed ellipse in a planet's ring such as the shape of Saturn's ring seen from the Earth (Fig.8) (Fukuda & Kakihana (2006))

(9) From the algebraic expression $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where is the length of a major axis and b is the length of a minor axis), you can draw an ellipse.

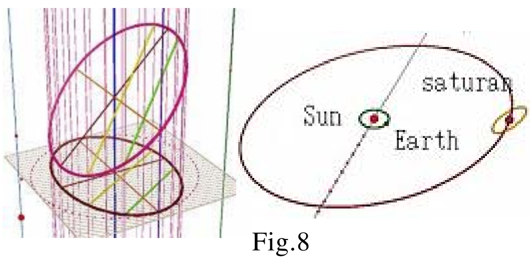

Fig.8

2.2 Step 2: Let's verify these shapes are any real ellipse from the definition of an ellipse.

(1) The definition of an ellipse

There are three definitions of an ellipse.

1) Definition 1 (Fig.9): An ellipse is the locus of a point P of a plane whose sum of

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distances from two fixed points F and F' is constant; that is, PF+PF'=2a.

2) Definition 2 (Fig.10): A possible shape derived from cutting the surface of a cone. This definition was shown by Apollonius in "Conics".

3) Definition 3 (Fig.11): There is a line L and a point F on a plane not on line L. When you put a point H on line L and move it, the locus of point P which is always PF:PH=e:1 is a

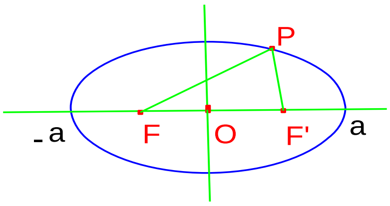

Fig.9

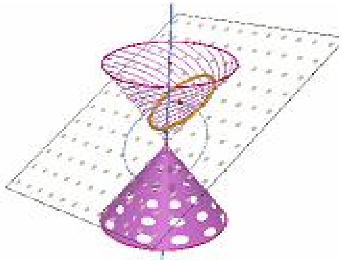
Fig.10

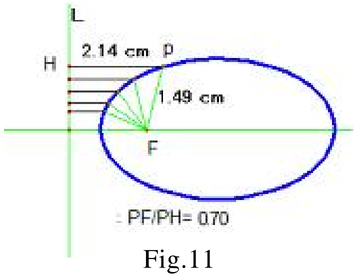

Fig.11

conic. (PH is the distance from line L to point P.) When e is less than 1, this conic is an ellipse. The proportion e of PF and PH is an eccentricity so line L is a directrix and fixed point F is one of the foci.

(2) Why the ellipses in Section 2.1 above are real ellipses.

Now, we will identify and explain each type of ellipse.

1) The reason for paragraphs (1) to (4):
When you move point P on figure 12, you find that the sum of the distance of PF1 and PF2 is always the length between the centers of the two spheres (Dandelin's Theory).

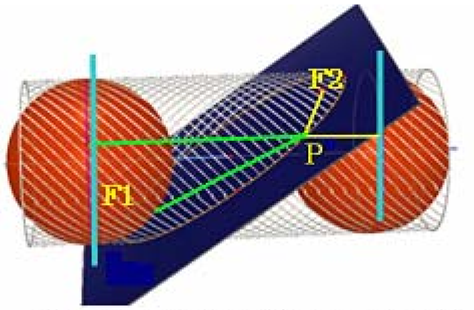

Fig. 12 :PF1+PF2=the distance between the centers of two spheres

2) The reason for (5), folding a round paper (Fig. 13).
Line m is a perpendicular bisector of PQ when Q is put on point P. Then, R is an intersection of line m and line segment OQ. OR+PR=OR+QR=OQ
The sum of OR and RP is always constant (the radius of the circle).

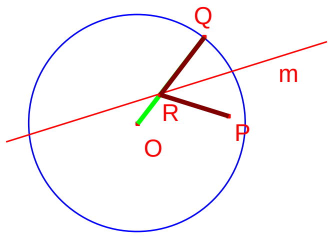

Fig.13 folding a circle paper

3) The reason for (6), you get an ellipse when you compress a circular cylinder (Fig.14).
We define each point as the following:
a: the length of the outside circle's radius
b: the length of the inside circle's radius
P: the point of compression
P'(t,0): an intersection of a perpendicular from point P to the x-axis.
Q: An intersection of line PP' and the circumscribed circle.

The coordinates of point Q are $(t, \sqrt{t^2 - a^2})$

The coordinates of point P are $(t, \frac{b}{a}\sqrt{t^2 - a^2})$ as P'Q : PP' = a : b

You eliminate t from $x = t, y = \frac{b}{a}\sqrt{t^2 - a^2}$, then you get the algebraic

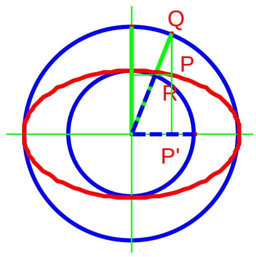

Fig.14 compressing a circular cylinder

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expression $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for point P.

4) The reason for (7) is simply the definition of an ellipse.

5) The reason for (9) is that this expression $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an ellipse (Fig.15).

Now, we define the coordinates of each point as shown in Fig.15.
Then, we think about the locus of point P which is given as PF+PF'=2a.

$$PF+PF'=\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$$
$$\sqrt{(x-c)^2+y^2}=t-\sqrt{(x+c)^2+y^2}$$
$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

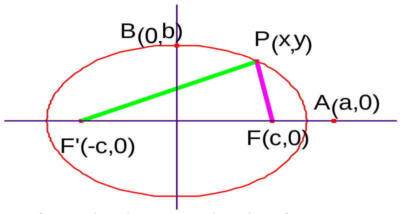


Fig.15 by an algebraic expression

In this expression, replace $\sqrt{a^2-c^2}$ with b, and you get $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Therefore, an expression for the locus of point P which is always PF+PF'=2a is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

2.3 Step 3: How to construct an ellipse

There are many ways to draw an ellipse. We will show four methods.

(1) Method 1 (by the definition)(Fig.16)

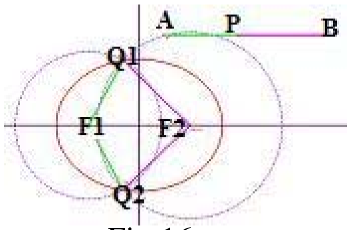
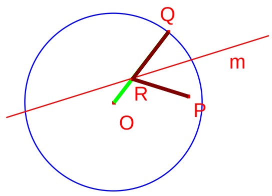


Fig.16

1) Draw a line segment AB. Put a point P on it.
2) Draw two points F1 and F2 (F1F2＜the length of AB）.
3) Draw a circle whose center is F1 and radius is AP
   Draw a circle whose center is F2 and radius is BP
4) Draw intersections of circles F1 and F2 and name them Q1 and Q2.
5) When you move point P on line AB, points Q1 and Q2 draw an ellipse.

(2) Method 2 (using an idea from origami)(Fig. 13)



The same as Fig.13

1) On a round piece of paper, mark the center as O and put a point P inside the circle.
2) Draw a point Q on the circumference of the circle.
3) Draw a perpendicular bisector m of points Q and P.
4) Draw an intersection R of OQ and line m.
5) When you move point Q, point R draws an ellipse.

(3) Method 3 (similar to compressing a circular cylinder) (Fig.17)

1) Draw two circles with the same center O. Their radii are OA for the outside circle and OB for the inside circle.
2) Draw a point P on the outside circle and draw a line segment

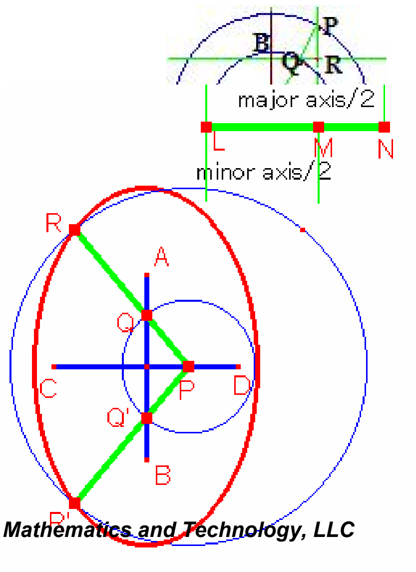


Fig. 18

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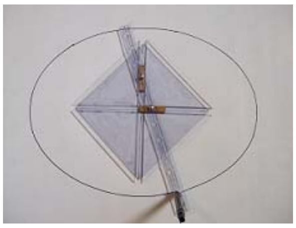
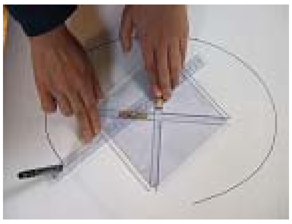
OP. Then, mark point Q at the intersection of OP and the circumference of the inside circle.

3) Draw two perpendicular lines from P to OA and a chord through Q to OB. Then, mark their intersection as R

4) When you move point P on the circle, point R draws an ellipse.





(4) Method 4 (similar to an ellipse compass as seen in the pictures right) (Fig. 18)

1) Draw two orthogonal line segments AB and CD which must bisect each other.
2) Draw a point P on line segment CD.
3) Draw a circle whose center is point P and whose radius is MN. MN is on the line segment LN. LN is a line segment to fix the length of PR.
4) Draw two points Q and Q' intersecting circle P and line segment AB.
5) Draw a circle whose center is P and whose radius is LM.
6) Draw an intersection R of extended PQ and the circle. In the same way, draw an intersection R' of extended PQ' and the circle.
7) When you move point P, points R and R' will draw an ellipse. If you move point M, you can change the ratio of the minor axis to the major axis.

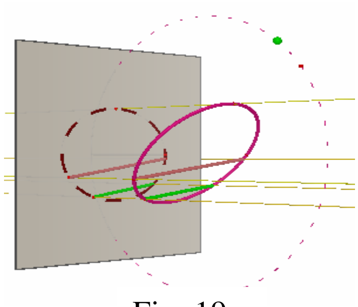


Fig. 19

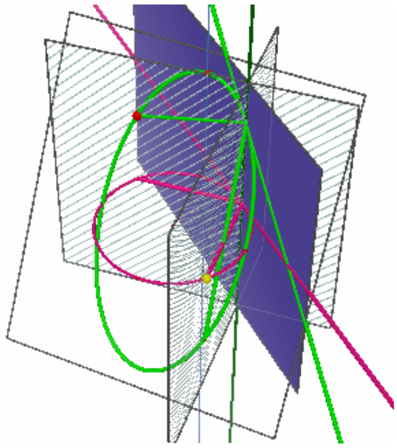


Fig. 20

2.4 Step 4: Further exploration

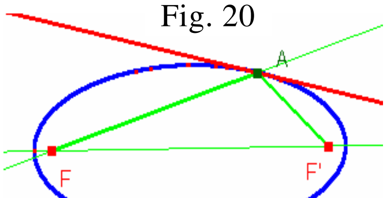
(1) Let's think about an ellipse is drawn as the shadow of a circle (Fig. 19).

a) Let's examine how to find the center of an ellipse by using a common characteristic between a circle and an ellipse.

1) Draw two parallel chords on a circle and draw a line through the midpoints of each chord. This line must be through the center of the circle.
2) In the same way, draw two other parallel chords on the circle and draw a line through the mid points of these chords.
3) The intersection of these lines is the center of this ellipse.

b) Let's see how to draw a tangent on an ellipse.

Look at figure 20. The rule that an angle of incidence is



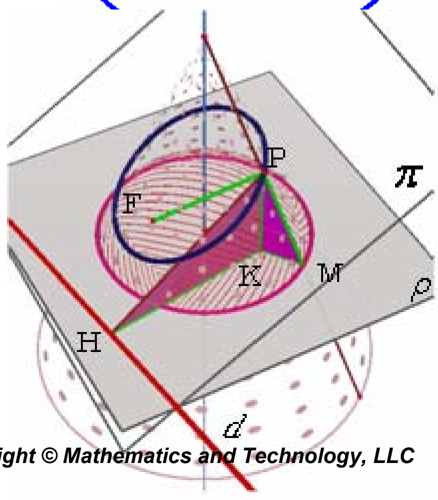


Fig. 22

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equal to an angle of reflection can be seen on the circle. The same rule applies to an ellipse. This rule gives us a method for drawing a tangent on an ellipse (Fig 21). A tangent is a bisector of exterior angle FAF'.

c) Let's find a directrix on an ellipse in 3 dimensions.

An ellipse is one of the cut planes of a cone.

In Figure 22,

α : an angle between the axis of the cone and the generatrix

β : an angle between a plane π and the axis

PM* $\cos\alpha$ = PF* $\cos\alpha$ =PK　　PF=PK/ $\cos\alpha$,

PH* $\cos\beta$ = PK　　PH=PK/ $\cos\beta$

PM=PF, then　e =PF/PH= $\cos\beta$ / $\cos\alpha$　(constant)

This is because an ellipse defined the locus of the point as PF:PH=e:1 （e (eccentricity) is constant）. You can find the directrix in this figure (line *d*) as the intersection of plane π and plane ρ. When β =90deg. that is, $\pi // \rho$, the directrix disappears and e is equal 0.

(2) Some construction problems.

a) The center of an ellipse is given, the minor and the major axes, the length axis, and the focus can be constructed (Fig. 23).

1) Draw a circle with a center O of this ellipse and intersections A, B, C, D of the ellipse and the circle.
2) Draw a line through the midpoint of AB and CD and intersection P1, P2 of this line and the ellipse. In the same way, you can draw points Q1 and Q2. The line segment P1P2 is the major axis and Q1Q2 is the minor axis.
3) Draw a circle whose center is a point Q2 and radius is half of the major axis.
   The intersection of this circle and the major axis are foci F1 and F2.

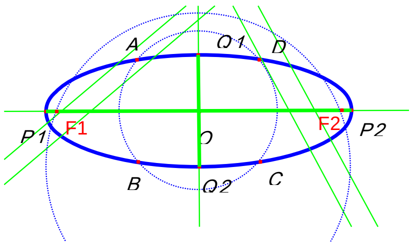

Fig. 23

b) If given two foci and a point A on the ellipse, an ellipse can be drawn (Fig. 24).

1) We draw a circle with radius FP.
2) Put point P' on the circle and draw a perpendicular bisector of P'F'.
3) the intersection with this line and P'F is point Q
4) The locus of point Q is an ellipse.

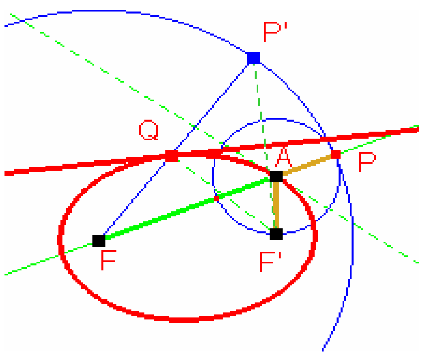

Fig. 24

(3) Let's find the shortest path.

Question: You are standing at point A in a round swimming pool.

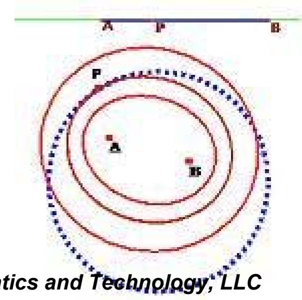

Fig. 25

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You want to swim to your friend at point B and, on the way, you would like to put your sunglasses on the side of the pool. Find the shortest path from A to B passing through point P at the poolside.

This is the swimming pool problem posed by Cuoco (1995).You can see that AP+PB is a function of P as P moves around the pool. To solve this optimization problem, you need to draw many contour lines for $AP + BP = 1, 2, \cdots, k, \cdots$, and find the shortest value of k. Each locus is an ellipse (Fig. 25).

3. Discussion and Results

The idea of a circular conic was discovered in ancient Greece where its basic features were studied. Later, Galileo showed that the locus of an emitter is a parabola. Johannes Kepler and Isaac Newton then realized that an ellipse and a hyperbolic curve were the paths of celestial spheres and comets. Ellipses have a long history. To understand the history of science, the idea of conics is very important. (Toyama, ibid.). At the same time, we can observe these functions as the shape of light cast on the ground by a flashlight, the shape of a fountain, etc. You can see many interesting conics problems in Wasan (Japanese ancient mathematics). In addition, since Cartesian coordinates were discovered, these functions have been studied as quadratic curves. There are many examples of ellipses in daily life and they make very interesting material for exploring. From one definition of an ellipse, we derive many methods for drawing an ellipse. Even when you move certain points on the figure, you could still find the characteristics of an ellipse in each case. These days, the interest in teaching spherical geometry – both as a subject and as means of interest in geometry has been growing (Elwyn, 1999). And you can see many web pages about ellipses and see a moving ellipse developed by Flash (software developed by Macromedia). But these materials are not available for students to explore by themselves. We have produced an electronic book which includes the process of exploring an ellipse by using Microsoft Word. We linked all figures produced by Cabri Geometry II Plus and Cabri 3D so that students can move each figure in the textbook. By moving a figure, students' ideas can be elicited and shuttled between a two dimensional and a three dimensional world. We found the following results through these activities:

1) Even though the starting points for exploring ellipses are different, that is, starting from analytic geometry, a cut plane of a circle cylinder or the locus point which has a constant sum of the distance from two fixed points, students will systematize the characters of an ellipse by relating each expression.
2) The idea of ellipses can be deepened and expanded by shuttling between two and three dimensional expressions. It is very difficult to understand the reason why the cut plane of a cylinder makes an ellipse or why Definition 2 of an ellipse is equal to Definition 3 (Figs. 10 and 11). However, when we think about them as we observe figures drawn by 3D software and manipulate them ourselves, we can understand them (Figs.12 and 22). If an

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ellipse casts the shadow of a circle (Figs. 19 and 20), we get an idea about how to construct the ellipse's center or its tangent. It is difficult for us to imagine a three dimensional figure. When we are confused about a 3D figure, we can gain an understanding by drawing a cut plane of the 3D figure on 2D software.

3) When you look at spatial figures drawn in a plane from one direction, you may not always understand. For example, it is very hard to visualize a spatial figure such as when drawing a tangent on a sphere. You can observe the construction of the tangent on the figure as it is drawn by 3D software and you will understand how to draw it.
4) Through this e-textbook, students actively experience geometry and geometric functions in an integrated learning environment. Students observe a function on a geometric figure and their "function sense" (Kakihana & Fukuda, 2004, 2005) is cultivated because they think about functions by manipulating a figure, observing it, and relating it to numeric data and drawings.

