

Mathematics Teaching and Learning with Technology

Part I

Using Videotaped Components of Lesson Study to Build Communities of Practice for Prospective Mathematics Teachers

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Abstract: Two tiers of pre-service mathematics teachers participated in an exploratory investigation in which components of lesson study were used to develop, teach, refine, revise, re-teach, and evaluate a unit on mathematical relations. Twelve fourth-year prospective mathematics teachers served as the instructional developers, planners, analyzers, and instructors for the unit, and twenty-five, second-year future elementary school mathematics teachers participated as students. Videotapes of the lesson study and the instruction showed that the fourth-year pre-service teachers were modestly successful in developing their second-year peers' understanding of the properties of a mathematical relation, primarily by using arrow diagrams and relations on sets of people to give meaning to the properties. Analysis of the videotapes and written lessons showed that the participants who taught the lessons were prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. The fourth-year pre-service mathematics teachers prepared a final paper reflecting on their experiences with the lesson study. A review of the final papers showed that these participants valued: (1) the opportunity to experience the lesson study process within the setting of an academic classroom, (2) the opportunity to work in a group to develop and improve a lesson, and (3) the experience of observing, and in some cases teaching, a lesson that incorporated the joint efforts of other participants. In retrospect, most of the participants also indicated that they would include more examples in the instructional unit, more examples of equivalence relations, more mathematical examples, and more examples generated and constructed by the students. Increasing class participation was also a concern expressed by most participants.

1. Introduction

Lesson study, a collaborative professional development approach that originated in Japan, is gaining widespread popularity in mathematics education (see [1]). In the lesson study process, a group of teachers meet as a team to set goals and to carefully craft and collaborate on the design of a lesson. Once the lesson is designed one of the teachers in the group teaches the lesson, while the other group members observe the lesson. Later the whole group evaluates and reflects on the lesson discussing ways to reteach it as a means of making it more effective and stimulating for the students. The revised lesson is taught, observed, and reflected upon a second time.

Chokshi and Fernandez [2] believe that lesson study provides a large potential influence on the impact of the professional development of teachers as it creates a professional knowledge among teachers, while offering a connection between educational policy and practice. Hiebert and Stigler [3] concur with this notion and describe lesson study as an institutionalized teaching improvement

system built on research and the idea that teaching is a complex, cultural activity. Particularly, lesson study enables an activity structure that affords shareable and upscales knowledge for teacher professional development.

2. Prior Knowledge and Expectations of the Lesson Study

As part of their senior-level mathematics education seminar, twelve pre-service mathematics teachers reviewed research literature on the lesson study process in preparation for planning and implementing of an actual lesson study. Prior to preparing an instructional unit on mathematical relations, the twelve future teachers prepared discussion papers that outlined their goals, objectives, and expectations for the lesson study. These papers allowed the future teachers to express their personal concerns before undertaking the lesson study.

Within these papers, the pre-service teachers discussed their desire to work well within each study group, to involve students in the instructional activities, to enable students to clearly understand the instructional content, and to enable the future teachers to understand how to evaluate their own and other's teaching. In the discussion papers the future teachers also expressed reservations about only being able to observe the lessons on videotape instead of being present at the lesson and the time-consuming nature of the lesson study process. The future teachers also wrote about their expectations of viewing different types of effective teacher behavior and of their anticipation of using the lesson study process to learn how to adapt and revise a lesson before re-teaching it. In addition, the future teachers who were scheduled to teach the first lesson related their nervousness in teaching a topic that they did not feel they fully grasped, while the future teachers who were scheduled to teach a second (a revision of the first) lesson wrote of their hopes to improve the quality and presentation of the lesson and to use the lesson study experience to make themselves better prepared teachers.

3. Participants

This study involved two tiers of participants. Twelve pre-service mathematics teachers who were enrolled in a senior-level mathematics education seminar planned and implemented the lesson study. Twenty-five students who were enrolled in a second course of mathematics for elementary school teachers were divided into groups and served as the subjects of the study.

4. Process

During their mathematics education seminar, five of the future teachers jointly prepared a lesson plan on relations and equivalent relations that included the lesson's relation to NCTM Standards, teaching procedures, worksheets, and assessment. Two of the pre-service teachers then taught the lesson to a group (Group 1) of prospective elementary school teachers.

The lesson was videotaped and later viewed by all twelve of the pre-service mathematics teachers who wrote reaction papers to the lesson. During a subsequent class the future teachers discussed ways to improve the lesson based on results from the in-class worksheets, assessments, and from debriefings by the two participants who taught lesson.

Two other seminar participants, neither of whom had written the first lesson plan nor taught the first lesson, prepared a second, revised lesson plan and taught the lesson to another group (Group 2) of prospective elementary school teachers. After the videotaping of the second class, all twelve future teachers viewed the tape and wrote reaction papers. The videotape of the class instruction and the pre-service teachers' reactions were discussed during the next seminar meeting. The prospective teachers were then instructed to write a summary paper of their overall views on lesson study and on their lesson study experiences.

5. Instructional Materials for Lesson One

The instructional materials used in the first lesson included an overhead transparency that described a relation on a set X as “any set of ordered pairs in which the first and second components are from X.” Reflexive, symmetric, and transitive properties of a relation were also described for a relation on a set X, and an equivalence relation was described as any relation on X that satisfies these three properties. The lesson also included three tasks on activity sheets for the students to complete.

During the first lesson students were given the definition of a relation and an explanation of its properties. They were then given an example which asked them to indicate whether the “relations on the set of all people are reflexive, symmetric, or transitive” and instructed to complete the following table.

Table 1 Table of Relations and Properties of Relations from the First Activity Sheet

Relation	<u>Reflexive</u>	<u>Symmetric</u>	<u>Transitive</u>
“is an ancestor of”			
“is a different age than”			
“has the same income as”			
“knows a telephone number for”			

The second activity sheet asked the students to determine if the relations given by three arrow diagrams were reflexive, symmetric, or transitive as in the next diagram.

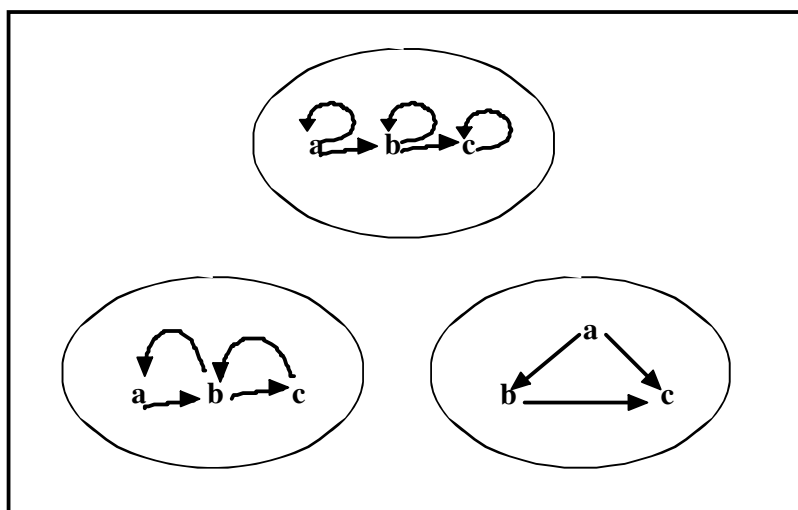


Figure 1 Diagrams of Relations from the Second Activity Sheet

The third activity sheet asked the students to draw ovals and construct arrow diagrams that showed relations that were: (1) reflexive and symmetric, but not transitive; (2) reflexive and transitive, but not symmetric; and (3) symmetric and transitive, but not reflexive.

6. Instructional Materials for Lesson Two

The refined second lesson plan also included definitions of the reflexive, symmetric, and transitive properties as well as a definition of an equivalence relation. However, a few changes were made based on discussions during the seminar class. The second lesson plan included a paper copy of a transparency sheet containing a definition of a relation along with four examples of relations: “has the same decimal value as”, “is bigger than”, “ $x^2 + y^2 = 1$ ”, and “ $x + y < 1$ ”. For the relation “has the same decimal value as”, an ordered pair member of the relation was presented with the statement “(1 / 2, 2 / 4) is a relation”. For the relation “is bigger than”, the example was the statement “5 is related to 3 but 3 is not related to 5”. For the relation “ $x^2 + y^2 = 1$ ”, the example included the statement that “(1, 0) is a relation, but (2, 2) is not”. For the relation “ $x + y < 1$ ”, the example contained the statement “(-1, 0) is a relation, but (2, 5) is not”. A table similar to the one given to the first group was used except that the relation “is taller than” replaced “is an ancestor of” and the relation “knows a phone number for” replaced “knows a telephone number for”. Also included with the second lesson plan were the same second and third activity sheets that dealt with arrow diagrams.

7. Results

After both Group 1 and Group 2 students had completed the lesson on relations, they completed test items that used arrow illustrations to represent the reflexive, symmetric, and transitive properties. The test items particularly asked students to illustrate situations that were 1) symmetric and transitive, but not reflexive; 2) reflexive and symmetric, but not transitive; and 3) reflexive and transitive, but not symmetric. Results showed that students responded with illustrations that were arrow diagrams.

In Group 1, 60% of the students illustrated a situation that was symmetric and transitive, but not reflexive; while only 3 of the 6 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Also, 30% of Group 1 students illustrated a situation that was reflexive and symmetric, but not transitive while only one Group 1 student illustrated a situation that was reflexive and transitive, but not symmetric.

In Group 2, 30% of the students illustrated a situation that was symmetric and transitive, but not reflexive, and each of the 3 correct illustrations depicted the transitive property for a relation on a set of 3 elements. Half of the Group 2 students correctly illustrated a mathematical situation that satisfied the reflexive and symmetric properties, but not the transitive property. Forty percent of Group 2 students illustrated a situation that was reflexive and transitive, but not symmetric.

8. Students' Reactions to the Lessons

Students from both Groups 1 and 2 were asked to write their reactions to the lesson on relations, their properties, and equivalence relations. In particular, they were asked to comment on what aspects of the lesson had been most helpful to them and to give an overall assessment of the lesson. When asked which aspect was most helpful, 40% of the participants from Group 1 responded favorably to the diagrams; whole 20% noted the examples and 10% responded each to the explanations and the overhead transparency. Group 2 also responded favorably to the diagrams (40%); followed by 17% for examples and 8% for the explanations. No participants in Group 2 rated the overhead transparency as being helpful. Notably 70% of the students from both Group 1 and Group 2 rated the class presentations as being very good.