4. Conclusion

In this study, we showed that activities for exploring ellipses are exciting. As a case study, we will give these materials to students in a class and let them try these activities. We will then investigate how effective the materials are in fostering a spatial sense in our students.

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Mathematics Teaching and Learning with Technology

Part II

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How learning and teaching of Mathematics can be made interesting: a case study

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Abstract: In this paper, we estimate the true proportion of mathematics educators and teachers at undergraduate / post graduate level in Karachi, Pakistan making Math courses interesting. We use random sampling of 75 students of engineering and commerce studying in three different institutes/universities namely University of Karachi, Usman Institute of Technology (UIT) and Karachi institute of Economics & Technology (PAF-KIET). For developing a 95% confidence interval to estimate this true population proportion, we use normal distribution. Furthermore, we investigate with the help of students' responses obtained from the sample data how mathematics teachers at undergraduate/post graduate level make their courses interesting – by their dedication, by giving logical reasoning and concrete examples or by making complex mathematical methods accessible to students giving them know-how of mathematical software.

1. Introduction

The way students learn and the approaches they adopt to study do have a major influence on their achievement. There is a general agreement that there are two fundamental approaches to learning: deep and surface, as originated from the research initiated in '70s. Students who assume a deep approach to their learning are intrinsically motivated and search for meaning by integrating new information into existing knowledge while surface learners are extrinsically motivated and are more inclined towards the mere reproduction of facts and theories, as Dr. Zarrin Siddiqui says in [6]. Mostly all observers of the education process, whether they are scholars, policy – makers or institutional administrators, point to "teacher" quality as the most significant institutional determinant of academic success [1]. In this paper, we present a study made on mathematics teachers, teaching at undergraduate/post graduate level.

Mathematics is important for students of all disciplines since it equips them with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills and ability to think in abstract ways. Mathematics is a creative discipline [2] so its' learners and teachers also need to be creative.

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In this paper, we work in two dimensions: at first, we find an estimate of the true proportion of mathematics educators and teachers at undergraduate/post graduate level in Karachi, Pakistan making Math courses interesting. Secondly, we study how do math educators make their courses interesting and then we sum up our findings and give concluding remarks.

2. Working Methodology

Mainly, we use statistical tools from [3], Ms-Excel and Curve Expert to make our journey easy. For meeting our objectives, we randomly selected a sample of 75 under/post graduate students from three different institutes/universities in Karachi, Pakistan, namely, University of Karachi, Usman Institute of Technology (UIT) and Karachi Institute of Economics & Technology (PAF-KIET). We obtain students' responses with the help of a short and simple questionnaire. Each student independently completes the questionnaire, which is based on a Likert – scale [4] of 1 to 5.

After gathering data, we use z-distribution [3] to obtain a 95% confidence interval to estimate true population proportion of math educators/teachers making their courses interesting at under/post graduate level. For finding quantitatively, how math teachers make their courses interesting, we later use correlation and regression [3], F-statistic and t-statistic for their test of hypothesis.

2.1 Descriptive Statistics & Interval Estimate

One of the opinions, the questionnaire asked is, "Math – teachers make their course interesting". In response to this opinion, students circle their rating on a Likert-Scale [4] of 1 to 5, where "1" indicates strongly disagree; "2" disagree, "3" neither agree nor disagree, "4" agree and "5" strongly agree. Table 2.1.1 and figure 2.1.2 shows the distribution of students' ratings that our survey resulted in:

Table 2.1.1

Ratings	No. of Students
1	8
2	12
3	25
4	17
5	13
Total	75

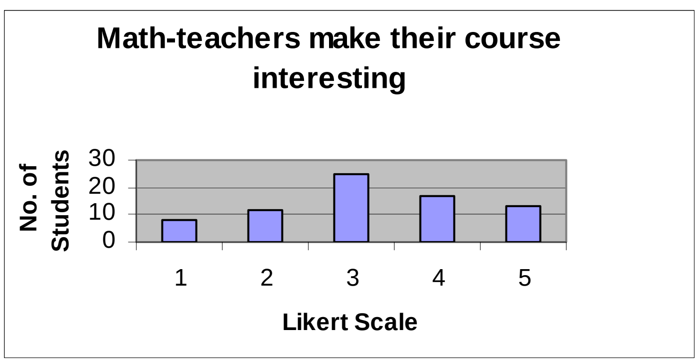

Figure 2.1.2

We define a random variable [3] Y as the rating of students for the said opinion and hence find that this variable is a Binomial Random Variable [3] to a reasonable degree. We define the responses 3, 4, 5 as

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our success and consider the students' responses 1, 2 as failure. According to this criteria, we find that the sample proportion of success and sample proportion of failure, denoted by $\hat{p}$ and $\hat{q}$ respectively, as

$$\hat{p} = \frac{55}{75} \Rightarrow \hat{p} = 0.733\overline{3}$$

$$\hat{q} = \frac{20}{75} \Rightarrow \hat{q} = 0.266\overline{6}$$

Using z-distribution and a 95% confidence interval, we find that the actual or true proportion and corresponding population percentage of all math-teachers making their course interesting at under/post graduate level in Karachi, Pakistan, denoted by p and π respectively, is

$$\begin{array}{l} 0.63325 < p < 0.833416 \\ OR \\ 63.33\% < \pi < 83.34\% \end{array} \qquad (2.1.3)$$

2.2 How Math -Teachers make their course interesting – Combined Effect

Our next and main target is to investigate how math-educators/teachers make their course interesting: By

(1) Dedication?
(2) Giving students logical reasoning and concrete examples? Or by
(3) Making complex math-methods accessible to students through giving them know-how of math-softwares?

For answering this investigation, we use correlation and regression analysis [3]. We assume

y as defined in Section 2.1
x_1 as average of students' ratings on math-teachers making their course interesting by their dedication
x_2 as average of students' ratings on math-teachers making their course interesting by giving students logical reasoning and concrete examples
x_3 as average of students' ratings on math-teachers making their course interesting by making complex math-methods accessible to students through giving them know-how of math-softwares

and now we present Table 2.2.1 as follows. Note that original data is not presented to reduce length of paper.

Table 2.2.1

Y	x_1	x_2	x_3
1	2.625	2	3.625
2	3.25	2.5	3.75
3	3.72	3.6	3.64
4	3.588235	4	3.470588

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5	3.538462	4.615385	4.076923

Using Ms-Excel we obtain regression statistics in Table 2.2.2, Table 2.2.3 and Table 2.2.3.

Table 2.2.2

Regression Statistics	
Multiple R	0.991654
R Square	0.983377
Standard Error	0.407715

Table 2.2.3

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-2.62039	4.810904	-0.54468	0.682488
X 1	-0.23112	0.891112	-0.25936	0.838444
X 2	1.500994	0.389265	3.855971	0.161541
X 3	0.370478	1.054801	0.35123	0.784969

Table 2.2.4

ANOVA					
	df	*SS*	*MS*	*F*	*P-value*
Regression	3	9.833768	3.277923	19.71898032	0.163704012
Residual	1	0.166232	0.166232		
Total	4	10			

Finally we have the regression equation as follows:

$$\hat{Y} = -2.62039 - 0.23112X_1 + 1.500994X_2 + 0.370478X_3 \qquad (2.2.5)$$

2.3 How Math – Teachers make their course interesting - Individual Effect

In this section, we present individual simple regression analysis, one by one for each factor of how math-courses are made interesting by teachers at under/post graduate level in Karachi, Pakistan, as it gives us a thoughtful result in the end.

2.3.1 By Dedication

Table 2.3.1.1 shows the first two columns of Table 2.2.1 while Figure 2.3.1.2 shows the scatter plot of the two variables. Then we show simple linear regression summary in Table 2.3.1.3 and Table 2.3.1.4 as follows.

Table 2.3.1.1

y_i	x_1
1	2.625
2	3.25
3	3.72
4	3.588235
5	3.538462

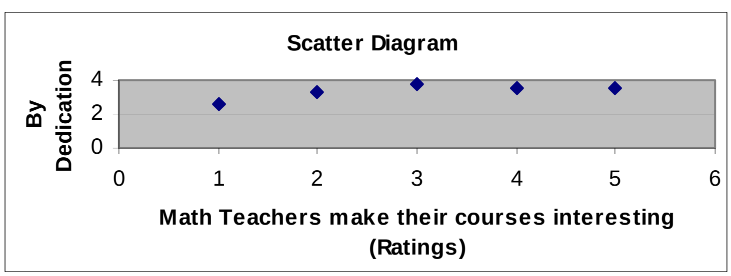

Figure 2.3.1.2

Table 2.3.1.3

Table 2.3.1.4

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	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-6.46987	4.372969	-1.47951	0.235559
X 1	2.83161	1.298725	2.1803	0.097316

Regression Statistics	
R	0.783
R Square	0.613088
Standard Error	1.135652

So the individual effect of dedication on math teaching can be reflected by the equation:

$$\hat{Y} = -6.46987 + 2.83161 X_1 \qquad (2.3.1.5)$$

2.3.2 By giving students logical reasoning and concrete examples

Table 2.3.2.1 shows the first and third column of Table 2.2.1 while Figure 2.3.2.2 shows the scatter plot of the two variables. Then we show simple linear regression summary in Table 2.3.2.3.

Table 2.3.2.1

y_i	x_2
1	2
2	2.5
3	3.6
4	4
5	4.615385

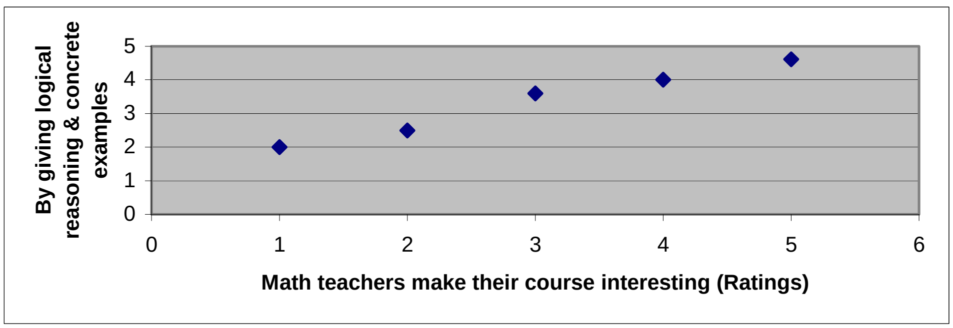

Figure 2.3.2.2

Table 2.3.2.3

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.85892	0.435085	-4.27256	0.023538
X 2	1.453429	0.125066	11.6213	0.001369

Regression Statistics	
R	0.989075
R Square	0.978269
Standard Error	0.269138

Hence the individual effect of math teachers giving logical reasoning and concrete examples to their student on math teaching can be reflected by the equation:

$$\hat{Y} = -1.85892 + 1.453429 X_2 \qquad (2.3.2.4)$$

2.3.3 By making complex math-methods accessible to students through giving them know-how of math-softwares

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Table 2.3.3.1 shows the first and last column of Table 2.2.1 while Figure 2.3.3.2 shows the scatter plot of the two variables. Then we show simple linear regression summary in Table 2.3.3.3.

Table 2.3.3.1

y_i	x_3
1	3.625
2	3.75
3	3.64
4	3.470588
5	4.076923

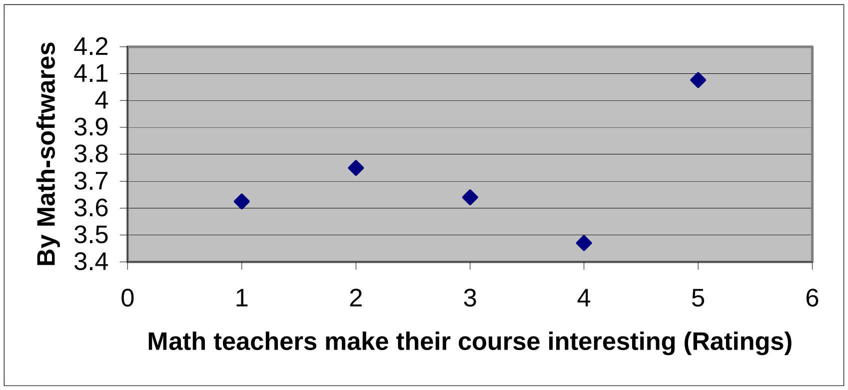

Figure 2.3.3.2

Table 2.3.3.3

Regression Statistics				
R	0.43544			
R Square	0.189608			
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-8.27294	13.47542	-0.61393	0.58268
X_3	3.036479	3.624337	0.837803	0.463636

The individual effect of math teachers making their course interesting by making complex math-methods accessible to students through giving them know-how of math-softwares on math teaching can be given by the linear equation:

$$\hat{Y} = -8.27294 + 3.036479 X_3 \qquad (2.3.3.4)$$

By looking at figure 2.3.2.2, we use 3rd degree polynomial in Curve Expert and obtain figure 2.3.3.5 as follows, which contains the standard error and correlation coefficient as well:

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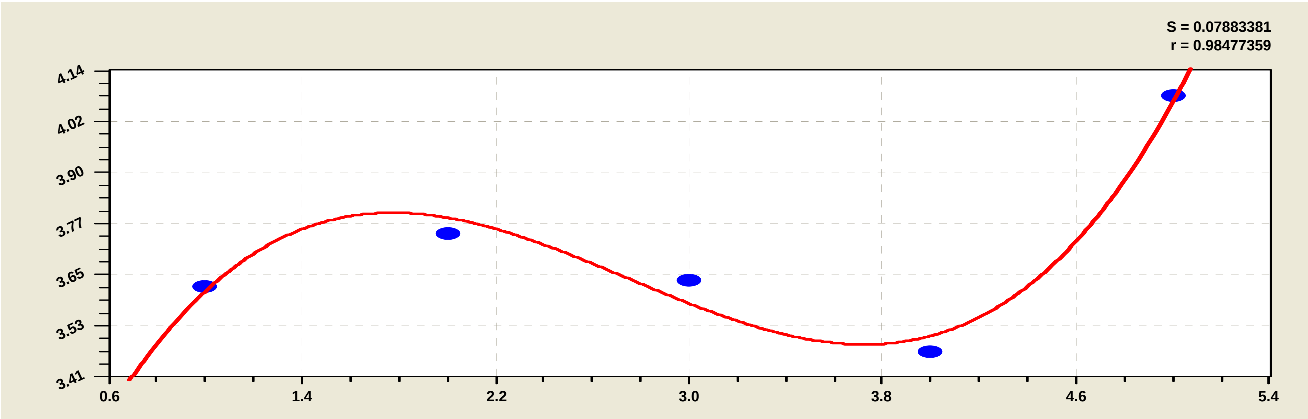

Figure 2.3.3.5

The cubic equation hence obtained is:

$$\hat{Y} = 2.5617552 + 1.6631353X_3 - 0.69354182{X_3}^2 + 0.084228917{X_3}^3 \qquad (2.3.3.6)$$

2.4 Suitability of Results and Interpretations

2.4.1 About Interval Estimate

Our interval estimate found in (2.1.3) satisfies all assumptions mentioned in [3]. This interval actually says that based on our sample results the probability, that the true population percentage of math teachers/educators making their course interesting at under/post graduation level in Karachi, Pakistan is in between 63.33% to 83.34%, is 0.95, with margin of error [3] of 10% approximately, which is indeed a good result.

2.4.2 About Models

2.4.2.1 Model (2.2.5)

We use p-value [3] stated in Table 2.2.4 and apply F-test [3] and hence conclude that the model is not suitable at a level of significance of 0.05, though multiple correlation coefficient is very close to 1.

2.4.2.2 Model (2.3.1.5)

We observe that the correlation coefficient between math teachers making their course interesting and achieving this goal by dedication is nearly 0.78 [2.3.1.4]. Using critical region [3] of $t > t_{0.1}$, that is, $t > 1.633$ [3], we conclude that the two variables are positively linearly related with each other. As dedication of a math teacher increases, he / she starts making his/her course more interesting and vice versa. This positive linear relationship is good since approximately 61.31% [2.3.1.4] variations in math teaching are explained by math teachers' dedication.

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2.4.2.2 Model (2.3.2.4)

We observe that the correlation coefficient between math teachers making their course interesting and achieving this valuable target by giving their students logical reasoning and concrete examples is nearly 0.989 [2.3.2.3]. Using critical region [3] of $t > t_{0.5}$, that is, $t > 2.353$ [3] or using p-value [2.3.2.3], we conclude that the two variables are positively linearly related with each other. As math teacher explains with logical reasoning and concrete examples more, he / she starts making his/her course more and more interesting and vise versa. Further, this positive linear relationship is very strong and almost perfect since approximately 97.83% [2.3.2.3] variations in math teaching are explained by math teachers' logical reasoning and giving concrete examples, which shows the suitability of the model very clearly.

2.4.2.3 Model (2.3.3.4)

We observe that the correlation coefficient between math teachers making their course interesting and achieving this target by making students aware of the math softwares is nearly 0.4354 [2.3.3.3]. Using critical region [3] of $t > t_{0.1}$, that is, $t > 1.633$ [3] or using p-value [2.3.3.3], we conclude that the two variables are not linearly related with each other. As only 18.96% [2.3.3.3] variations in math teaching are explained by their giving their students know-how of mathematics. Therefore this model is not suitable for explaining variations in math teaching by making use of math softwares.

2.4.2.4 Model (2.3.3.6)

From Curve Expert, we find that the correlation coefficient between math teachers making their course interesting and achieving this target by making students aware of the math softwares is nearly 0.985[2.3.3.5]. By letting $(X_3)^2 = Y_1$ and $(X_3)^3 = Y_2$ [3] and using F-test for multiple regression, we conclude at 0.01 level of significance that the two variables, math teachers making their course interesting and achieving this target by making students aware of the math softwares, are strongly correlated with each other. This cubic relationship is a very strong and good relationship and hence this is a very suitable model for explaining variation in math teaching by making use of math softwares.

3. Our Findings

Our inference from sample results and survey shows that

(1) About 63% to 83% math teachers/educators make their course interesting in Karachi at under/post graduation level though no lab sessions, Math-Education workshops/programmes or academic assistants are given to them for their support by universities/institutes as compared to faculty members of other disciplines like Engineering, Commerce or Computer Science Departments. This indicates whatever hard work they do and however they struggle is all their own effort. Because of the capabilities and abilities that mathematics has blessed them with, they are quick learners and without any such support, they try to do their job well.