9. Seminar Participants' Overall Comments and Discussion of Their Experiences with the Lesson Study

At the conclusion of the viewing of videotape of the second lesson, the pre-service mathematics teachers were asked to prepare a final paper describing their experiences with the lesson study and any overall comments they had about the study. A review of the final papers showed that the participants valued:

- (1) Opportunities to experience the lesson study process within the setting of an academic classroom,
- (2) Experiences of working in a group to develop and improve a lesson, and
- (3) Opportunities to observe, and in some cases teach, a lesson that incorporated the joint efforts of the seminar participants.

The participants also reported that they had added to their repertoire of teaching skills by observing and discussing the videos of the lesson. A majority of the seminar participants also reported that they saw improvements in the lesson implementations in moving from the first to the second lesson.

On the other hand, seminar participants expressed some disappointment that their lesson did not engender more classroom participation and student involvement. They also noted the criticism levied by some participants about the preparation and classroom performance of participants who taught or prepared the plans for the first or second lesson. Nearly half the seminar participants made reference to examples that were presented in the first lesson as being confusing and in need of being reviewed for clarity before the lesson. A major thrust of the suggestions for improvement was for pre-typed overhead transparencies with the definitions, for more explanations and student involvement, and for more examples of relations and their properties.

Overall, most participants indicated that they would include more examples in the lesson, including more examples of equivalence relations, more mathematical examples, and more examples generated and constructed by students in the classroom. Increasing class participation was also a theme of the comments from most of the seminar participants.

10. Conclusion

According to Lewis, Perry, and Hurd [4], “Lesson study is not just about improving a single lesson. It’s about building pathways for ongoing improvement of instruction (p.18)”. This was exactly the situation that these pre-service mathematics teachers experienced. They developed working relationships within their groups that allowed them the opportunity to collaborate and reflect on their mathematics lessons. The prospective teachers showed evidence of enhancing their lesson study experiences by not only following a cohesive lesson study cycle that included both mathematical and pedagogical knowledge, but also exhibiting collegial qualities that supported learning.

Another benefit of the pre-service teachers’ lesson study activities was the involvement in collaborative tasks that helped them systematically develop their understanding of recent educational reform. The topic of educational reform constituted a sizable component of the seminar’s content, but the reality of implementing classroom reform was very concretely encountered by participants through their involvement with the lesson study activities. Analysis of the videotapes and written lessons showed that the participants who taught the lessons were more prone to tell their students about relations without providing tasks for the students to conjecture and construct their own relations. During the seminar participants’ commentaries on the video of the second lesson, it was apparent that most had come to recognize the need to involve students more actively in the construction of relations through experimentation and mathematical reasoning.

The future teachers’ implementation of lesson study was not without challenges. In particular, some of the future teachers expressed an anxiety of being videotaped and of making their teaching public as well as finding ways to agree on common ground for joint lesson planning. They also experienced some of the challenges to lesson study that relate to research skills such as posing researchable questions, designing classroom experiments, specifying types of evidence for collection, and interpreting and generalizing results.

Since lesson study is explicit in virtually all areas of the curriculum in Japan and elsewhere, it is certainly feasible to extend the use of the components used in this study to other content in mathematics teacher education. Successful extensions to other content are apt to require less instructional time and effort by selecting topics that are relatively straightforward for prospective teachers to teach as demonstration lessons.

Hiebert and Stigler [3] concluded their review of lesson study by noting that lesson study’s chances of success in the U.S. were closely tied to society’s willingness to recalibrate its expectations for change by adopting a long-term improvement strategy that guarantees more effective teaching, not 1 year in the future, but 20 years in the future. The results of this study to use components of lesson study with pre-service teachers bear out this long-range view. The pre-service teachers enrolled in the senior seminar had an advantage in that they did not have some of the ingrained assumptions and solo practices associated with practicing teachers.

Whether or not lesson study will become an important technique for instructional improvement in mathematics education remains to be seen. However, its methodology of working collaboratively to plan, observe, and redesign a lesson certainly warrants innovative implementations.

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Development of the Web resources for Interactive Lessons in Geometry

- Based on the mathematics standards in Iran -

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Abstract: The purpose of this paper is to develop an interactive web resources for the mathematics curriculum concentrated on Geometry. Based on the result of our previous surveys (Behnoodi, M., & Moriyama, J., ATCM 2006) we designed the web resource into eight types of contents categories. A) Lesson planning, B) Dynamic and interactive component, C) Printable worksheets, D) Explanation in details, E) Encourage the students in challenging mathematics learning, F) Projects and presentations, G) Questions and answers, and H) How to use the web resource and math software. By these web resource it is expected that the teachers access virtual sections by printing or using simulations and active functions which can be changed by students. Teachers can see the results of students' progress at the end of each lesson by checking the answers of the questions or the degree of difficulty of each section.. Even more, they can make new simulations by themselves after they read our guide manual for software used in the web resource.

1. Introduction

The National Council of Teachers of Mathematics (NCTM, 1989) recognizes the importance of geometry and spatial sense in its publication “*Curriculum and Evaluation Standards for School Mathematics.*” Spatial understanding (3-Dimension imaging) is necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions concerning two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Children who develop a strong sense of spatial relationship and who master the concepts and language of geometry are better prepared to learn number and measurement concepts, as well as other advanced mathematical topics.” (p. 48)

The geometry textbooks of the new system of secondary education in Iran differ dramatically from the old ones in regards to their aims, visions, content, approach, and educational purpose. Four hundred and eighty mathematics teachers participated in a nationwide professional development program conducted to facilitate the implementation of the new changes. (Gooya, Z., 2007, p331).