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(2) Although model (2.2.5) is not suitable [2.4.2.1] but we would like to comment what it still shows about the overall effect of the factors influencing math teaching that if a math teacher is neither dedicated at all, nor he/she gives logical reasoning / concrete examples to the student and nor give the students know-how of math softwares in order to make complex math methods accessible to students, then such a math teacher/ educator is a total failure in his field. In making his/her course interesting he will have to spend time to explain students logically, give good and concrete examples and use math softwares. In fact explaining to the students with logical reasoning and showing them examples increases students' ability more as compared to using math softwares and this finding of ours matches with what Madeleine Chowdhury says that a student capable of visualizing a problem situation is far better off than a student utilizing a calculator or computer to do the visualization [5]. Technology actually assists a student in sharpening students' visualization skills but it is not a substitute to analytic thinking.

(3) Talking about individual effects, we again notice from model (2.3.1.5), (2.3.2.4), (2.3.3.6) and (2.3.3.4) that all three factors play a positive role in development of students' understanding in mathematics. The greater the math teacher is dedicated to his/her subject, the greater he/she is successful in making his/her course interesting. The greater the math teachers' ability to transfer his/her knowledge through logical reasoning and visualizing the problem by giving concrete examples, the greater success he/she achieves in math teaching. The more the math teacher updates the students with math softwares, the more he/she is successful in increasing students' understanding. But again we would like to remark that the ability of a math educator to enhance thinking skill in a student is the most significant tool/factor in success of his/her teaching skills.

(4) According to model (2.3.3.6), making use of math softwares is really good for such students who feel mathematics is entirely uninteresting or boring course (surface learners [6]) or for such students who love doing mathematics and whose understanding in the subject is very high (deep learners [6].

4. Concluding Remarks

We remark in the end that math teaching can be made interesting by all three factors: dedication, giving logical reasoning and concrete examples and giving know-how of technology in terms of math softwares to students. But no doubt that technology and math softwares do assist to learners of mathematics of all types, whether surface learners or deep learners but it is the responsibility of the math educators to understand differences in students' learning and necessarily equip them accordingly with visualization of abstractness in mathematics by means of good reasoning, examples and graphics.

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Mathematics Problems and Real Life Scenarios

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Abstract: Creating problems that demonstrate the power and relevance of mathematics has always been a time consuming challenge. Here we demonstrate that the development of such problems can be efficiently aided with the use of video sharing websites, which require students to extract their own data to understand and solve problems based on real-life scenarios.

1. Introduction

The creative design of mathematics problems that demonstrate the power and relevance of mathematics can be very time consuming and difficult. Ideally, a good problem will motivate students to see how mathematics can be used to solve a problem and hence help to develop an appreciation of the power of mathematics. A good problem should therefore at least manifestly motivate the use of mathematics as well as allow the student to extract relevant and meaningful data to be used in the context of some theory or topic. This would greatly help to clarify to students the relevance of a mathematical topic as well as develop the art of seeing what is needed (and not needed) in order to carry out some required analysis.

There are a number of efficient ways to construct interactive content for use over the web [1, 2] using various practical tools. However, there are very few tools to help with the development of interesting problem sets or tools to help with the development of problems based on real-life scenarios.

Popular video repositories, such as YouTube [3] and MySpace [4], contain a vast amount of video resources that can be exploited to develop interesting educational resources and assessment sets. Video clips of films, songs and many other forms of popular culture can be readily linked to and used to form the basis of some topic and/or analysis. The novelty here is that, at first sight, many of the video resources seem to have little to do with learning but, nonetheless, can be exploited to demonstrate the real-world relevance of a topic, as well as be the basis of some formal analysis.

In this paper, we show how a new fully on-line mathematics course exploits video sharing websites to illustrate a number of elementary and important mathematical concepts.

2. On-Line Mathematics Course

From January 2007, SIM University students are required to take 10 credit units of a university core (130 credit units comprise an ordinary degree and 170 a honours degree). Of the 10 credit units, a 3-credit unit course is a fully on-line mathematics course “Thinking with Mathematics”.

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Thinking with Mathematics is a fully on-line course that reviews a number of important and essential mathematical topics. The course starts with gentle reminders of a number of fundamental principles and reinforces such principles by applying them to solve a variety of problems. The course will be particularly useful to students who have not studied a quantitative subject for some time, and is an ideal reminder of a number of core mathematical skills that students will find useful both in life and in their further studies at UniSIM. Note that students require access to the Internet in order to take the course.

The core of the course is a collection of on-line videos that carefully reviews and reminds students of the following 14 topics:

1. **Order of Arithmetic Operations:** Students will understand the correct order of arithmetic operations required for numerical calculations.

2. **Algebra and Equations:** Students are reminded of some standard simplification techniques and the solution of simple equations.

3. **Fractions:** Students will develop an understanding of how to simplify and combine fractions, as well as appreciating the application of fractions for solving problems.

4. **Powers:** Students will be taught to raise various numbers and mathematical expressions to an integer power, and understand what happens to negative numbers when raised to some even or odd power.

5. **Percentage**: Students will develop a firm understanding of the meaning of percentage and be able to apply percentages to solve a variety of problems, as well as using percentages to describe proportions and understand their relationship with fractions.

6. **Ratio and Proportion:** The basic ideas of ratios and proportions are fully discussed and students are exposed to a variety of related problem. Students will then develop a firm and clear understanding and how to describe and problems related to ratios and proportions.

7. **Averages:** The mean, median and mode of a set of data values are reviewed and applied to various data sets. Students will develop a firm understanding of the various measure of central tendency and learn to apply them with confidence.

8. **Graphs:** Graphs and charts come in various forms and students are exposed to a variety of ways for expressing information. Students then develop an appreciation in displaying information in efficient and different but equivalent forms.

9. **Area and Volume:** Students are reminded of how to calculate the area of a number of basic areas; such as a square, rectangle, triangle and circle. Students then develop an understanding of the calculation of more complicated areas in terms of simpler areas.

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10. **Patterns:** An important theme within all branches of mathematics is to see number patterns and to generalise. Students will develop some basic number pattern skills, which will help to develop their inductive reasoning.

11. **Substitution:** The important process of mathematical substitution is developed here, giving students important skills in using various kinds of formulae.

12. **Scientific Notation:** To develop a basic understanding of science and engineering, students need to master expressing very large and very small numbers in a convenient and efficient way. Students will learn how to express such numbers in a convenient and commonly used way, and also how to combine numbers expressed in scientific notation.

13. **Estimation:** Make sensible estimates based on uncertain data

14. **Problems and Review:** A number of more challenging problems are reviewed showing students that such problems can be solved using basic techniques. Students will then appreciate that many seemingly complicated problems can be solved using basic but important techniques.

Each of the 14 topics is carefully reviewed in great detail starting from very basic ideas and hence no technical prior knowledge is assumed. The videos can be used as both an introduction as well as reminders of the 13 topics.

Following each review video, there are additional detailed videos that consider problems that go beyond the basics, as well as many practice problems where students can practice their understanding and check their answers.

3. Course Links

Thinking with Mathematics comes with many detailed video-based worked examples and problem sets. To help show the real world relevance of many topics, we felt it would be ideal to have some activities that require students to extract their own data to solve problems.

The well known, and much talked about site YouTube [3] has a vast and remarkable repository of video resources, which can be exploited to motivate students and to help with the development of interesting real-world examples and problem sets.

Thinking with Mathematics uses a number of links to YouTube, which students need to study closely in order to extract relevant data and information to solve problems. A number of specific themes, topics and example include:

- The average speed of cars [5]
- The average of a set of numbers
- Checking quantitative statements made by popular songs
- Time and motion analysis
- Estimation

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- Checking the real-world possibility of film scenes.

The examples mentioned exploit YouTube video clips, which are part of popular culture and hence help to show the real-world relevance of a topic. For example the link to the YouTube video [5] requires students to extract both distance travelled and time intervals of three cars and compare their average speed. The video is fun to watch, and in some sense is better than actually being at the race track since the video gives an all round view of the race, which is not possible if you were actually located at the race track!

An interesting variation of this approach can be found in [6]

4. Stability of Hyperlinks

There are some obvious concerns with regard to relying on external repositories; the most obvious is the removal of videos. However, experience has shown that most removals are quickly restored!

A much more worrying prospect is that hyperlinks may soon require permission. A Texas judge [7] has made a ruling that linking to a Web page violates copyright. Declan McCullagh News.Com reports that SFX Motor Sports has won a case against Robert Davis, the operator of Supercrosslive.com, a Web site, which linked directly to audio files for motorcycle races on SFX's Web site.

Web sites often link to other sites, no permission is required and no laws are infringed. However, a 2000 case confirmed the legality of "deep linking" into any page of a Web. This is worrying since if you take away the ability to link then you have taken away the fundamentals and full potential that makes the Web work.

SFX claimed that Supercrosslive.com's direct links to audio files on its website threatened its business, because listeners could bypass the advertisements on SFX's website. It is argued, however, that if SFX intended to prevent people from going directly to audio files, a small change in its website server configuration could have stopped them from doing so.

Clearly, if files had been copied and reposted then SFX would have an open-and-shut copyright case, but many don't agree where the copyright law applies in this particular case.

The judge's decision is currently under appeal with the large web players such as Google and Yahoo watching the case closely as the final outcome of the case will effect how the web can be used. With regard to educational repositories, or sites that can be exploited as such, the outcome will have serious legal consequences and the use of hyperlinks will need to be used with greater care. However, copyright issues may vary from country to country!

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5. Conclusion

It has been shown, in particular, that a fully on-line mathematics course can exploit popular video sharing repositories to create interesting learning resources that help to show the real-world relevance of a mathematical topic. The problems can not only be fun but also allow students to extract relevant information or data to solve problems without much additional effort required by the instructor or course developer.

References to popular culture or popular video repositories can help to motivate students to learn what they may have previously perceived as being a remote uninteresting topic. Links to video clips based on popular culture help to bring topics alive and manifestly demonstrates the real-world relevance of mathematics in all walks of life.

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The Dynamic Geometry Software as an Effective Learning and Teaching Tool

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Abstract: This article describes how the use of dynamic geometry software has helped preservice teachers develop their abilities in three aspects: 1) challenging problem solving; 2) mathematical modeling; and 3) constructing student-centered teaching projects. The examples given indicate that for some of the challenging problems that are presented to students, it is almost impossible or very hard to manually make correct drawings. To overcome this difficulty, the use of dynamic geometry software seems to be critical, or at least very desirable. In addition, the use of the software can stimulate students' insight of problem solving and provide an easy and convincing way of verifying the solution. Moreover, students can construct accurate visual representations to model real world situations very efficiently by using transformations in dynamic geometry software. This can save time significantly so that students can concentrate on more conceptual oriented tasks. Good teaching projects that take advantage of dynamic geometry software can also effectively enhance school children's mathematics learning.

The Math Education program at New York University (NYU) offers *a Technology in Mathematics Learning and Teaching* course. The primary purpose of this course is to enable middle and high school preservice teachers to experience learning and teaching mathematics with technology. A secondary purpose is the review and enhancement of the subject area knowledge in the algebra, geometry, trigonometry, probability and statistics areas. Students entering the math education program come with diverse backgrounds and understandings, including substantial knowledge and understanding in some areas of subjects and severe deficiencies and misconceptions in others. Overcoming the difficulties and building upon strengths are essential to maximal learning and achievement of expected outcomes in the program.

This course focuses on computers as a primary technology and the most beneficial software in mathematics education. Because geometry has been a week spot in school mathematics teaching, and also because the Geometers' Sketchpad (GSP) is one of the most excellent and powerful mathematics software packages, the use of GSP in learning and teaching geometry is a major part of the course, and one of the objectives of this course is to improve pre-service teachers' mathematical reasoning and proof abilities. More specifically, the course aims at increasing pre-service teachers' geometric thinking by one van Hiele level (van Hiele, 1986). According to van Hiele, students progress in geometric thinking through a taxonomy of levels, and progress from one level to the next higher level is dependent on the nature of the instruction provided to students.

Many authors (see [1], [2], [4], [6], and [12]) have written articles to show how the use of GSP has facilitated students' geometry learning, and eased the burden and increased the joy of teaching geometry. Consistent with these authors, this article will mainly describe how the Technology course at NYU has helped students develop their abilities in three aspects: 1) challenging problem solving; 2) mathematical modeling; and 3) constructing student-centered teaching projects that take advantage of the use of GSP.

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1. The Use of GSP and Challenging Problem Solving

According to the NCTM Professional Standards (see [10]), in order to develop students' problem solving and mathematical reasoning abilities, the teacher should select worthwhile mathematical tasks for lessons. These tasks must engage students' intellect, develop students' mathematical understanding and skills, and stimulate students to make connections and develop a coherent framework for mathematical ideas. As the instructor of the Technology course for preservice teachers at NYU, I (the author) tried to present such worthwhile tasks, which are increasingly challenging problem solving and mathematical reasoning endeavors in the GSP environment. (For clarity, in this article, preservice teachers are referred to as "students", and secondary school students are referred to as "children".)

Promoting Visual Representation

Making a drawing (visual representation) is an important problem solving strategy, especially for geometry problems. Without a drawing (more accurately, a correct drawing), it is very hard, if not impossible, to solve a geometry problem. (A correct drawing refers to one that may or may not be exactly accurate, but close enough to the accurate construction so that it correctly represents the mathematical relationship(s) described in the related problem and therefore can help stimulate students' insight into the problem.) Sometimes, however, for a geometry problem, especially a challenging geometry problem, it is not easy to make or construct a correct drawing. The following problem is an example.

Problem: Give equilateral triangle ABC with an interior point P, such that $\overline{AP}^2 + \overline{BP}^2 = \overline{CP}^2$, *and with an exterior point Q such that* $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$, *where points C, P, Q are on a line. Find the lengths of* $\overline{AQ}$ *and* $\overline{BQ}$ *if* $\overline{AP} = \sqrt{21}$ *and* $\overline{BP} = \sqrt{28}$ (see [8]).

When students began to make a drawing for this problem, most of their drawings were similar to what is shown in Figure 1. However, more serious thinking led students realize that their drawings were far from "correct". While exploring the solution to this problem, students tried different ways to approach it. A few students did think of rotating quadrilateral APBQ -60˚ around point B, resulting in the drawing shown in Figure 2.

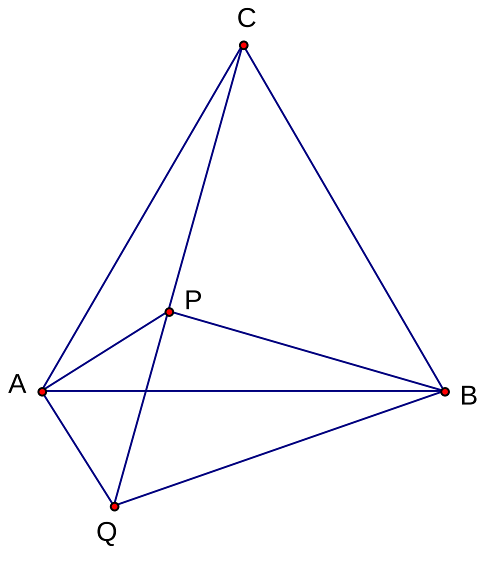

Figure 1. Incorrect drawing

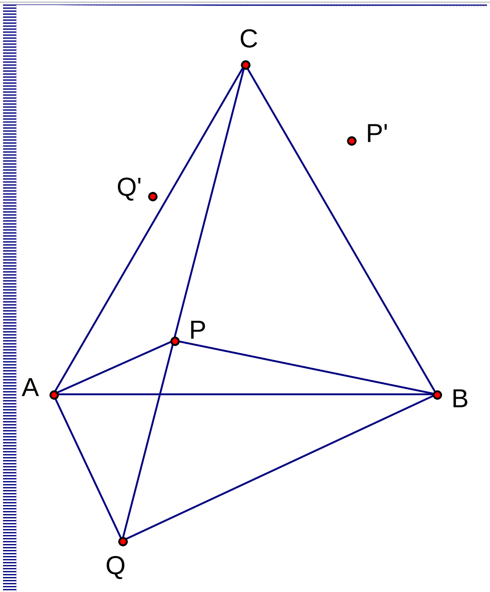

Figure 2. The drawing after a rotation

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Based on the given condition $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$ and some logical reasoning, students realized that ∠CQ'Q should be 90˚, but visually ∠CQ'Q is far from a right angle. This huge inconsistency alerted students that a more accurate drawing was necessary. However, no matter how much they tried, they found it was difficult to manually make a correct drawing for this problem.

This difficulty can be easily overcome by utilizing GSP. Students could display the values of $\overline{AP}^2 + \overline{BP}^2 - \overline{CP}^2$ (v1) and $\overline{AQ}^2 + \overline{BQ}^2 - \overline{CQ}^2$ (v2) by using the "Length" and "calculate" functions in the "Measure" pull down menu. Then they could drag points P and Q respectively until both v1 and v2 were very close to 0. A resulted drawing is shown in Figure 3. It is easy to see the sharp contrast between the incorrect drawing (Figure 1) and the drawing made with GSP (Figure 3). The contrast indicates how misleading an incorrect drawing could be. Due to intuition, however, locating Point Q (that is actually so distant away from both point C and Point P) correctly without using the dynamic geometry software would seem very difficult, if not impossible.

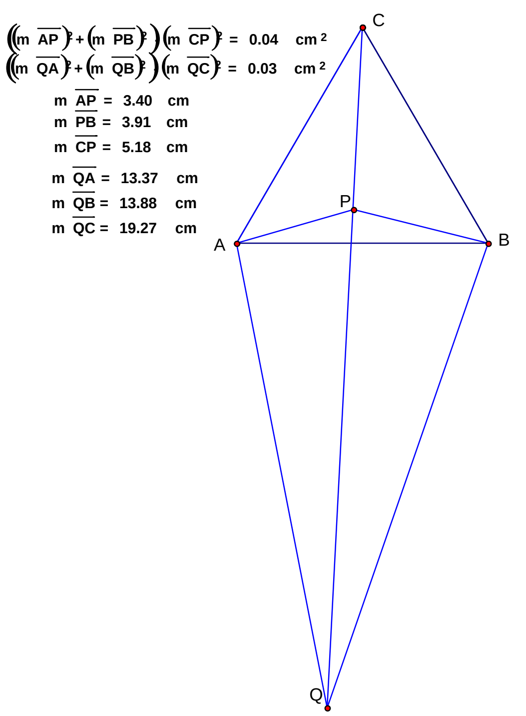

Figure 3. A correct drawing

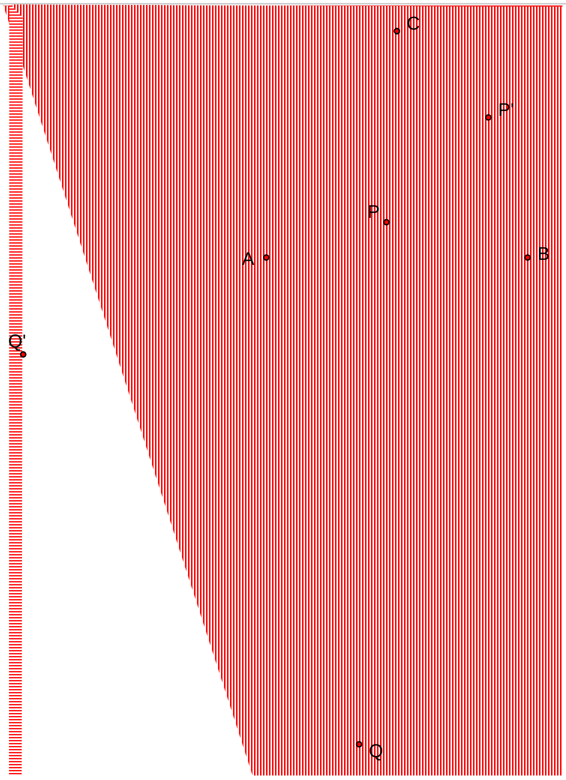

Figure 4. The drawing after a rotation

While most students could make a correct drawing in GSP, a few stronger students, who had gained a sound understanding of the relationships embedded in this problem, were able to even make exactly accurate drawings (constructions).