During the last decade, major advances have occurred in the development and use of instructional technologies, such as hypertext-based computer software and multimedia presentation systems.

When properly used, instructional technology has the potential to produce significant gains in both student achievement and attitude towards geometry.

As technology is now in routine use by students within mathematics classrooms, this technological advancement motivates students to use technologically based educational learning tools to better understand mathematical concepts. Technology transforms what is possible in the teaching and learning of mathematics (Instructional Technology in Mathematics education, IT 1171, The University of Pittsburgh). "The advent of computers and calculators in the classroom facilitates a new approach, one where the focus is on reasoning with a variety of representations and understanding the relationships among those representations" (Dugdale et al., 1995, p. 330). Therefore, it is important to provide the next generation of teachers with opportunities to experience firsthand mathematics learning activities that incorporate technology into their teaching.

The purpose of our research is to develop a web resource for teachers and students based on NCTM standards, mathematics standards in the new geometry textbooks used in Iran and the result of our online survey. In developing their curriculum in mathematics education we attempted to clarify the teachers' needs and *viewpoints* toward web resources of the internet (Behnoodi, M. & Moriyama, J., ATCM 2006). By conducting an online survey, we gathered feedback from mathematics teachers from various countries including Iran and Japan. The teacher's expectations of our mathematics web site were:

- A) Lesson planning
- B) Dynamic and interactive activities
- C) Printable worksheets
- D) Detailed explanations of geometrical concepts
- E) Challenging geometry problems for advanced students
- F) Projects and presentations
- G) Questions and answers
- H) How to use web resources and math software

2. Concept for Development

The results of the survey suggest that lesson planning with dynamic and interactive activities is the most important need as far as the teachers are concerned. Lesson planning should contain dynamic and interactive components and activities should sometimes be controlled or manipulated by the students who are the active learners in the lesson. The students can answer questions and immediately see the results of their answers. Some extra detailed explanations are necessary for better understanding as well as more difficult questions for those students who are functioning at higher mathematics levels. In this way, the students are engaged in challenging mathematics learning in addition to mastering the different technological software used in the web site. The students become fully engaged and can even create some new parts by themselves and submit them to the website to possibly be linked for usage by other users. Further more, if the teachers have interesting teaching methods in their classrooms they can record their speech and actions then send it for linking. The site also contains a forum for questions and answers.

An additional advantage of this site is the software training provided can be used for making the applets and animations. When software is designed for use across a variety of mathematical topics,

it can be designed for many different applications and the user can examine the area of mathematics in which the software will be used and develop lessons that promote the type of learning on which they will focus. A dynamic geometry program allows the user to construct, measure, and manipulate what is displayed on the screen, providing immediate feedback as the object changes size or shape.

The main advantages of this kind of web site are, *free and easy access* without asking ID or password, printable worksheets, and unlimited use for any movies, animations or applets in education by teachers and students. Students interactively discover the properties of geometric figures and literally create geometry for themselves. In order to teach students how to use the software, we teach how to produce a variety of activities which integrate the computer skills forming a parallel with the geometry content.

By doing the activities in this web site, the students learn, practice, and apply rotations, translations and reflections and link mathematics to space and form in the world around them. In this way students investigate two-dimensional and three-dimensional space by exploring shape, area, and volume; studying lines, angles, points, and surfaces; and engaging in other visual and concrete experiences.

3. Methods for Development

When we started to make some applets by *Java* we thought mathematics teachers may find programming is too difficult in some cases as most of them are not professional in computer programming; therefore, we searched for an easier way. *Cabri Java* was the software that answered our needs; it was easier for some of the teachers to find it difficult. Furthermore it can be compatible to transfer into html pages as a Java applet. Some of the animations which were produced by *3D-Studio Max* could be shown as 3-dimensions shapes in Geometry unlike conventional textbooks. By using *Ulead Gif Animator* we could change the AVI files to Gif files and the volume decreased. For capturing a movie like power point slide show to AVI files we used *Camtasia recorder* and then used *Cyber link Power director* to change the AVI files to WMV files. For quiz and examination we used Java script; it can calculate the examination results at the end. Students can compare their answers if they click on each explanation; a new page will open and explain the solution method by animation or simulation. We can save all printable worksheets in PDF file format without any changing the original one in each computer. With well-designed review, and practice software, the role of technology is to reinforce skills through an optimal sequence that assures certain predictable outcomes (Hooper & Hokanson, 2000).

4. Developed Web Resources

The structure of the site map (Figure 4.1) contains five base branches, “about us; online lectures in geometry grade 10; resources; learn software and contact us”. Online lectures contain the interactive lessons in geometry according to the NTCM and Iran standards focused on high school 10th grade text book. This part contains four sessions; Reasoning, Pythagorean Theorem, Similarity and Solid Geometry.

The webpage producer tried to use samples depending on daily life sciences with lesson planning, applets, animations, video streaming, printable worksheets and some tips for teachers to use in their class. Project and presentation that we introduce to students and teachers and the other activities done by the students from all over the world can be joined and collaborated with each

other. Teachers can share their teaching through movie as a presentation for common users, and links for more usage. In addition the different components of each lesson are arranged separately in special folders in order to allow the user to access the applets they need. If they want to print all the worksheets, they can find all those worksheet in the separate worksheet folder.

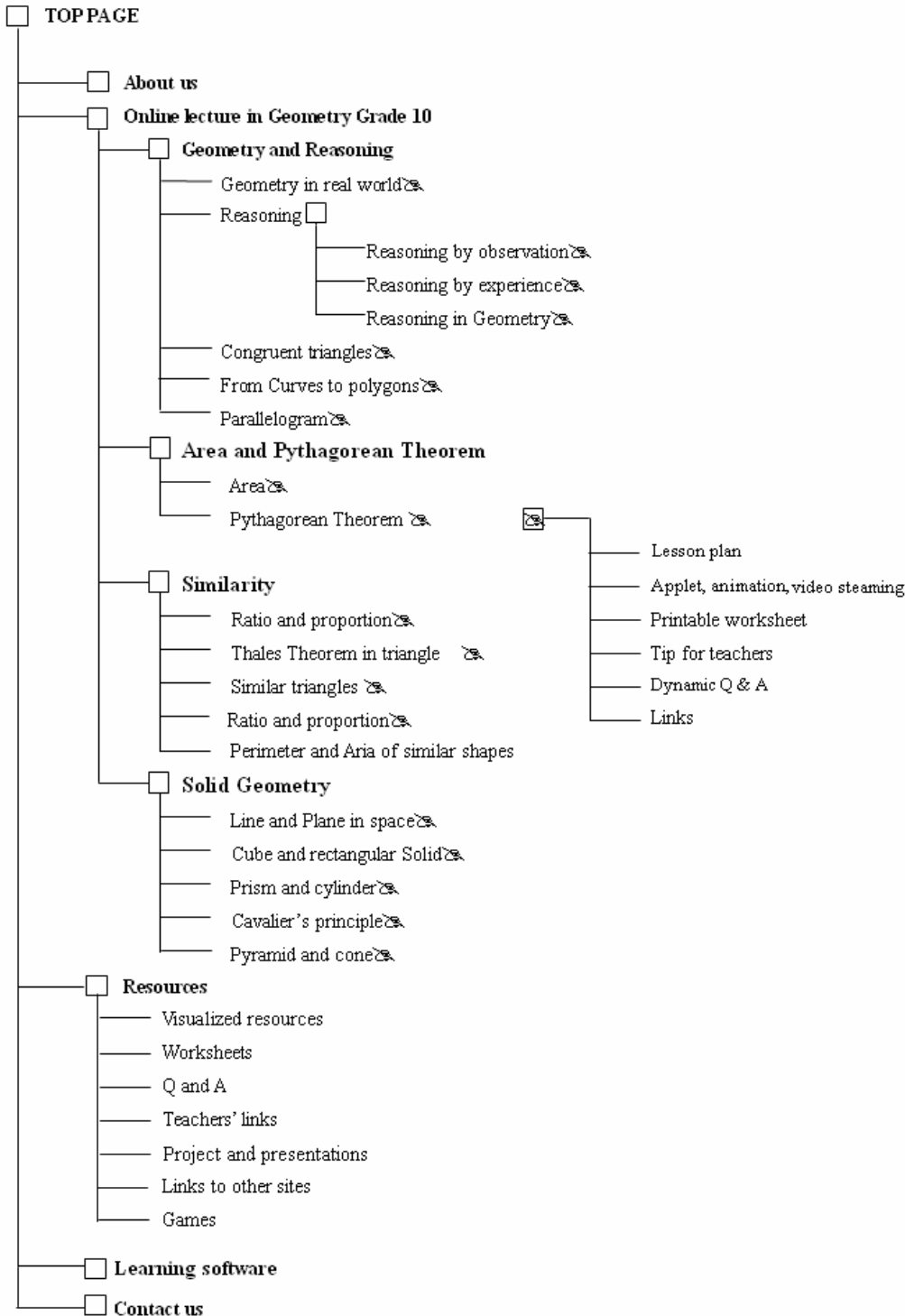


Figure 4.1 The structure of site map

Samples of developed web resources

Sometimes by observation and practical experiments we can see the proof of theorems.

As shown in (Figure 4.2) the unit squares can be moved from one part to the other part and students can recognize the truth of that theorem. Of course later they will learn Geometrical and Algebraic proof. One of the effective ways in teaching is visualizing the problems for easy understanding. 3D-Max is the software that we used for most of the 3D shapes that can not be recognized on the black board.