Stimulating Insights for the Solution

The most important role that GSP plays is to facilitate students' thinking. When students work on a problem, they usually would try different methods to approach the problem. The GSP dynamic movement and measurement features can help students to either confirm a method so that

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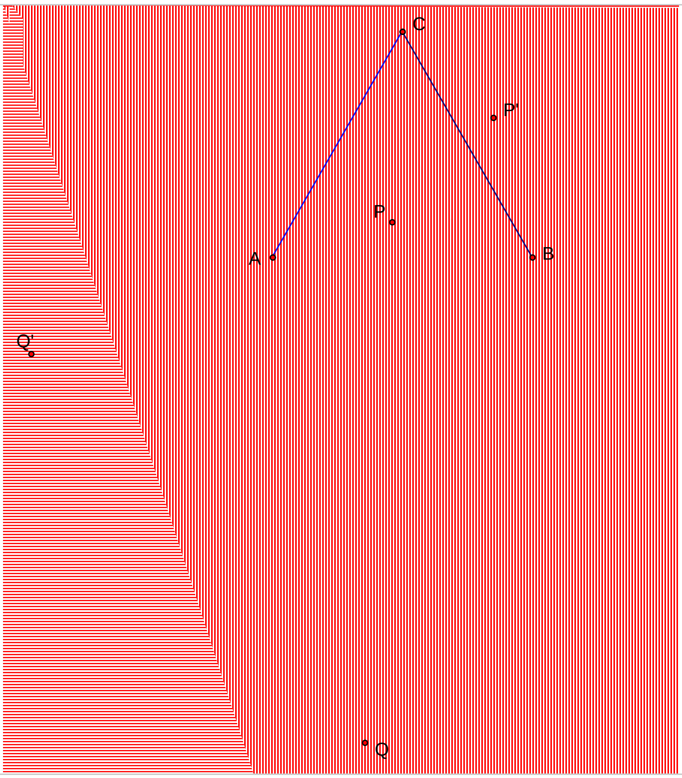
students know continuing to use it will most possibly achieve the solution, or provide contradicting feedback that either reveal the infeasibility of a method or stimulate a new angle of thinking.

In the problem we just discussed, for example, some students thought that since points C, P, and Q are collinear and there are quite a few triangles in the figure, there might be one or more pairs of similar triangles. They first guessed that ΔCPA and ΔCAQ are similar. However, when they measured ∠CAP and ∠CQA, they found the two angles were not congruent at all. They then checked other pairs of triangles and got similar results. So they learned that trying to find some numerical relationships through using similar triangles was not feasible. So they moved on by trying different approaches. A few students thought of the numerical relationships derived from the two given conditions $\overline{AP}^2 + \overline{BP}^2 = \overline{CP}^2$ and $\overline{AQ}^2 + \overline{BQ}^2 = \overline{CQ}^2$. They reasoned that this had to do with the Pythagorean Theorem and right triangles. There were no right triangles in the current figure and so auxiliary lines should be constructed to form right triangles. They also thought of the given original equilateral ΔABC, and reasoned that 60˚ angles should be considered. Thus, they made -60˚ rotations of ΔPAB and ΔQAB around point B, and quickly found newly formed right triangles (see Figure 4). They were very excited about the discovery, and knew they were on the right track. This new approach can be found without the software, but it would be much more difficult and time consuming.

The following shows a complete solution that one of the students worked out (with minor help from the author):



Solution:

Rotate quadrilateral PAQB -60^o around point B. Because ΔABC is an equilateral triangle, we have quadrilateral P'CQ'B, with ΔCP'B as the rotation image of ΔAPB and ΔCQ'B as the image of ΔAQB.

Construct segment $\overline{PP'}$. Since $\overline{BP'} = \overline{BP}$ and ∠PBP' = 60^o, ΔBPP' is an equilateral triangle. So $\overline{PP'} = \overline{BP} = \sqrt{28}$. We have $\overline{CP'} = \overline{AP} = \sqrt{21}$, so $\overline{CP}^2 = \overline{AP}^2 + \overline{BP}^2 = \overline{CP'}^2 + \overline{PP'}^2 = 21 + 28 = 49$. Therefore ΔCPP' is a right triangle by the converse of the Pythagorean Theorem, and ∠CP'P = 90^o. Hence ∠CP'B = ∠CP'P + ∠PP'B = 150^o. This implies that ∠APB = 150^o. With a similar reasoning, we can prove that ∠CQ'B = 30^o, which implies that ∠AQB = 30^o. Therefore PAQB is a cyclic quadrilateral, as its opposite angles are supplementary.

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In ΔAQP*, by the Sine Theorem,* $\dfrac{\overline{AQ}}{\sin(<APQ)} = \dfrac{\overline{AP}}{\sin(<AQP)}$. *But* $\angle AQP = \angle ABP$ *because they are the inscribed angles on the same arc AP, so* $\dfrac{\overline{AQ}}{\sin(<APQ)} = \dfrac{\overline{AP}}{\sin(<ABP)}$ ==> $\overline{AQ} = \dfrac{\overline{AP}}{\sin(<ABP)} \bullet \sin(\angle APQ)$.

Now let's find $\dfrac{\overline{AP}}{\sin(<ABP)}$ *and* $\sin(\angle APQ)$. *In* ΔAPB*, by the Cosine Theorem,* $\overline{AB}^2 = \overline{AP}^2 + \overline{BP}^2 - 2\bullet\overline{AP}\bullet\overline{BP}\bullet\cos(150°) = 21 + 28 - 2\sqrt{21}\bullet\sqrt{28}\bullet(-\dfrac{\sqrt{3}}{2}) = 91$, *and so* $\overline{AB} = \sqrt{91}$. *By the Sine Theorem,* $\dfrac{\overline{AP}}{\sin(<ABP)} = \dfrac{\overline{AB}}{\sin(<APB)} = \dfrac{\sqrt{91}}{\sin(150°)} = \sqrt{91}\,/(\dfrac{1}{2}) = 2\bullet\sqrt{91}$.

In ΔAPC*, by the Cosine Theorem,* $\cos(\angle APC) = \dfrac{\overline{AP}^2 + \overline{CP}^2 - \overline{AC}^2}{2\bullet\overline{AP}\bullet\overline{CP}} = \dfrac{21+49-91}{2\bullet\sqrt{21}\bullet 7} = \dfrac{-21}{14\bullet\sqrt{21}} = \dfrac{-\sqrt{21}}{14}$ ==> $\sin(\angle APC) = \sqrt{1-\cos^2(<APC)} = \dfrac{5\bullet\sqrt{7}}{14}$. *Since C, P, and Q are on a line,* $\sin(\angle APQ) = \sin(180^o - \angle APC) = \sin(\angle APC) = \dfrac{5\bullet\sqrt{7}}{14}$.

Therefore, $\overline{AQ} = 2\bullet\sqrt{91}\bullet\dfrac{5\bullet\sqrt{7}}{14} = 5\bullet\sqrt{13}$.

Using a similar method for finding the length of $\overline{AQ}$*, or using the Cosine Theorem in* ΔAQB*, we can find that* $\overline{BQ} = 3\bullet\sqrt{39}$.

Verifying the Solution

Another important role that GSP plays is that it can provide a good and easy way to verify the solution. In the problem that was discussed above, when the final results were found, students were not sure if they were correct answers, especially as they didn't look like "neat" or straightforward answers numerically. However, GSP dynamic measurement feature makes checking the answers quite easy and convincing. For those who constructed exactly accurate drawings, they were ready to do the verification. For others, the author provided such a drawing (construction). Students measured segments AP and AQ, calculated the ratio AQ/AP, and multiplied this ratio by $\sqrt{21}$, *which is the length of segment AP. The result was approximately 18.03, and it was independent of the drawings constructed by different individuals. By a quick calculation (through GSP or a calculator),* $5\bullet\sqrt{13} \approx 18.03$. Thus, the length of segment AQ was verified. The length of segment BQ was verified in a similar way.

2. The Use of GSP and Mathematical Modeling

Different from general problem solving, mathematical modeling refers to processes of dealing with (usually real-world) situations that comprise information which might be incomplete,

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ambiguous, or undefined, with too much or too little data (see [3]). It "has to do with an open-ended problem that can be solved in a variety of ways or has many different solutions" (see [7]). One powerful feature of mathematical modeling is that students can make assumptions to simplify a complex situation so that they can start to build a model. Students will then continue to make sense of the related information, elicit and work with the embedded mathematical ideas, and modify and refine their models (see [3]).

Children need to develop mathematical modeling abilities to function effectively in a world that is demanding more flexible, creative, and future-oriented mathematical thinkers and problem-solvers (see [3]). To make this happen, teachers need to have these abilities in the first place. Therefore, in the Technology course, a certain number of matheamtical modeling problems were presented to the students. The use of GSP was found to be very effective in students' exploration of the modeling problems.

Among other problems, the students explored the following real life situation:

A new restaurant will be opening in Manhattan this summer. The owner has enlisted you to help design the layout for the tables in the restaurant. Because the space is limited, she wants to maximize the number of people she is able to seat. The dining area of the restaurant is a rectangular space 40 feet by 80 feet. Each table needs a 2.5 feet border for waiters and patrons to walk between the tables. Your task is to write her a letter and explain what kinds of and how many tables she should order.

It can be a complicated process to figure out how many tables of different shapes and sizes should be ordered and to design the layout for these tables to maximize the number of people the restaurant is able to seat. However, as just mentioned, the students can make assumptions to simplify the situation so that they can start to build a model that they can refine later. The situation can be simplified at first by considering only one kind of table (a table of a certain shape and size, e.g., a 3 foot by 4 foot rectangular table seating 4 people or a circular table with circumference 3π feet, seating 2 people). Students can do their exploration taking advantage of technology. GSP can be used very effectively here because its transformation features allow students to quickly layout "tables". This exploration is an excellent opportunity for students to solidify their understanding of and apply proportionality when they work with the dimensions of a rectangle and the circumference, radius, and diameter of a circle. The outcome of the exploration would be a graphical representation of a uniform table layout with associated calculations. A comparison between two or more such cases will give the maximum (relative to the students' explorations) number of people that the restaurant is able to accommodate.

Figure 5 shows part of a student's modeling solution to the Restaurant problem from which we can see the GSP transformation functions were used intensively. The graphic representation was constructed accurately, and more importantly, the student was able to save time from the details of the layout and concentrate on analyzing the mathematical relationships embedded in the layout and real life considerations to achieve deeper, conceptual understanding.

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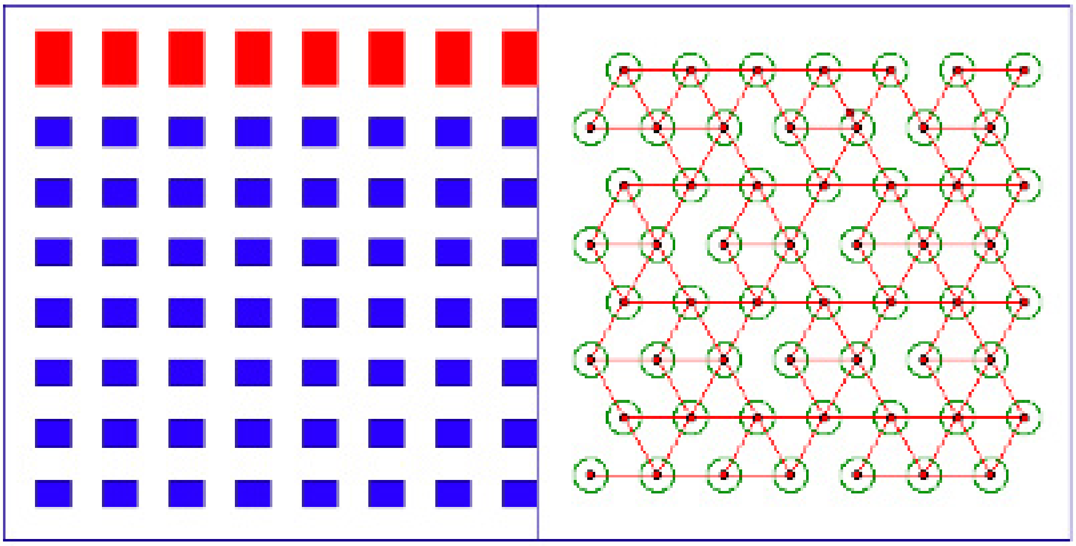

Above is a scale model of the restaurant, in which 1 inch is equivalent to 10 feet. Therefore, the 40 foot by 80 foot restaurant space is 4 inches by 8 inches long.

After a Google search for the phrase "seats 10 to 12 people," I found the Web site of a catering company that listed a bunch of different table sizes and how many people can sit at each table (see attached).

A 4-to-6 person rectangular table was 4 feet by 30 inches (2.5 feet). Thus, I figured that a standard 2-person table would be 2 feet by 2.5 feet.

My first thought was that the most convenient setup would be to have just that one type of table: the 2-person 2' x 2.5' table. That way, two or more of these tables could be put together to seat parties of 4, 6, 8 or more.

I figured that restaurant patrons would not want to be seated with people they didn't know. Some restaurants have mostly large tables and do this, but the vast majority tend to seat each party separately. I tried to make the model fit this requirement.

If the restaurant were filled only with these tables for two, I asked myself which orientation would allow you to put more of the tables in the restaurant?

Figure 5. Part of a student's modeling solution to the Restaurant problem.

It is not impossible to explore and model this problem situation without dynamic geometry software, but it would be much more inconvenient and time consuming. In addition, this results in less time spent on conceptual oriented tasks.

3. Creating Teaching Projects That Take Advantage of the Use of GSP

The technology course was designed for prospective teachers. Eventually, students enrolled in this course will apply what they have learned to their classroom teaching. Therefore, students should experience using GSP as both a learning tool and a teaching tool. For this reason, students were requested to create different projects for a high school geometry class in which the use of GSP was required or at least encouraged. The projects required children to construct their own sketches and to write a proof justifying their observations. Each project integrated previously learned material with new properties or ways of thinking about proof. The goals for the projects were:

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- To guide children to learn more about using GSP as a learning tool through constructing their own sketches;
- To emphasize the versatility of the theorems and properties that children have studied by applying them to new situations;
- To empower children to draw conclusions based on observation and justify their statements with rigorous mathematical proof.

Assuming that the children had not yet taken on the tasks of both making new observations and proving them by themselves, the projects were required to have guiding questions to help to lead them through the process. The children were familiar with the GSP construction tools, but they were not experts. Therefore, in describing the steps for construction, the projects had to be very clear.

The guiding questions for the proof should be specific enough to point out important parts of the construction, yet open enough that the children would have to figure things out on their own, playing with the dynamic nature of the software and answering the questions for themselves. If the children got stuck, their teacher would prompt them to take different measurements, and then drag different corners of their shapes. Then the teacher would ask more guiding questions: What changed? What stayed the same? Why is that? What do the different shapes share or have in common?

One of the most powerful outcomes of the projects would be the integration and application of previously learned theorems and properties to new conjectures.

Many projects met these requirements and were considered to be good projects by both the author and the whole class of students. The following project gives an example.

"A Twist on the Pythagorean Theorem"

We are familiar with Pythagorean Theorem and we have proven it in several ways. Let's think more about the idea of "squared" geometrically rather than algebraically. What if instead of computing the side squared we computed the side equilateral triangle-d or the side regular hexagon-ed or the side regular pentagon-ed. Can we construct other regular polygons (with any number n of sides) besides squares for which this equation will hold?

You will use the Geometer's Sketchpad to illustrate this question, and then you will prove your observation using the formula for the area of a regular polygon.

Follow these steps twice – with different values for n. Create these on two different pages within the same file. To do this, go to File -> Document Options and choose Add Page -> Blank Page. Name your pages by the name of the n-gon featured. If you can make it general enough, you only have to write one proof.

1. *Construct a right triangle using the "construct perpendicular" feature of the Geometer's Sketchpad.*
2. *Choose a number of sides n for a regular polygon. Use the formulas discussed in class to compute the angle measurement of each interior angle of your polygon.*
3. *One at a time, rotate the sides of your right triangle n times to create a regular n-gons on the three sides of your triangle*
4. *One at a time, select all of the vertices of an n-gon and find its area.*
5. *What is the relationship between these areas? Is it what you would expect, using the Pythagorean Theorem as a model? Use Measure -> Calculate to verify this relationship, and drag a vertex of the right triangle to verify that it is generally true.*

Your Proof:

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What you have observed can be proven using the area formula for a regular polygon. To keep your work as general as possible, you will use variables instead of the numbers that you can find through measuring on Geometer's Sketchpad. Work out this proof on your own, and then type it into your Sketchpad file. In addition to this text, you should display the calculations done in steps 4 and 5 above.

The students who created this project also provided a GSP file giving a sample solution (Figure 6). This solution was not perfect, but the general idea was correct and clear.