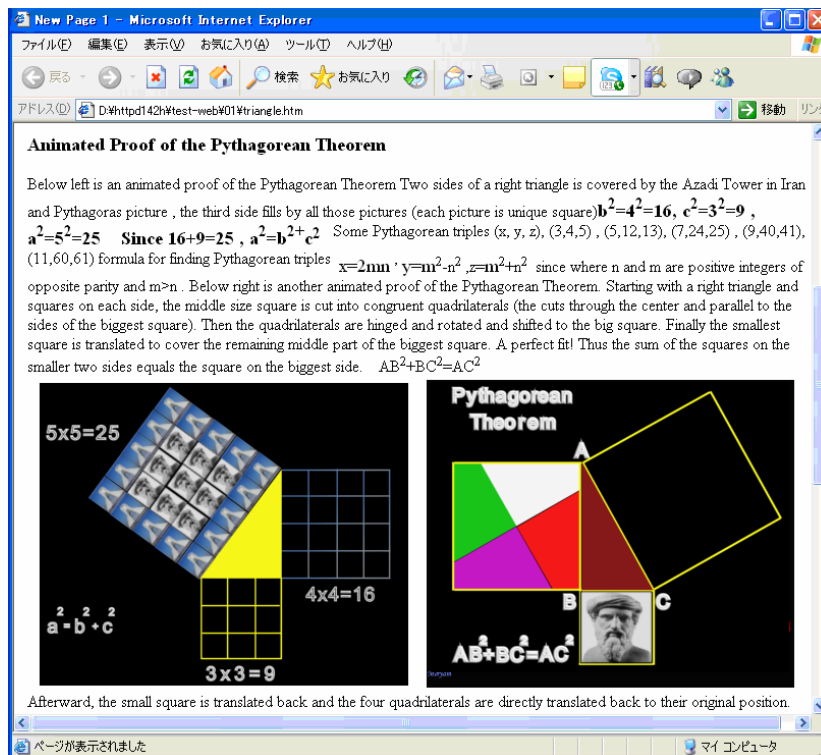


Figure 4.2 A sample of GIF animation created in AVI by 3D-Max

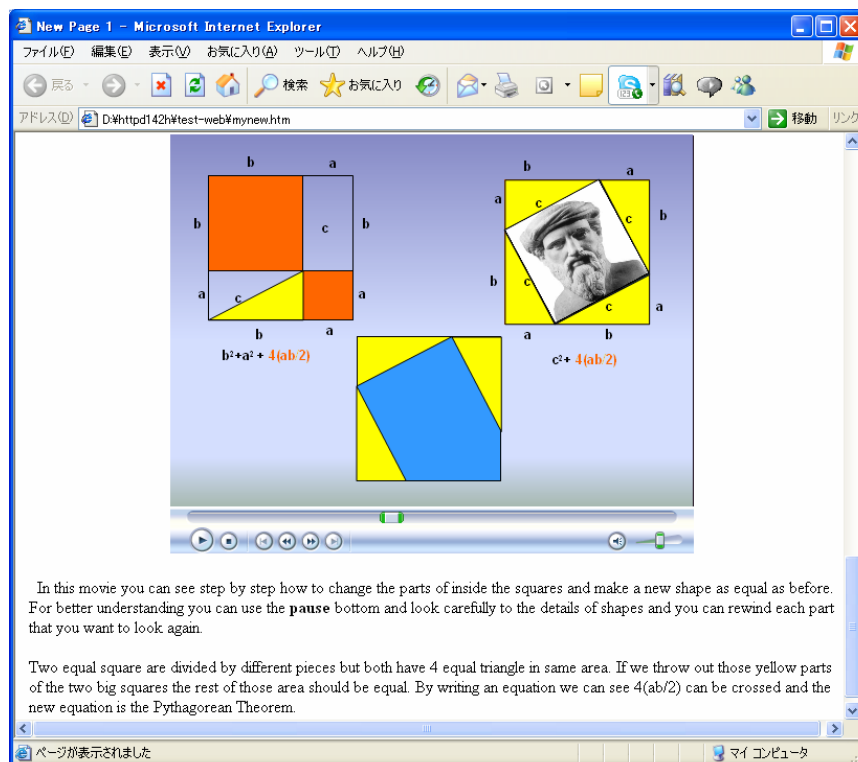


Figure 4.3 A sample of video streaming from power point show

In (Figure 4.3) this video streaming is made by PowerPoint presentation and after that we made a movie of the PowerPoint show by the screen captures as an AVI file. Using of video streaming makes it Embed in html file, allows students and teachers to control the display. They can pause and talk about each part that they think need some explanation. Even more the voice and explanation can be recorded on each part.

Sometimes by observing the movement of a shape, changing the data and comparing those data, the students can see the differences or equalities between the shapes. In this sample (Figure 4.4), It can be seen that four triangles, side by side are equal. So the students, before proving have an imagination of can image the equalities before proving. In many case the students can see the results while they type the data on the webpage. This applet is made by Cabri Java and can be read in html pages. One possible advantage is, Cabri Geometry II that it doesn't need to make programming. Therefore, a mathematics teacher can easily learn how to work by this software. By Cabri Geometry III software, the teachers can make some 3-dimensional demos that can not be replicated on the traditional blackboard.