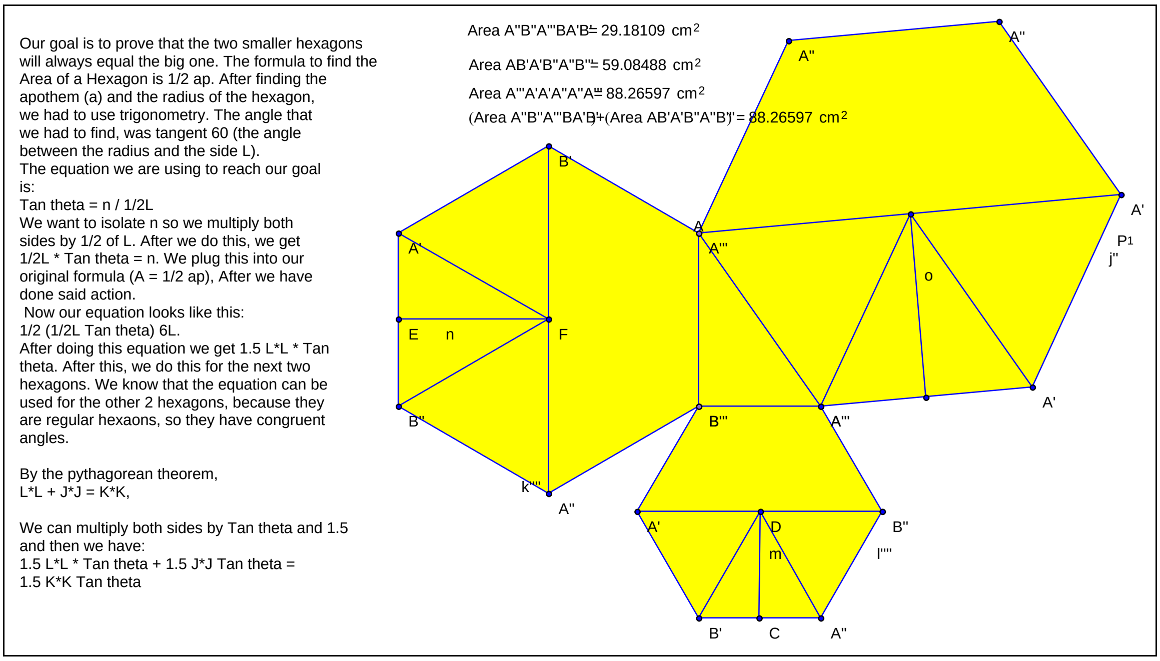

Figure 6. A GSP file along with the project *"A Twist on the Pythagorean Theorem"*

From this project, we can see that children were required to be engaged in the hands-on and minds-on exploration activities that took full advantage of the power of GSP. This student-centered project was fun, challenging, and different from the usual work out of the textbook. It was a powerful blend of discovery with integration and application of previous knowledge, which I believe is at the heart of what mathematics really is.

4. Conclusion

My experience in teaching the Technology course at NYU indicates that the preservice teachers have benefited from exploring mathematics and mathematics teaching with technology and especially GSP. The examples given above point out that for some of the challenging problems that are presented to students, it is almost impossible or very hard to manually make correct drawings. To overcome this difficulty, the use of dynamic geometry software seems to be critical, or at least very desirable. In addition, the use of the software can stimulate students' insight of problem solving and provide an easy and convincing way of verifying the solution. Moreover, students can construct accurate visual representations to model real world situations very efficiently by using transformations in dynamic geometry software. This can save time

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significantly so that students can concentrate on more conceptual oriented tasks. Good teaching projects that take advantage of dynamic geometry software can also effectively enhance school children's mathematics learning.

I administered a pretest and a posttest at the beginning and the end of the course, using Choi-koh's (see [1]) instrument to assess the students' van Hiele levels of geometric thinking. An initial analysis provided evidence that almost all students enrolled raised their geometric thinking by at least one van Hiele level by the end of the course, with most of them having progressed to van Hiele level 3 (abstract/relational) or 4 (deduction). Further research is needed to assess the causal linkages between the use of GSP and these outcomes. As NCTM indicates in its Standards documents (see [9], [10], and [11]), the utilization of technology in the learning and teaching of mathematics is essential. The place of technology as tools for learning and teaching mathematics is rapidly increasing, and the diverse uses are multiplying. It is, therefore, important to institute broad use of technology into the mathematics education programs. A technology course is necessary and strongly encouraged to be included in any secondary mathematics teacher preparation program.

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Converting 'More to Less' and 'Less to More': Designing Self-Determined Learning Environments within Minimalist Instruction

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Abstract: Emphasizing the genesis of heuristic processes and students' ability of to develop intuition and mathematical ideas within constructivist approach can hardly be reached without a systematic planning of the learning environments. In learning situations, however, students often hope to have freedom to choose the problems that they want to solve within continuous self-evaluation instead of relying on guidance by the teacher. Based on results of the ClassPad project the article suggests that a viable pedagogical framework to fulfil strong demands of constructivist view of teaching and learning might be to convert systematic planning to minimalist instruction within self-determined learning environments. Concepts and procedures can be constructed by students themselves – and other way around - well-known concepts can be applied in one form or the other one. Minimalist approach seems to enrich student's mathematical profile but also lead to higher cognitive performance.

1. Introduction

When planning and creating learning environments the teacher usually encounters a conflict between conceptual and procedural knowledge types[1]: how much students should understand before they are able to do, and vice versa ([14]). Concerning technology-based learning, the first challenge arises from the structure of the topic to be learned, whereas the other is caused by the instructional variables required for technology use. The Finnish TIMSS and PISA results (cf. [24, 42]) refer to the fact that students seem to learn mathematical as well as technical skills effectively outside school. This forces us to ask if there is something wrong inside school as far as the question "how to learn" is concerned, and which factors in our education are important for the development of thinking abilities. If the main task of education is to promote a skilful "drive" along knowledge networks so as to support pupils in making use of their rich activities outside school, it seems appropriate to look for a suitable balance between the two knowledge types.

The logical relation between the two knowledge types in the *developmental approach* is based on a *genetic view (G)* (i.e. procedural knowledge is necessary for the conceptual) or a *simultaneous activation view (SA)* (i.e. procedural knowledge is necessary and sufficient for conceptual knowledge). It seems appropriate to claim that the goal of any education should be to invest in conceptual knowledge from the very beginning. If so, the logical basis of this *educational approach* is the *dynamic interaction view (DI)* (i.e. conceptual knowledge is necessary for the procedural), or again the simultaneous activation view.

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[1] I adopt the following characterizations of Haapasalo and Kadijevich [16]:
- *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems.

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2. MODEM framework for systematic planning and assessment

Kadijevich [21] points out that students should, among other things, be educated to respect the following two requirements: (1) when utilize mathematics, don't forget available tool(s); when making use of tool(s), don't forget the underlying mathematics; and (2) to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working with different representations (algebraic and graphical, for example). These demands can be realized if the teacher has fundamental know-how of the relation between conceptual and procedural knowledge. According to Rittle-Johnson and Koedinger [38], the two knowledge types seem to develop iteratively, where a change of problem representation influences their relation [39]. Such development was assumed in the pedagogical model developed within the *MODEM*-project[2]. This model of the interplay between the two knowledge types makes use of spontaneous procedural knowledge as well as the simultaneous activation principle (Figure 1).

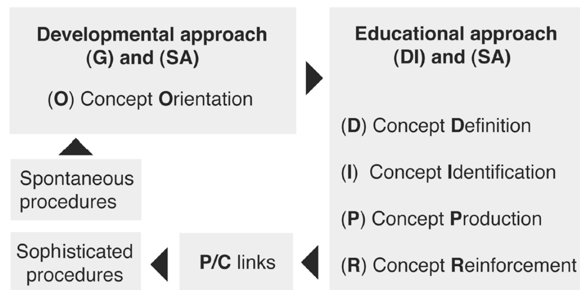

Figure 1. Interplay between the approaches.

When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side of Figure 1. On the other hand, when offering students opportunities to construct links between representation forms of a specific concept, the focus is on the right-hand box, in which the stages of mathematical concept building are illustrated.

Orientation (O) basically utilizes a developmental approach: the interpretations of the situation can be based on mental models of the pupils, coming, more or less, from their naïve procedural ideas. These act like a wake-up voltage in an electric circuit that triggers another, much more powerful current to be amplified again. The procedural and conceptual knowledge types start to support each other. The example in Figure 2 is taken from the ClasPad project.

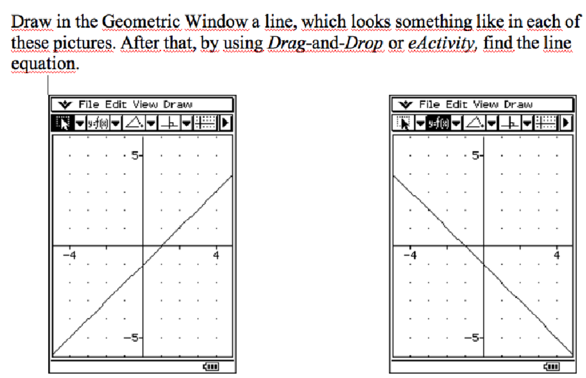

Figure 2. An orientation task with ClassPad.

The role of the Concept *Definition (D)* is to offer students opportunities to make their own investigations, to express the results of their investigation especially in verbal forms in each case, and to argue about these results within the collaborative teams and between the teams. As a result of social construction, a definition for the concept is born, meaning that students try to fix the relevant determiners of the concept in verbal, symbolic and graphic forms. Especially in the phases of orientation and definition, creative thinking and productive work is needed.

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[2] See [14] and http://www.joensuu.fi/lenni/modemempe.html. To see the systematic approach of the MODEM-framework, the CAL–software is freely downloadable at http://www.joensuu.fi/lenni/programs.html

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The next phases of concept building utilize the principle of dynamic interaction. The idea is to give students a sufficient number of opportunities to construct concept attributes and procedural knowledge based on them. In the phase of *Identification (I)* we have to give students opportunities to train themselves in recognizing concept attributes in verbal *(V)*, symbolic *(S)* and graphic *(G)* forms. For this we need six kinds of tasks *(I)*: *IVV, IVG, IVS, IGG, ISS* and *ISG.* During the learning process, the teacher must be ready, if necessary, to begin with tasks that require distinguishing between only two elements before going on to the identification of several elements. Fugure 3 illustrates task types *IGS* and *IGG.*

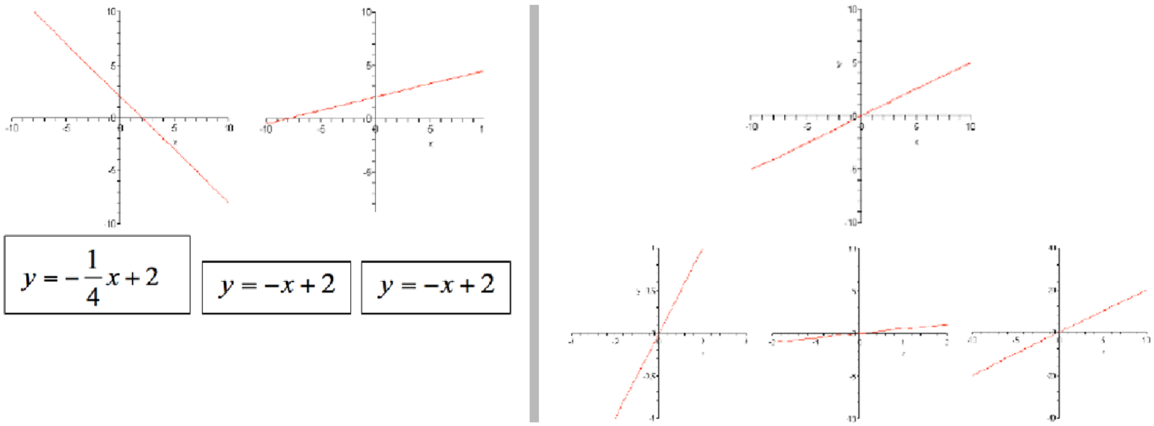

Figure 3. Identification task IGS (on the left) and IGG (on the right).

In the phase of *Production (P)* we have to give pupils the possibility to produce from a given presentation of the concept another representation in a different form. The development of production *(P)* requires nine combinations: *PGV, PGS, PGG, PSG, PSV, PSS, PVS, PVV* and *PVG*. The tasks of identification and production must be achievable without any complicated processing of information on the student's part. Figure 4 represents task types PGG and PGS (even though the tasks can be solved as other task types by using *Drag-and-Drop* activity or *eActivity*).

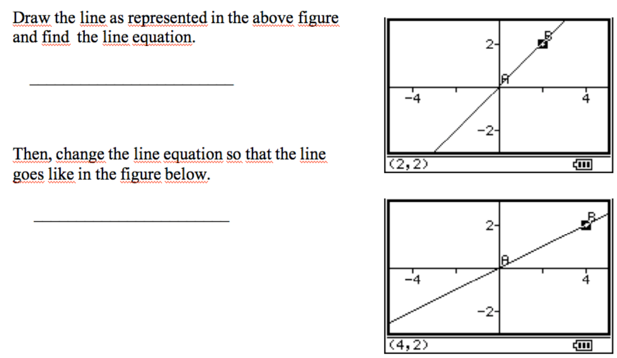

Figure 4. Production tasks *PGG* and *PGS.*

In the phase of *Reinforcement (R),* the goal is to train and utilize concept attributes and to develop procedural knowledge to be used in problem solving and applications. The following task of our ClassPad project can be classified in this category:

> GSM operator A offers free calls for total fixed price 17,90 € pro month. Operator B does not charge any basic fee whereas cost for every call is 0,10 €/min. Can you make a graphic representation to compare which of these offers might be the most suitable one for you?

Relating different representations cannot only support the development of conceptual knowledge [33], but also relate procedural and conceptual knowledge [14, 17, 22, 28, 41]. Because of that, in order to coordinate the process and object features of mathematical knowledge, multiple forms of representation are to be utilized and connected, especially with the aid of modern technological tools.

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3. Human perspective

In the research in mathematics education the following four areas have been neglected (cf. [20]): promoting the human face of mathematics; relating procedural and conceptual mathematical knowledge; utilizing mathematical modelling in a humanistic, technology-supported way; and promoting technology-based learning through applications and modelling, multimedia design, and on-line collaboration. Even though these defects greatly stimulate us to use modern technology in all its forms, today even small pocket computers can be utilized for the first three of the above-mentioned demands (cf. [17, 23, 37]). One such small computer is ClassPad made by Casio (see http://www.classpad.org). ClassPad's drag-and drop technology allows the student to manipulate mathematical objects between two windows, illustrating two different forms of mathematical representation. In many cases this means forming links between conceptual and procedural knowledge, which is a relevant perspective in which to evaluate the long-term suitability of educational technology. Before representing the Class Pad project, I discuss why there might be an essential dilemma between minimalist instruction and systematization.

Zimmermann's ([43]) study of the history of mathematics reveals eight main motives and activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years. It is appropriate to take this network of activities illustrated in Figure 5 as an element in our theoretical framework for the structuring of learning environments and for analyzing student's cognitive and affective variables. Within these activities our ClassPad study focuses on "changing representation" which is not only a powerful thinking tool to enhance problem solving processes (cf. [35]) but it might also promote links between procedural and conceptual knowledge.

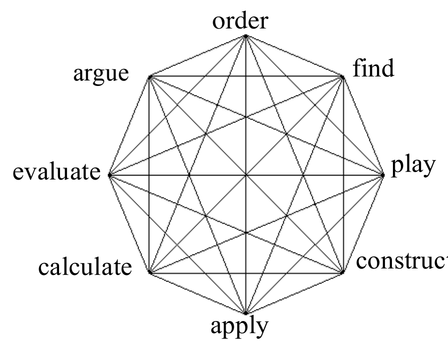

Figure 5. Activities which proved to be successful in producing new mathematics.

4. Minimalism

It is the right-hand half of Figure 5 that emphasizes creative human activities, which very often run optimally without any external instruction or demand. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *minimalist instruction*, introduced by Carroll [3, 4], is crucial not only for teachers but also for those who write manuals and help menus for the software. Carroll observed that learners often tend to "jump the gun". They avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulty recognizing, diagnosing, and recovering from their errors. We next pick up the following characteristics of minimalist instruction (cf. [25, 29]) keeping in mind the fostering of problem-solving abilities. These features of *minimalism* include several varieties of constructivism, offering also instructional assumptions (cf. [6, 8, 9, 31, 36])[3]:

[3] The assumptions, characteristics and methods of minimalism are almost totally opposite to those of Gagne [15].

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- Learning is modelled and coached for students with unscripted teacher responses.
- Learning goals are determined from real tasks stressing doing and exploring.
- Errors cannot be avoided and should be used for instruction
- Learners construct multiple perspectives or solutions through discussion and collaboration.
- Learning focuses on the process of knowledge construction and development of reflexive awareness of that process.
- Criterion for success is the transfer of learning and a change in students' action potential.
- The assessment is ongoing and based on learners' needs.

When looking for minimalist instruction, the strategy may primarily be procedural or conceptual. This polarization has been studied by several studies [1, 7, 30, 40].

5. The ClassPad project

The aim of the *Stage 1* of the *ClassPad project*[4] was to get experiences of what kind of impact minimalist instruction causes among students of 8th grade, how they could use ClassPad voluntarily on their free time during their summer holiday, and how these activities would shift each of the eight components in Figure 5 concerning the following questions:

(1) How strong does each component appear in student's view on mathematics (mathematical profile)?
(2) How strong is the student in doing mathematics on each component (identity profile)?
(3) How strong is the impact of computers in making of mathematics on the components (ICT-profile)?

Data for each of these three types of *profiles* (in Figures 8-9) was collected by using a web-based questionnaire[5]. Students had the opportunity to use - if they wanted to do so - ClassPad during their summer holiday. But there was not any kind of duty to use it. The only duty was to write a portfolio of their working according to certain guidelines. Immediately four students volunteered for this study and after a few days the amount increased to fifteen. Those students were given a leaflet including some problems connected to the basic features of linear function, which is one of the most important concepts in the curriculum of the 9th grade. Before giving ClassPads at the end of May to those students, a cognitive test was administered to the students concerning the topic, and profiles were measured by using a web-based questionnaire. After the summer holiday these measurements were repeated in August. Furthermore, students were interviewed to get more detailed information by their answers and experience. The cognitive tests consisted of the sub-phases of the mathematical concept building in the right-hand box of Figure 1, utilizing multiple representations between verbal, symbolic and graphic forms on the levels of identification and production.

In *Stage 2* the whole class (N=22) including the above-mentioned voluntary students studied the basic features of linear function as a part of their normal school curriculum of 9th grade just by using ClassPad. All mathematics student teachers were involved in the project to make their educational studies. Besides the affective variables above, students' cognitive results were measured at the end of the working period by using the same test as above. The aim was to find out (1) how ClassPad can be used in classroom when studying topics of normal school curriculum; (2) how does working with ClassPad change students profiles; (3) how does working with ClassPad affect students cognitive scores, and (4) what could be the optimal way to combine minimalist instruction and a systematic pedagogical model.

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[4] The background, aims and methods of the ClassPad project have been described in detail in [10].
[5] To measure each of the so called Zimmermann-profiles, the questionnaire used Likert-type scaling; see http://jnor.joensuu.fi/eronen/ztest

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The results of Stage 1 suggest that doing mathematics with ClassPad, even during a short period of time outside the classroom, shifted student's mathematical identity within these motives and activities into positive direction, opening new progressive ways to arrange teaching and learning of mathematics. Because these results can be found in more detail in Haapasalo (2007, pp. 3-5), we therefore next restrict myself in collecting the most interesting features and results of Stage 2.

6. Interplay of systematization and minimalism

The learning material was planned according to the systematic MODEM- framework represented above. However, to follow the principle of minimalist istruction, different task types were organized to form a "problem buffet"[6]. Students had freedom to "jump the gun" by choosing from this buffet any problem that they wanted. To go for linear function within the above-mentioned reinforcement example, one student team, for example, initially selected the quite complicated problem on optimizing mobile phone costs. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set. This happened to consist of 'identification tasks' – the first and lowest level of the concept building within the systematic MODEM framework, which was on the basis of the planning of the learning environments. This example shows that a sophisticated interplay between a systematic and minimalist approach can be achieved even by simple pedagogical solutions. Note this important feature of minimalism: We did not want to regulate students' work by recommending them an easier sub-problem, for example. Instead of that it was students' internal motivation that regulated their task choice. Figure 6 represents the path of another student team, which selected different kinds of problem types from the buffet. Recalling the abbreviations in Figure 1 we notice that this team did not utilize the MODEM framework in an optimal way. The team went directly to *P*SG tasks and also selected from that list those eleven tasks (#1 - # 11) more or less randomly (as it was the case within 'orientation tasks'). Students evidently liked the amazing drag-and-drop function, which automatically performed the *P*SG action.

→	***P***SG (#1)	→	***P***SG(#5)	→	***P***SG(#3)	→	***P***SG(#7)	→	***O*** (#3)
→	***O*** (#1)	→	***O*** (#2)	→	***P***SG(#2)	→	***I*** VG(all)	→	***P***SG(#6)
→	***I*** VS(all)	→	***I*** SS(all)	→	***R*** (#1)	→	***R*** (#1)	→	***R***(#1)
→	***P***SG(#4)	→	***P***SG(#9)	→	***P***SG(#8)	→	***P***SG(#11)	→	***P***SG(#10)
→	***P*** VV(all)	→	***P*** GV(all)	→	***P***SG(#12)	→	***P*** GG(all)	→	***P*** VV(all)

Figure 6. An example of a "classpath" when selecting tasks.

Järvelä's study [19] suggests that there are three different types of learners, and that the instructions should be tailored to meet these needs. *Conceptually-orientated* learners aim to learn things by advancing from conceptual knowledge towards procedural knowledge. *Procedurally-orientated* learners act in the opposite way – they advance from procedural knowledge towards conceptual knowledge. *Procedurally-bounded* learners concentrate only on procedural knowledge. Encountering a procedurally-bounded learner is much more likely than meeting a procedurally- or conceptually-orientated one. Even though the first two learner types seem to produce ideal students, the example in Figures 8-9 shows that even they can be woundable.

[6] Just to note that it perhaps would be even more optimal if students could be actively involved in planning and organizing the buffet.

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7. Students' cognitive performance

Students' scores in all test items were significantly higher after the working period than in the pre-test (see Figure 7). The most remarkable changes were in the tasks types (*P*VV), (*P*SV), (*P*VS), (*P*SS) and *(I*VV). Furthermore, in the most test items students' scores were significantly higher than those of students in conventional school teaching, revealed by *MODEM 1* study in 1987 by using the very same test (cf. [13]). However, it is appropriate to mention that after their working period with ClassPad students also worked two hours with a CAL software mentioned in footnote 2. Even though this probably caused a positive effect on learning results, we felt that it was our responsibility to give students opportunity to use this software, which was available at the school.

Recalling one of the main results of the *MODEM* studies that the production of a verbal form (i.e. *P*GV, *PSV* and *P*VV) and identification between verbal forms (*I*VV) have the highest reliabilities when measuring the conceptual understanding (see [13]). The fact that the lowest scores appeared in the production from graphic to symbolic form (*P*GS), for example, might refer to that drag-and-drop activities need a careful re-consideration when learning with ClassPad.

ClassPad work without any textbooks within interaction between minimalism and systematization was also successful concerning students' cognitive development. Students scored in all test items significantly better after the ClassPad working than in the pre-test. They also showed remarkable procedural skills not only connected to the linear function but to other function types. I skip introducing these results in more detail and refer to [10]. From the viewpoint of problem solving it is appropriate to mention that students clearly liked the feeling that they had reached action potential, which was described to be one of the main aspects in assessment within minimalism. What this philosophy concerns in all, students seemed to like a free architecture of learning without any pre-set goals or tutoring from teacher's side.

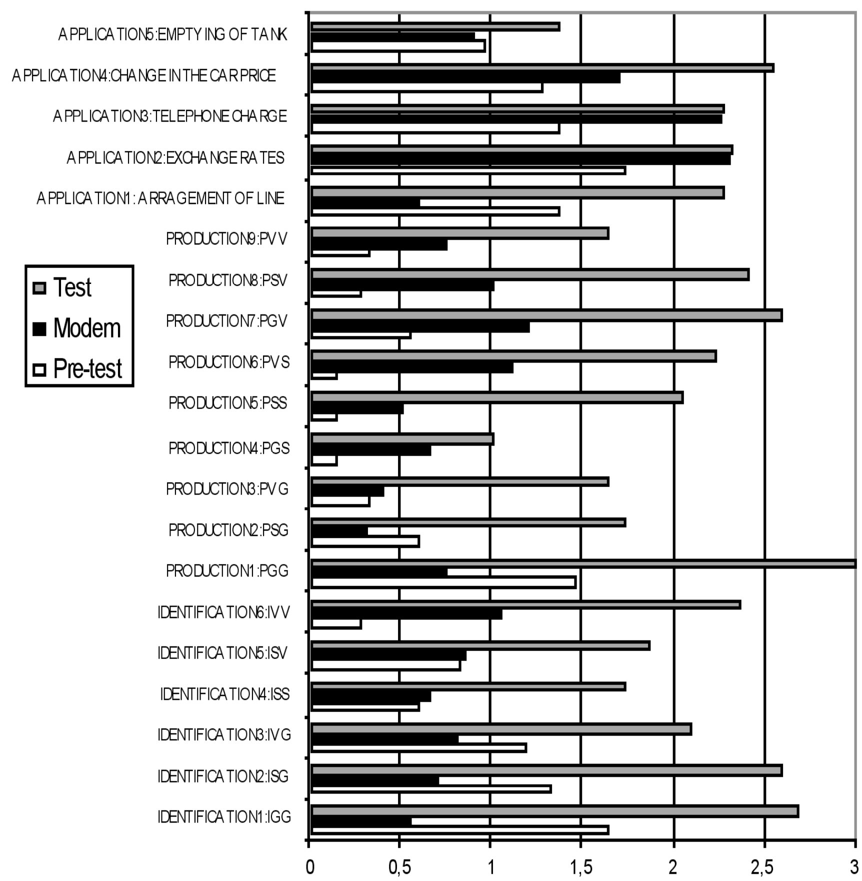

Figure 7. Students' cognitive results compared with those of MODEM 1 study.

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8. Variations in student profiles

When analyzing all students' profiles we noticed that there are some kinds of average shifts among the whole class: Especially the arguing component was stronger in students' answers in May than in January. Furthermore, after ClassPad work the certainty of students' answers increased. However, not any of those changes was statistically significant. Much more interesting findings came out by the analysis of students' individual responses. As an example let us take the case, when a procedure-oriented peer-teacher was teaching her procedure-bounded classmate. Figures 8-9 show that this kind of peer-teaching period quasi-enriched the profile of the peer-teacher but degenerated her mathematical self-confidence. Interestingly the profiles of those two students seem to run in opposite directions. The most interesting – even alarming – finding is that at the same time when the classmate seems (perhaps wrongly) to think she could find, apply and argue better than at the beginning, her peer-teacher's own self-confidence in making mathematics seem to degenerate. This kind of finding might be extremely interesting not only when thinking about the fostering of problem-solving abilities but also when considering the mental problems among teachers[7]. It is – again – a symptom that behaviorist way of teaching can damage both student and teacher.

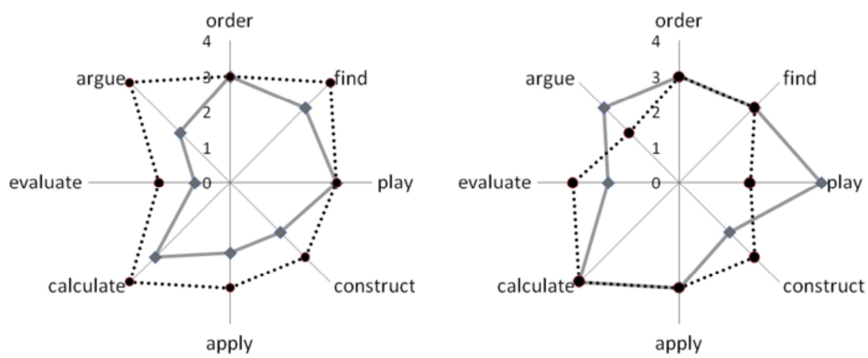

Figure 8. Mathematical profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the working period.

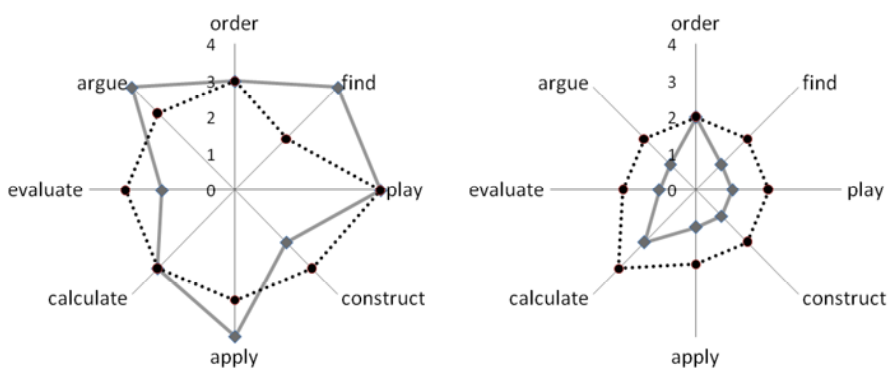

Figure 9. Identity profiles of the peer-teacher (on the right) and her classmate (on the left) at the beginning (dashed) and at the end of the working period.

[7] When typing "teacher's mental problems", Google suggested more than 3 million sites 10th of January 2007.

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9. Closing remarks

By looking the relationship between technology and mathematics education from five perspectives (links between conceptual and procedural knowledge, metacognitions, sustainable components of mathematics making, interplay between systematic approaches and minimalist instruction, and learning by design), [15] suggests that instead of speaking about 'implementing modern technology into classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society'. This means emphasizing more the making of informal than formal mathematics within the framework of eight main activities mentioned above.

The findings presented in this article give support to many modern features and trends of using educational technology. They are in accord with the trend that focus of technology-based learning has been shifted from a technology-oriented viewpoint to a humanistic view, stressing cognitive, affective and social variables involved in the learning process. Sophisticated hardware and software offer us real opportunities for changing our educational paradigm in very radical ways. Therefore teachers and students should be made aware of the technological developments occurring outside the classroom. If students' mathematical profiles can be shifted into more creative directions on a larger scale, this might increase their self-confidence and confirm their decision to use modern technology[8]. Even though the imagination of the user of a progressive technical tool might limit invention of environments within simultaneous activation, for example, most operations can be complicated to carry out without first learning the basic routines for using the equipment. The ClassPad manual consists of about 700 pages, which is a non-optimal user interface in the sense of Carroll [3] and Norman [32]. The versatility of the tool can hinder finding the essential elements that should be altered and tested because each of the intelligent menus actually contains a huge amount of conceptual (mathematical) knowledge. The fact that students in the ClassPad project expressed no particular difficulties with mathematical issues (but rather with more or less technical ones) points to many interesting pedagogical problems connected to minimalism. As Pesonen et al. [34] showed when stressing the importance of reflective tutoring, moving from an old study-culture towards a modern technology-based one is full of cognitive, emotional and social problems.

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Making Mathematics Simple, Attractive, and Personal

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Abstract: An instructional activity has been created to help students make connection of different topics in mathematics. This activity is based on students' personal names or other words to arouse students' interest in learning mathematics. However, since this mathematics learning is an individualized activity, the assessment of students' performances seems very difficult. In this paper, we will discuss how we use the Excel Spreadsheet Models (ESM) for students to pose questions, discover new mathematics ideas and checking answers. The teachers can use this ESM to check and/or evaluate students' individualized mathematics homework or tests. With all the possible answers collected from the whole class, the students can identify patterns and define mathematics concepts. Thus the purposes of learning mathematics are not just graded students, but to offer opportunities for students to work cooperatively, individually, creatively, and successively.

1. Introduction

Mathematics educators all over the world are trying to find a better way of teaching mathematics and for students to learn mathematics. However, students are always wondering; why do we need to study mathematics, how is it related to us? With the help of technology, plugging into a formula to compute the right answer is not the main learning process, what is important is that our students understand mathematics, know the connections among different topics and are aware of the relevance of mathematics to every one of us.

In order to make learning mathematics meaningful, challenging and exciting to the students, we have created an instructional activity to help students to discover some of the mathematical concepts by themselves. This activity is named as "my own mathematics" or "Math DNA" (See [3]). However, since this mathematics learning is an individualized activity, the outcome of students' performances seems very difficult to evaluate. We know that spreadsheets can be used as problem-posing tool (See [1]) and with the help of technology we can do mathematical modeling

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(See [8]), so we design the Excel Spreadsheets Modeling (ESM) system as a teaching and learning module. The teachers can use it to evaluate students' performance and the students can use it to pose problems, solve problems, and check their answers. Furthermore, we talk about how this ESM can be not merely a method of assigning grades, but can play a creative role in helping students discover the connections of different topics in mathematics.

2. Mathematics DNA

2.1 The Activity

This activity is very simple however the mathematics contents derived from this activity are abundant. First, we will explain how this activity is created. We simply convert the alphabet to numbers 1~10 given by the following Table 1, then perform binary operations on successive pairs of these numbers, and finally, the mathematics topics for different concepts will be created based on what types of binary operations are used. For example, based on the four operations, we can introduce the integers, the fractions, the decimals, and the real number system. For example, using an ordered pair concept we can relate to curve plot and produce the geometric content activity and talk about geometrical transformations. Since this activity is based on the individual person's name, it turns out to be individualized mathematics DNA.

Table 1: Assignments between alphabets and numbers

1	2	3	4	5	6	7	8	9	10
a	b	c	d	e	F	g	h	i	j
k	l	m	n	o	P	q	r	s	t
u	v	w	x	y	Z				

2.2 Excel Spreadsheets Models

The activity itself is simple correspondence among numbers and the alphabet. However, the mathematics contents are abundant. Based on each individual's name, we have different numbers. With numbers we can create more numbers by four operations. We can create some points, with points we can draw figures. With figures, we can identify geometrical shapes. With shapes, we can talk about properties of different shapes and geometrical transformations. The Excel Spreadsheets Model we designed can help the students as well as teachers to explore all the mathematics we mentioned above. All we need to do is to key in the numbers into our pre-designed ESM sheets, the required operations and answers are produced by these models. For example, if the content intended to be taught is decimals, then students can create their decimal DNA by division. The students can

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identify different types of decimals by sorting and discussing the whole class's results. The teachers can pose more questions for students to go further into the field of number theory (See [6]).

3. Discussion

To better understand our ESM model, we use ATCM (the acronym of "Asian Technology Conference in Mathematics") as an example to illustrate how this activity can be used to teach different topics in mathematics. The letters A, T, C, M according to Table 1, correspond to numbers in ordered set X={1, 10, 3, 3}. These numbers are considered to be the basic numbers for ATCM. For the binary operations we need to create a second set of numbers. We start with the second number in the Set X ,which is 10, and continue with the numbers 3 and 3, at the end add the first number 1 to form the second set Y={10, 3, 3, 1}. In simpler words, what we do is we choose the next consecutive number to be our second number in our four operations.

3.1 Integers and Rational Numbers

When students find their basic numbers, they can use four operations to create more numbers. The following Table 2 shows how we use the numbers corresponding to ATCM to perform four operations in an Excel spreadsheet. Notice how the mathematics concepts evolve from integers to rational numbers automatically.

Table 2: Basic Four operations and coordinate points

#	x	y	x+y	x-y	x*y	x/y(Fraction)	x/y(Decimal)
1	1	10	11	-9	10	1/10	0.1
10	10	3	13	7	30	3 1/3	3.3333333…
3	3	3	6	0	9	1	1
3	3	1	4	2	3	3	3

Although this is an individualized activity, no uniform result is expected, but abundant results can help students to discover patterns, talk about their discoveries, and define the mathematical meanings. For example, the students find the different patterns in the results of division. For only four numbers, we notice that there are a unit fraction 1/10, whole numbers 1 and 3, and a mixed fraction $3\frac{1}{3}$. Similarly, they can also find that there exist whole numbers, terminating decimal 0.1 and repeating decimal 3.333… in other forms of division. We then can ask students to talk about why we need fractions and decimals and the pro and con of these two.

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3.2 The Powers and the Radicals

The mathematics content can go beyond the rational numbers. If we take other operations using our basic numbers, such as powers or roots, we create the real numbers in the following table.

Table 3: The Powers and the Roots

x	x^0	x^1	x^2	x^3	x^(1/2)	x^(1/3)	-x	1/x
1	1	1	1	1	1	1	-1	1
10	1	10	100	1000	3.1622777	2.154435	-10	0.1
3	1	3	9	27	1.7320508	1.44225	-3	0.3333333…
3	1	3	9	27	1.7320508	1.44225	-3	0.3333333…

Using the new number patterns created by this above table, the students can find out the zero power always ends up as unity. That will make students to understand why $a^0=1$ for all positive integers a. Students are interested in how many digits should be there after the decimal point. They can ask questions about whether or not all roots are repeating decimals? They can also explore how many digits do we need to write down for repeating decimals? What is the period of a repeating decimal? Or is it a non-repeating decimal? Etc. After the whole class discussion, the students will have a better understanding of real numbers and perhaps even go further into learning the concepts of number theory.