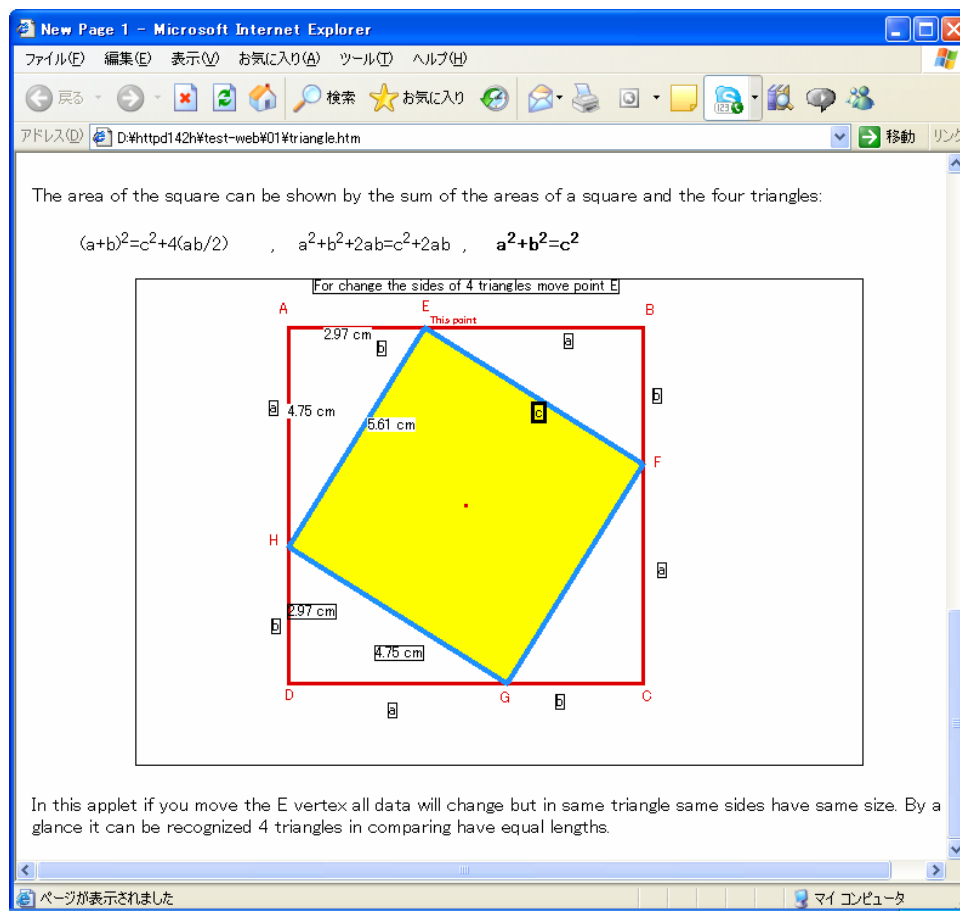


Figure 4.4 A sample of Java applet created by Cabri Geometry II

(Figure 4.5) shows two levels of worksheets. Level A is meant for common students and Level B is meant for discussion in a group training setting. The worksheets include samples related to daily life with colourful photos included. If students have a colour printer available, the questions and their components can be more attractive and aide in clearer understanding for the students. Some computers tend to change fonts or the layout upon printing, so all worksheets are made in PDF format and are printable. The users can copy these pages freely as long as by just acknowledging the name of the site for copyrights.