3.3 Geometry

Before talking about any mathematical concepts coming from this activity, we usually ask the students to design their own name logo. This name logo design will eliminate their mathematics anxiety. With the transformed rule given in Table 4, they can create basic geometric shape (Fig.3.1.1) and transformed shapes. If we combine four graphs overlapped in the same grid we have the combination shape (Fig.3.1.2). After they color it (using Little Paint in their computer) they can design their own name logo (Fig. 3.1.3 and Fig. 3.1.4) or make quilt blocks (See [2], [4], [5] and [7]). Figures 3.1.1 and Figure 3.1.2 are created with the Excel Model System.

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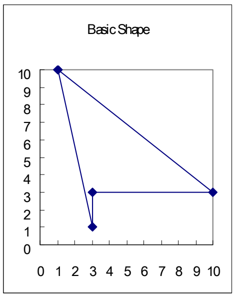

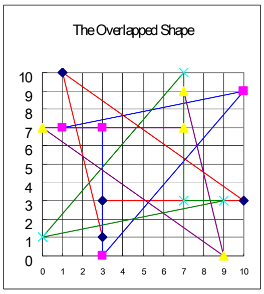

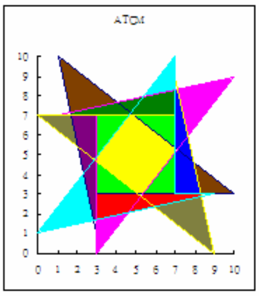

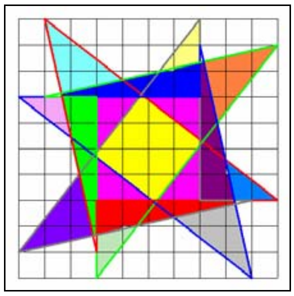

Fig. 3.1.1 Fig. 3.1.2 Fig.3.1. 3 Fig.3.1. 4

Figure 3.1: The various versions of the graphs of ATCM

Table 4: Points for Basic Shape and Transformed Shape

Basic Point (x, y)	1st Transform (y, 10-x)	2nd Transform (10-x, 10-y)	3rd Transform (10-y, x)
A(1 , 10)	A_1(10, 9)	A_2 (9 , 0)	A_3 (0 , 1)
B(10 , 3)	B_1(3, 0)	B_2 (0 , 7)	B_3 (7 , 10)
C(3 , 3)	C_1(3, 7)	C_2 (7 , 7)	C_3 (7 , 7)
D(3 , 1)	D_1(1, 7)	D_2 (7 , 9)	D_3 (9 , 7)

The students can draw and design their name logos based on the rules given in the ESM. They also need to talk about why the rules affect the orientations of the basic shape. What geometrical concepts are involved in the creations of new shapes? If they can identify the rotations of the basic shape causing the new shapes, the next question is to ask them to identify the center of rotation.

Table 5: The Points in ESM

Alphabet	X	Y	10-x	10-y	x
A	1	10	9	0	1
T	10	3	0	7	10
C	3	3	7	7	3
M	3	1	7	9	3
	1	10	9	0	1

The graphs of ATCM are created by ESM in the following figures, all students need to do is key in their basic numbers in the above ESM system and then let their imagination go wild.

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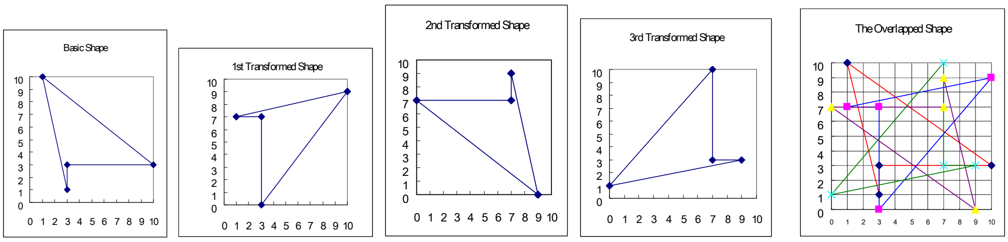

Figure 3.2: Basic shape, Transformed shapes and Overlapped shape

The students can copy their overlapped shape paste to Little Paint in their computer and design their logos.

3.4 Analytical Geometry

The students can explore analytical geometrical concepts such as vertices' points and intersection point, lines, parallel lines and perpendicular lines, positive and negative slopes, lengths of line segments. The students can also identify different geometric shape**s**. For example, how many squares are there? What types of transformations are needed from the basic shape to form the overlapped shape? And formulate proofs of the properties of geometric shapes, for example, can they prove that the center yellow equilateral really is a square?

3.5 Algebraic Concepts

In Table 6 the students can explore some basic algebraic concepts evolved from the whole class discussion about the analytical geometry concept. For example, we know that two lines may look as if they are perpendicular to each other or parallel to each other. Can we prove it? If there appear to be a square, is it really a square? How can we be sure? One of the main purposes of this activity is to discover that how mathematics concepts are integrated. These mathematics concepts are not unrelated. The students need to understand number sense, real numbers, fractions, decimals, geometry, algebraic reasoning in order to answer the question: Does there exist a square? The students need to know the sequence of knowledge. They need to know an equilateral is a four sided figure. If we have two opposite sides parallel to each other, then the figure is a parallelogram. If a parallelogram has two adjacent sides perpendicular to each other, then it is a rectangle. If two adjacent sides of a rectangle have the same length, then it is a square. Thus, the students need to find the line segment, the slopes, equation of lines and solve the system of equations, as we state in the following table. In order to prove a four sided figure is a square, the students have to solve systems of linear equations. After the vertices of the four sided figure are found, the students can then use the distance formula to find the lengths and the slope to find whether the four sides are parallel or perpendicular to prove it is a square.

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	A	B	C	D	E	F	G	H	I
1	Number	x	y	point	Coord	line	slope $m=\frac{y_1-y_2}{x_1-x_2}$	y-intercept $b=y-mx$	slope-intercept form $y=mx+b$
2	1	1	10	A	(1,10)	AB	- 7/9	10 7/9	y=-0.78x+10.78
3	10	10	3	B	(10,3)	BC	0	3	y=0x+3
4	3	3	3	C	(3,3)	CD	#DIV/0!	#DIV/0!	#DIV/0!
5	3	3	1	D	(3,1)	DA	-4 1/2	14 1/2	y=-4.5x+14.5
6		1	10						

Figure 3.3: The Excel Spreadsheet of slope-intercept formula

We key in the slope-intercept form of line equations function formula in Excel, “ ="y="&ROUND(H2,2)&"x+"&ROUND(J2,2) ” as Figure 3.3 indicates, the H column calculates the slopes and the J column is for y-intercepts. We create a template and our students only need to change the number column and put into it their own basic numbers. The students can find their own line equations as created by Excel. We use “="y-"&C2&"="&ROUND((C2-C3)/(B2-B3),2)&"(x-"&B2&")"” to create the two points form of line equations, and use the following function: “="y-"&C2&"="&ROUND(H2,2)&"(x-"&B2&")"” for point and slope form of line equations. One of the advantages here is students can learn quickly that the denominator can not be zero. They can not get the equations because they get an error message. However by noticing the points A and C, they realize that the x=3 in both points and hence the line formed by points A and C is a vertical line.

The next module in the ESM is to solve the system of linear equations. We have the ATCM as our example in the following Table.

Table 6: The formation of general form of line equation

Name	X	Y	Point	Coordinate	Line	Ax+By=C : (y1-y2)x-(x1-x2)y=x1(y1-y2)-y1(x1-x2)
A	1	10	A	(1,10)	AB	7x+9y=97
T	10	3	B	(10,3)	BC	0x+-7y=-21
C	3	3	C	(3,3)	CD	2x+0y=6
M	3	1	D	(3,1)	DA	-9x+-2y=-29

Table 7: Cramer’s Rule for solving systems of linear equations

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Ax+By=C	A	B	C	Denominator	x	y	Point of intersection	Coordinate
7x+9y=97	7	9	97	-49	10	3	B	(10, 3)
0x+-7y=-21	0	-7	-21	14	3	3	C	(3, 3)
2x+0y=6	2	0	6	-4	3	1	D	(3, 1)
-9x+-2y=-29	-9	-2	-29	-67	1	10	A	(1, 10)

Use the function “=I2&"x+"&J2&"y="&K2” in the ESM to get the general form of linear equation. In the ESM we key in function “=(I2*J3)-(I3*J2)” to get the denominator and the “=(K2*J3-K3*J2)/L2” for x value and “=(I2*K3-I3*K2)/L2” for y value.

3.6 The Connections of Different Topics in Mathematics

Because of the rotations of our basic shape, we have at least one square in each student’s logo, implicitly or explicitly. Thus one of our examination questions is to ask our students to prove that they have a square in their logo.

In the logo of ATCM designed above, we can identify a yellow square (See Figure 3.4) formed by four lines L1, L2, L3 and L4. Our task is to show that the yellow square is truly a square. In the following ESM system demonstrated by Table 8-1 and 8-2, we can justify that our assertion is valid.

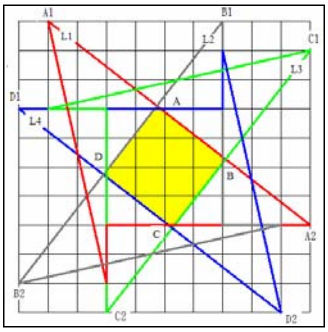

Figure 3.4: The Yellow Square

Table 8-1: I have a square

Points of 4 lines	X	Y	Coordinate	Line	Slope	A	B	C	AX+BY+C
A1	1	10	(1,10)	L1	- 7/9	7	9	97	7x+9y=97

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A2	10	3	(10,3)						
B1	7	10	(7,10)	L2	1 2/7	9	-7	-7	9x+-7y=-7
B2	0	1	(0,1)						
C1	10	9	(10,9)	L3	1 2/7	9	-7	27	9x+-7y=27
C2	3	0	(3,0)						
D1	0	7	(0,7)	L4	- 7/9	7	9	63	7x+9y=63
D2	9	0	(9,0)						

Looking at the ESM results, students can identify that Lines L1 and L4 have the same slope =7/9, while the slope of lines L2 and L3 is 9/7, hence they are parallel lines. So the equilateral ABCD formed by lines L1, L2, L3 and L4 is a parallelogram. At the same time the slope of lines L1 and L2 are the negative reciprocal of each other, hence they are perpendicular. Thus the parallelogram ABCD is a rectangle. Next step is to show the rectangle actually is a square. How can we do that? We do not know the vertices of ABCD. How can we demonstrate the four sides of ABCD are the same length? The ESM system helps us to find the answer, but the most important thing is for students to realize the connection for solving system of linear equations.

Table 8-2: I have a square

Linear equations	Denominator	x	y	Point of intersection	Coordinate	Segment	Length
$\begin{cases} 7x+9y=97 \\ 9x-7y=-7 \end{cases}$	-130	4.738	7.092	A	(4.74, 7.09)	AB	2 600/611
$\begin{cases} 9x-7y=-7 \\ 7x+9y=63 \end{cases}$	130	2.908	4.738	D	(2.91, 4.74)	BC	2 600/611
$\begin{cases} 9x-7y=27 \\ 7x+9y=97 \end{cases}$	130	7.092	5.262	B	(7.09, 5.26)	CD	2 600/611
$\begin{cases} 7x+9y=63 \\ 9x-7y=27 \end{cases}$	-130	5.262	2.908	C	(5.26, 2.91)	DA	2 600/611

4. Conclusion

This instructional activity has several characteristics: 1) it is personal; 2) it is multi-dimensional; 3) it is meaningful; 4) it is sophisticated; 5) it is real-life situated; 6) it is

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cooperative; 7) it is technological.

The spreadsheet is easy to use and almost instantaneously gets numerical simulations. It allows for visualization of an inductive proof, supports a transition from computing to a formal language of mathematics, and provides the answers of all the above mathematical problems quickly. As for teachers, checking students' answer sheets is not tedious work anymore.

The assessment uses different tests to check students' performances, are based on students' different levels of mathematics knowledge, and students are forced to learn instead of copying or cheating either on homework or tests.

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Exploring Ethnomathematics with the Geometer's Sketchpad (GSP): Thai Students' Weaving Projects

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Abstract

The purpose of this research study is to explore the connection of mathematics, arts and technology in the context of school mathematics in Thailand. The research study emphasizes on students' projects on the ethnomathematics particularly the weaving using the Geometer's Sketchpad (GSP). GSP empower students to use their ability to visualize and create graphical representation, which will enable them to develop their mathematical thinking skills, concepts and understanding.

Research findings show that through the use of GSP the students are able to illustrate the connection of geometry patterns and functions based on indigenous Thai designs such as Tean-Jok, Nam-Lhai, and Mud-Mee, and also to create new designs. In addition, students' projects on ethnomathematics reveal that their designs are woven into textiles and cloths. These findings display the implications of drawing in students' mathematics project through mathematics learning and commercial product.

Introduction

This research is a case study which emphasizes on students' projects on the ethnomathematics, particularly weaving using the Geometer's Sketchpad (GSP). GSP was introduced in Thailand since year 2000, and in year 2004 GSP was translated into Thai language and has been used widely in Thailand. After that more than 3,000 mathematics teachers have been trained to use GSP as a tool in their mathematics classes. There were 200 workshops conducted on the use of GSP by various agencies in Thailand. The Lab Schools Project (or Dream School Project) of the Ministry of Education, Thailand has set up twenty (20) GSP Training Centers in twenty Provinces throughout Thailand. These twenty Training Centers have trained mathematics teachers at primary and secondary mathematics levels. Where as The Institute for the Promotion of Teaching Science and Technology (IPST), and universities such as Chiangmai University, and Suan Sunandha Rajabhat University have trained lecturers on the use of GSP within mathematics contents at university level. In addition at least 50 workshops on the use of GSP have been conducted by various agencies in Thailand such as SEAMEO RECSAM (Southeast Asian Ministry of Education Organization, Regional Centre for Science And Mathematics), Ministry of Education Thailand under the World Bank loan project on "*Secondary Education Quality Improvement on Mathematics (SEQI): Module 3.*

After attending the workshops, many mathematics teachers consequently conducted action research in mathematics and have incorporated the use of GSP as a tool in teaching mathematics in their classes and have also used GSP in mathematics project-based learning approach. In addition teachers encouraged their students to use GSP in exploring ethnomathematics which are the connection of Mathematics, Arts and Technology in the context of school mathematics in Thailand.

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There are quite a number of research studies on the effectiveness of GSP in students' achievements. However, there are a few research evidences that show the effectiveness of using GSP in ethnomathematics. Hence this research is conducted as a case study.

Empowerment Through Tools: The Geometer's Sketchpad

The Geometer's Sketchpad is one of the dynamic mathematics software that provides opportunities for students to investigate and discover mathematics concepts in particular geometric patterns, functions and graph of trigonometric functions. GSP empowers students to use their abilities to create graphical representation, to enable them in developing their mathematical thinking skills, concepts, and understanding. In using GSP students learn by exploring, investigating and discovering.

Ethnomathematics: What is it?

D'Ambrosio (1985) defined Ethnomathematics as the kind of mathematics practiced among cultural groups such as national-tribal societies, labour groups, children of a certain age, professional classes, and so on. The International Study Group on Ethnomathematics was formed in 1985 by Gloria Gilmer, Ubiratan D'Ambrosio and Rick Scott. The purposes of the International Study group were to increase the understanding of the cultural diversity of mathematical practices, and to apply this knowledge to education and development (D'Ambrosio,U., 1985). The First International Congress on Ethnomathematics was held in 1998 in Granada, Spain. Since then, a variety of definitions of Ethnomathematics were described and the role of Ethnomathematics was much more than improvement of way of teaching. Many educators have given their attention to the subject and a lot of valuable work have been done. There were various online lists of publications relating to the ethonomathetics such as **www.ethnomath.org**. and the ISGEm Newstletter.

Relationship of Ethnomathematics, Thai Students' Weaving Projects and The Geometer's Sketchpad

Students' weaving project is one of the mathematics project-based learning approaches in Thailand. Students have to design and develop their mathematics skills that related to their daily lives. This idea was support by Masingila, J (1993), she said that it is my contention that the gap between doing mathematics in school situations and doing mathematics in out-of-school situations can only be narrowed after much is learned about mathematics practice in the context of everyday life.

In Thailand, mathematics project-based learning approach is employed in secondary schools. Mathematics project-based learning approach is one of the learning activities that shift away from the traditional classroom practices which are isolated, and teacher-centered. This approach emphasizes learning activities that are long-term, interdisciplinary, student-centered, and integrated with real world tasks to enhance learning. Students engage in project-based learning generally work in cooperative groups for extended periods of time, and seek out multiple sources of information. In addition, the project-based learning promotes collaboration among students, between students and the teacher, and between students and the community as well. The Thai Students' weaving projects provide opportunities for students to apply and integrate the content of different subject areas such as Mathematics, Arts and GSP to the production process.

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Research Process

This research is a case study that emphasizes on students' projects on ethnomathematics, particularly, weaving using the Geometer's Sketchpad (GSP).

The main purpose of the study is to explore the connection of ethnomathematics and mathematics contents to arts and technology in Thailand. Data of the study were collected from sample schools in December 2005 until July 2007 from Srisawat Witayakarn School; Nan province, Subprab Wittayakom School; Lampang province and Phana Suksa School; Amnaj Charoen province, Thailand. The 16 year-old students were the Upper Secondary level. Mathematics teachers in these schools implemented GSP as a tool in their classes. Their students used GSP in mathematics projects and worked together in small groups of three to four members.

Research Questions

1. In what way do ethnomathematics, GSP, and Thai weaving designs related to the mathematics curriculum in Thailand.?
2. What are the effects of Thai students' weaving projects on teaching and learning mathematics?

Research Finding

In this study, the researcher collected data from various resources such as classroom observations, students' project reports in ethnomathematics and the weaving designs, newspaper, commercial products of indigenous Thai weaving textile, Thai weaving textile designed by the students in this study and commercial products. Semi-structured interviews with the teachers and students were also conducted.

The summary of research findings are described as follows:

1. Ethnomathematics was included in teaching and learning in mathematics in Thailand.

The researcher examined the secondary mathematics curriculum and mathematics textbooks used in Thai classroom. She found that the ethnomathematics was utilized as a tool in teaching and learning mathematics. The pictures of indigenous Thai weaving textile were shown in mathematics textbooks and displayed as examples of mathematics contents in *Transformation* such as translation, rotation, and reflection. Thai culture and indigenous Thai weaving were taught and showed the relationship of cultures in teaching and learning mathematics. These examples inspired Thai students to explore more topics and issues on mathematics and the knowledge gained was employed in doing their weaving project.

2. *How students used GSP in exploring ethnomathematics and Thai weaving designs?*

The following examples were based on the students' project reports, the researcher's classroom observations, and interviews. The students' work show how they used GSP to construct the geometric pattern and Thai weaving designs. Examples of Thai Students' weaving projects on ethnomathematics with GSP are divided into two topics. They are:

- Indigenous Thai weaving textile; and
- Students used GSP to design Thai weaving textile.

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Indigenous Thai Weaving Textile

Research findings show that through the use of GSP, the students are able to illustrate the connection of geometric patterns and functions based on indigenous Thai designs such as Tean-Jok, Nam-Lhai, and Mud-Mee. The researcher retained the names of indigenous Thai designs in Thai language and used similar pronunciation.

Tean-Jok is the art of woving in the Northern province of Thailand; Chiangmai, Chiangrai, and Sukhothia. The Tean-Jok weaving design consists of geometric patterns such as triangles, squares, parallelogram, and parallel line. The *Nam-Lhai* is the art of woving in Nan province, Audtaradit province; and many other provinces in the North of Thailand. The indigenous Nam-Lhai design looks like the current or running water in the stream. The *Mud-Mee* is the art of weaving in in Amnajcharoen, Chiangmai, Lampang, Lopburee, and Supunburee provinces. The patterns of the Mud-Mee design portray the environment such as animals, flowers, currents, mountains, and mathematical pattern.

Based on researcher's interviews and observations, the students reveal that they are able to draw the pattern of the weaving design on paper using ruler and pencil. They can draw parallel lines, geometric shapes such as parallelograms, squares and lines as show in the Figures below. However they can not show the reflection rotate, or tessellation. In contrast with the use of GSP, the students can copy the picture of the indigenous Thai weaving textile: Tean-Jok and Mud-Mee, and can paste it in GSP. Later on, they can construct geometric pattern on top of the picture. The students used GSP menu to drag the picture or geometric shape until the shape fit with weaving pattern.

The following examples reveal how GSP can enhance students' understanding in mathematical contents and topics. The mathematical contents found are:

(1) Geometry and Symmetry

The students in the sample schools used GSP to construct the geometrical and symmetrical patterns from indigenous Thai weaving textiles as follows:

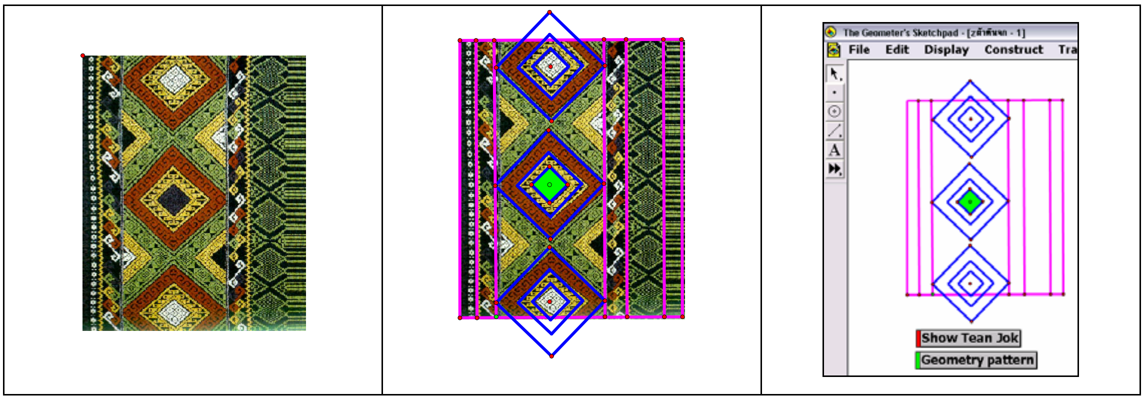

Figure 1: Students' work using GSP to construct the geometrical and symmetrical patterns of Thai weaving: Tean-Jok design

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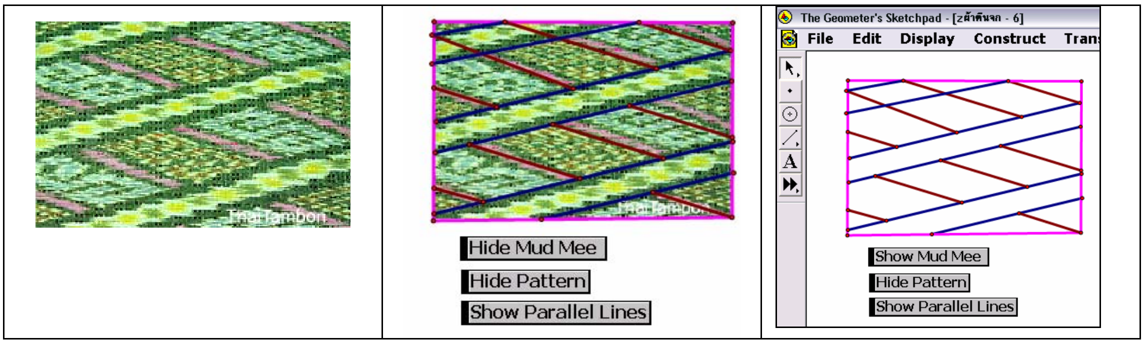

Figure 2: GSP, Geometrical Pattern and Mud-Mee Design

(2) Transformation: Translation, Reflection and Rotation and Tessellation

From the students' project reports, the students works show how they use GSP to illustrate the relation of translation, reflection rotation and tessellation, and the fabric designs. The examples of the works are shown in Figure 3.

Figure 3: The Relation of GSP, Translation, Reflection, Rotation and Tessellation

(3) Absolute Value Functions

According to the students' project reports, the researcher's interviews and observations students can connect the pattern of the weaving design and graphs of absolute value as shown in Figure 4.

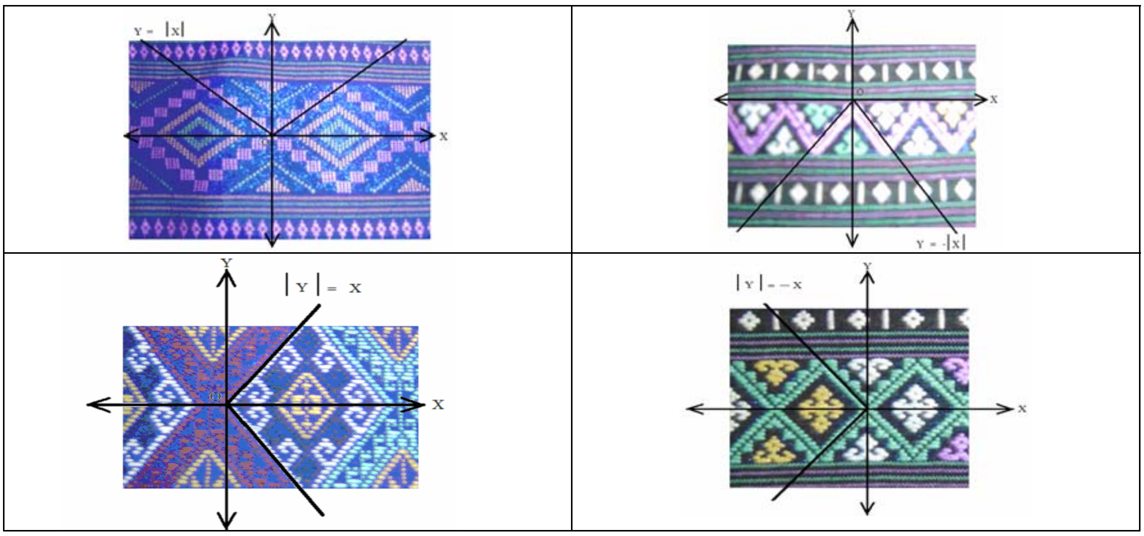

Figure 4: Graph of Absolute Value and the Weaving Design

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The following activities describe how GSP were used to enhance students' understanding in mathematical concepts on absolute value function and the weaving patterns. The activities were:

(1) Started the Geometer's Sketchpad program and chose *New Sketch* from *File* menu;
(2) Copied the pictures of indigenous Thai weaving textile and pasted at point *A*;
(3) Constructed graphs of absolute value as follows:
- Using Graph menu to construct parameter *a*, *b*, *c*, and *d*
- Enter function $f(x) = a - |b \times x|$; g(x) = - f (x); $h(x) = c - |d \times x|$; and q(x) = -h(x) and plot graph of their functions;

(4) Adjusted the parameter *a, b, c*, and *d* until the graphs fit with the weaving pattern as shown in Figure 5.

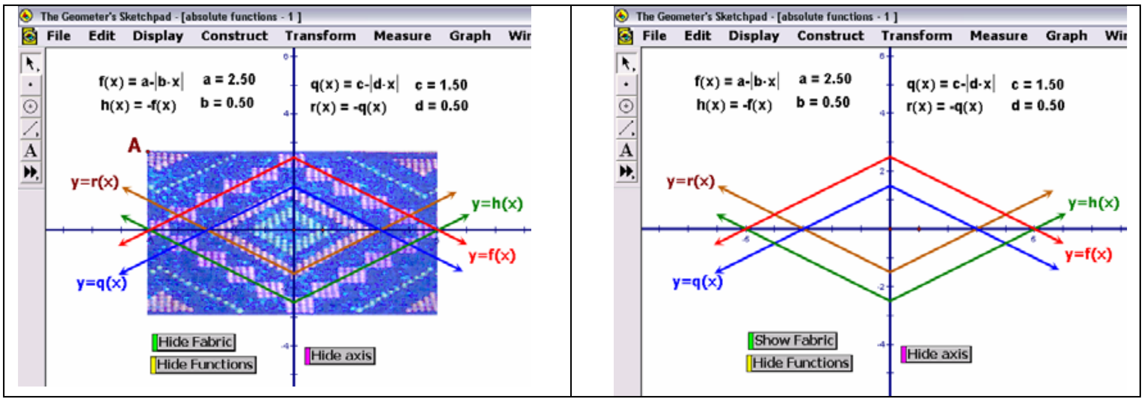

Figure 5 The relation of GSP and graphs of absolute value

Using GSP to Design Thai Weaving Textile

The research findings show that through the use of GSP the students can construct geometrical patterns and graphs of trigonometric functions to design Thai weaving textiles. The students can also create the names of their Thai weaving textile designs. Examples of Thai weaving textiles designed by students in this study were selected from the students' project reports as follows:

(1) Mud-Mee: Lai KarnKab and Mud-Mee: Lai Dok Mali are the new patterns designed by the students of Phana Suksa School, Amnaj Charoen Province, by applying knowledge on translation rotation and reflection to create these patterns.

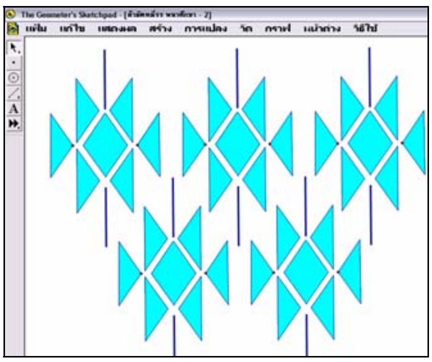

Figure 6: Mud-Mee: Lai KarnKab

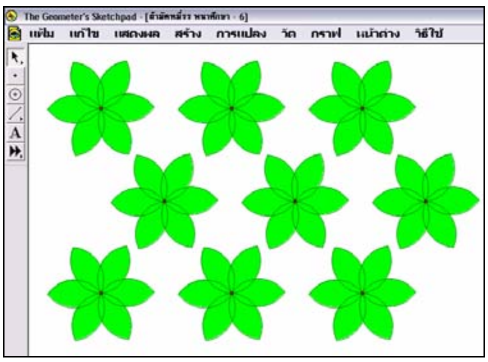

Figure 7: Mud-Mee: Lai Dok Mali

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(2) Thai weaving textile was designed by the students of Srisawat Witayakarn School, Nan Province Thailand. They applied knowledge on graph of functions and arts to create the patterns of Thai weaving textile design. The name of the designs are Samukkee-Klomklew, Klun-Obe-Kao and Kleau-Klun.

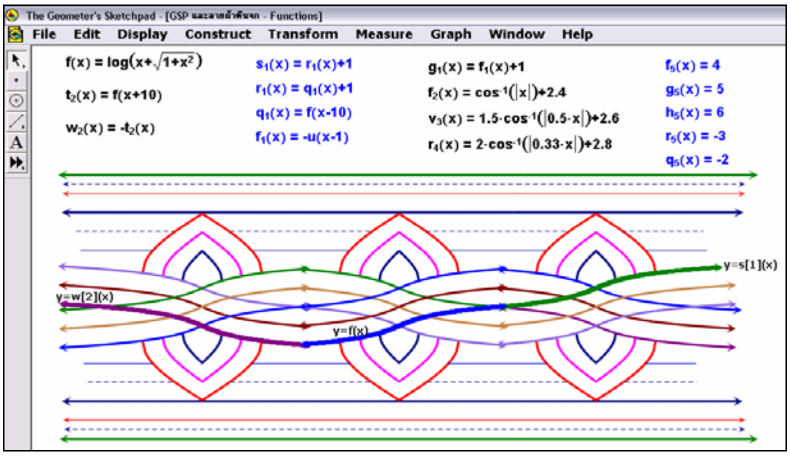

Figure 8: Samukkee-Klomklew Design and some of their Functions

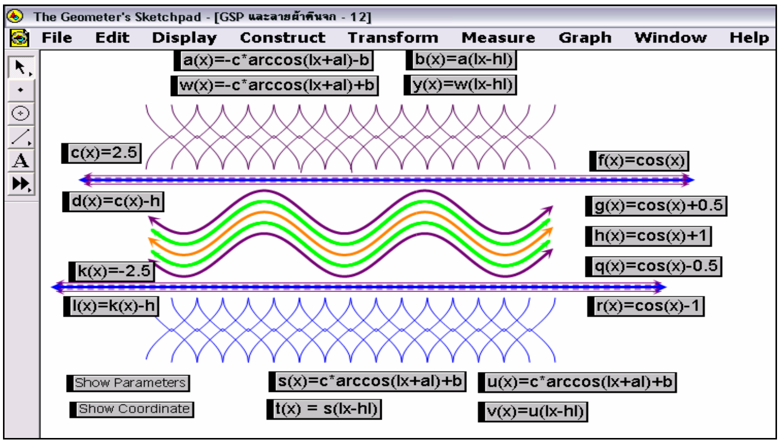

Figure 9: Klun-Obe-Kao Design and some of their Functions

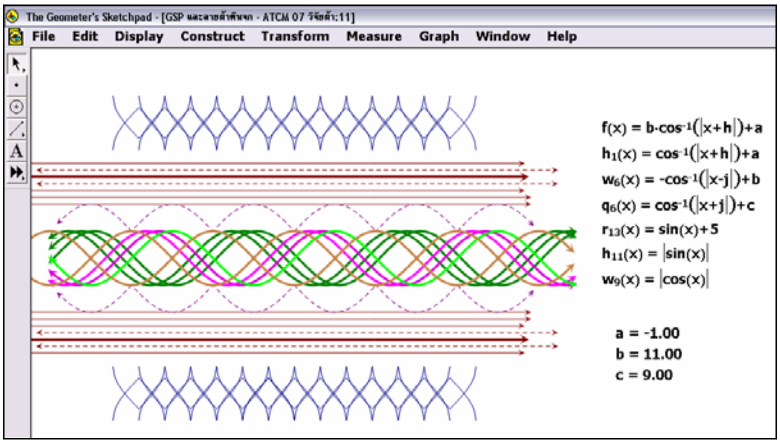

Figure 10: Kleau-Klun Design and some of their Functions

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(3) Thai weaving textile was designed by students of Subprab Pittayakom School, Lampang Province Thailand is showed as Figure 11. The students applied knowledge on geometry, translation, symmetry and arts to create the patterns of Thai weaving textile design named Tean-Jok.

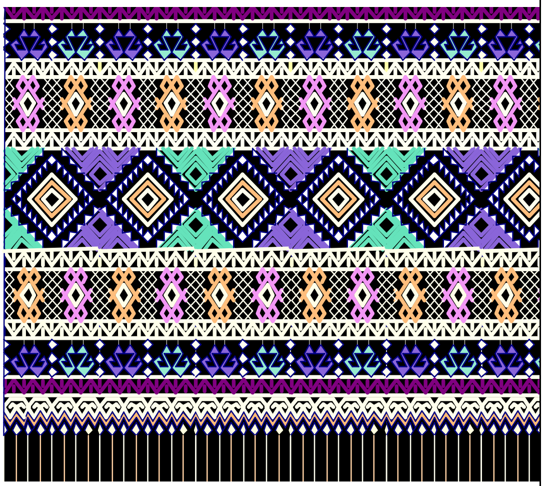

Figure 11: Tean-Jok Designed using GSP

Figure 12: Tean-Jok Textile: Commercial Product

3. *The effects of Thai students' weaving project on teaching and learning mathematics.*

Based on the findings, the researcher found out that the students can exhibit their relational understanding in mathematics which involves understanding structures and connections within concepts (Skemp, 1978). Results from students' semi-structured interview reveal that the after the students did their projects they liked to learn mathematics and they have more understanding on mathematical topics. The students can explain, know what to do and knew why they had to do. The students explained that it was fun in learning mathematics by this method. It was better than work only from the exercises in mathematics textbook. In addition, they have acquired positive attitude toward mathematics.

The teachers of Srisawat Witayakarn School, Nan Province Thailand said that the Thai weaving textile design named *Kleau-Klun* design received *Best award* of students' projects on Mathematics from the Ministry of Education in 2005. The schools and teachers who were involved in this project are also recognized by the Ministry. Since then, the students' mathematical projects on ethnomathematics using GSP have been used nationwide.

4. From Thai students' weaving project to Commercial Product.

Kleau-Klun, Mud-Mee: Lai KarnKab, and Tean Jok designs are students weaving and have become commercial textile products. The photographs of the textile are shown in Figure 13.

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Figure 13: Kleau-Klun Design and Commercial Textile Product

Conclusion

This paper shows that through the use of GSP the student are able to illustrate and also to create new designs in connection with geometric patterns and functions based on indigenous Thai designs such as Tean-Jok, Nam-Lhai, and Mud-Mee. It is clear that, with GSP students are able to visualize and create graphical representations to enable them to develop their mathematical thinking skills, concepts and understanding. This project can further facilitate learning among students establish useful connections in Ethnomathematics, particularly, weaving to life in the world outside classroom and to develop commercial products.

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