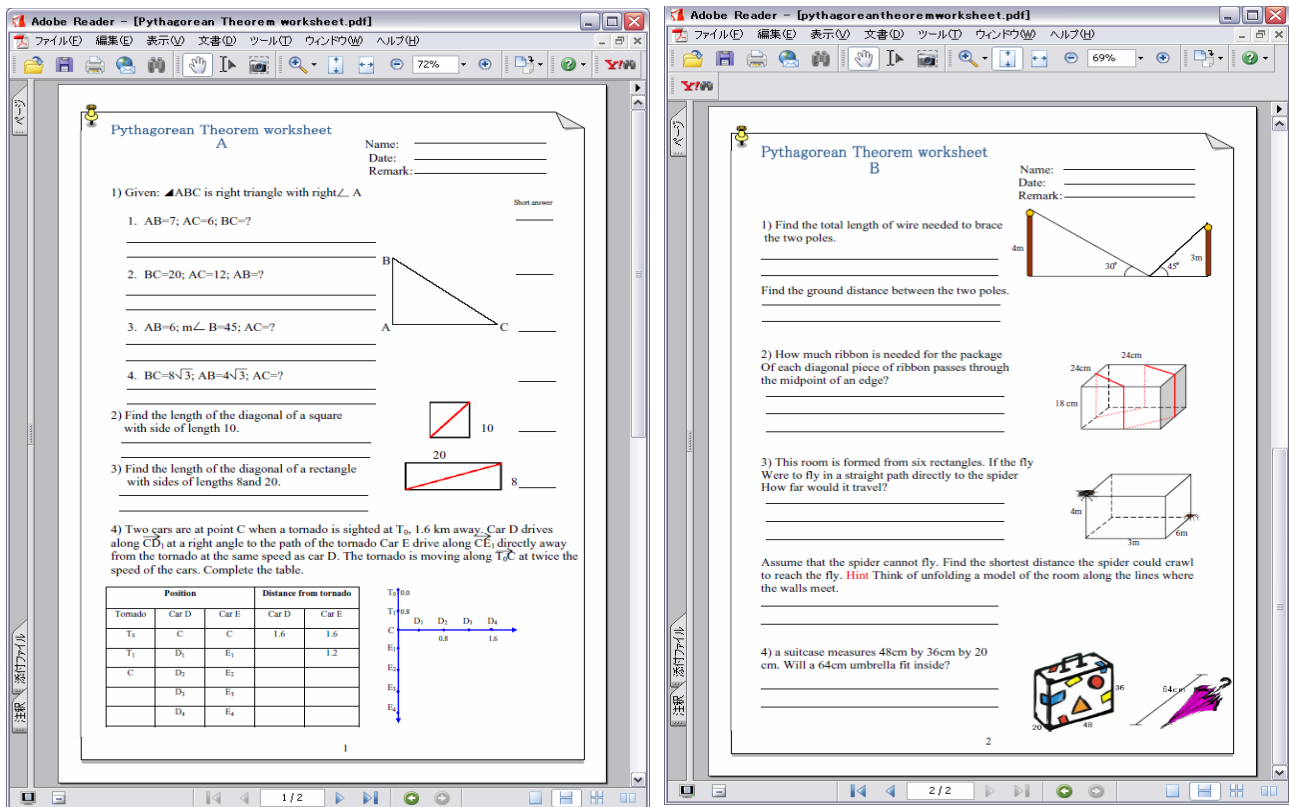


Figure 4.5 Two samples of printable PDF worksheet

As shown in (Figure 4.6), the dynamic question and answer sheets designed by Java Script are presented. There are two frames, for worksheet and answer sheet. In this kind of multiple choice answer sheet the students can answer each question by clicking on A, B, C, or D. After finishing the quiz they can see the results of their examination, and then by clicking on solutions they can compare their answers. In ordinary worksheets the students only observe a paper in black fonts and shapes which are often drawn by hand, however by using the computer generated illustrations the students gain a more accurate sense about questions by looking at the animations. They can come to understand the different situations of each question.

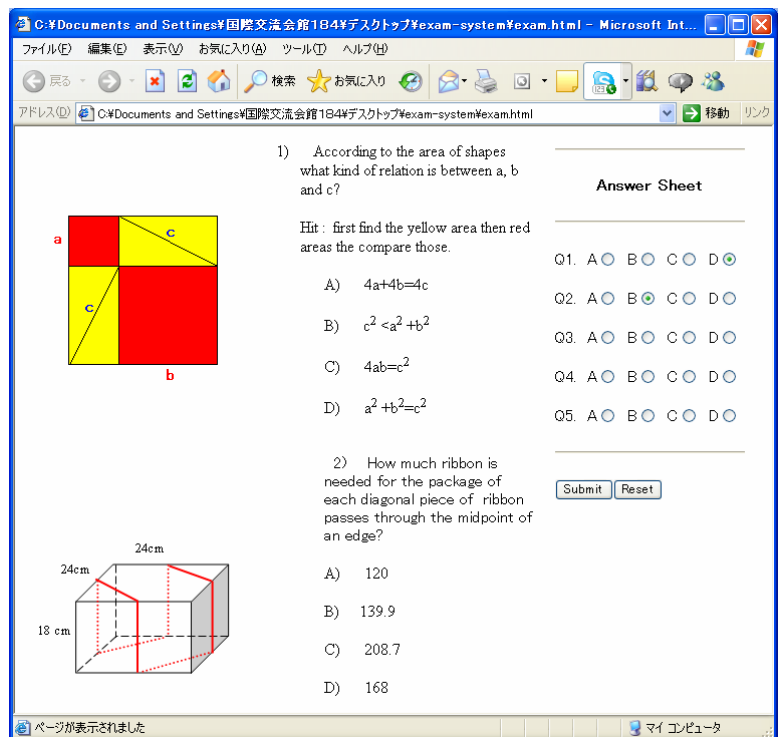


Figure 4.6 Dynamic Q & A by Java Script

For an extra challenge we made some additional worksheets for students who would like to push further into higher levels of research. (Figure 4.7) If the students solve these problems step by step by the end they can discover some new rules which correspond to the proof that can be seen in their text book. This allows them to look at problems with a different perspective. This part not only contains extra worksheets but also depending on the lesson contains some activities, puzzles and games to guess the answers, depending on their previous knowledge. Let us try further to introduce some famous unsolved problems for more thinking and finding a solution.

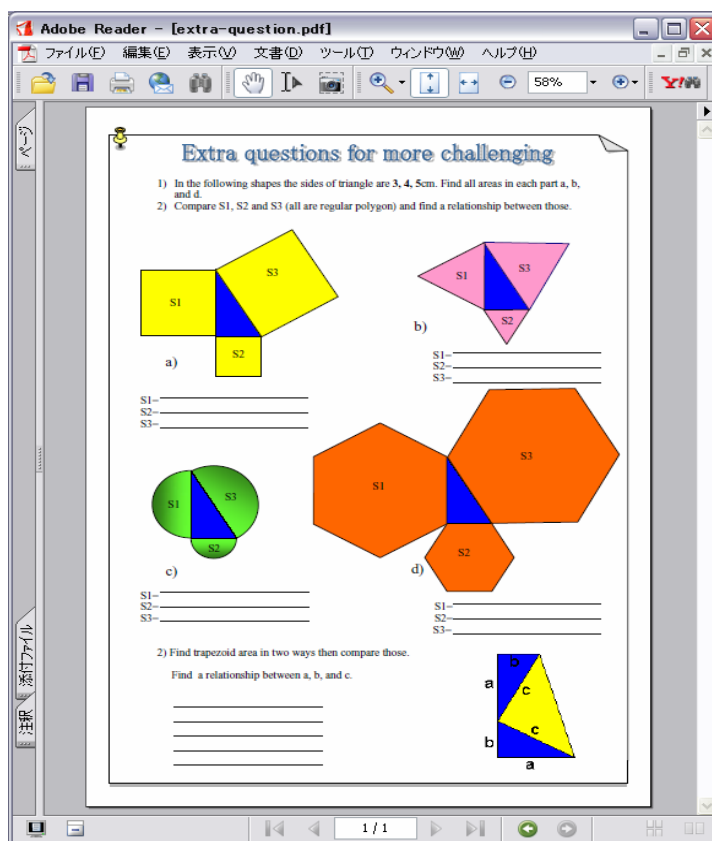


Figure 4.7 A sample of extra sheet for more challenge

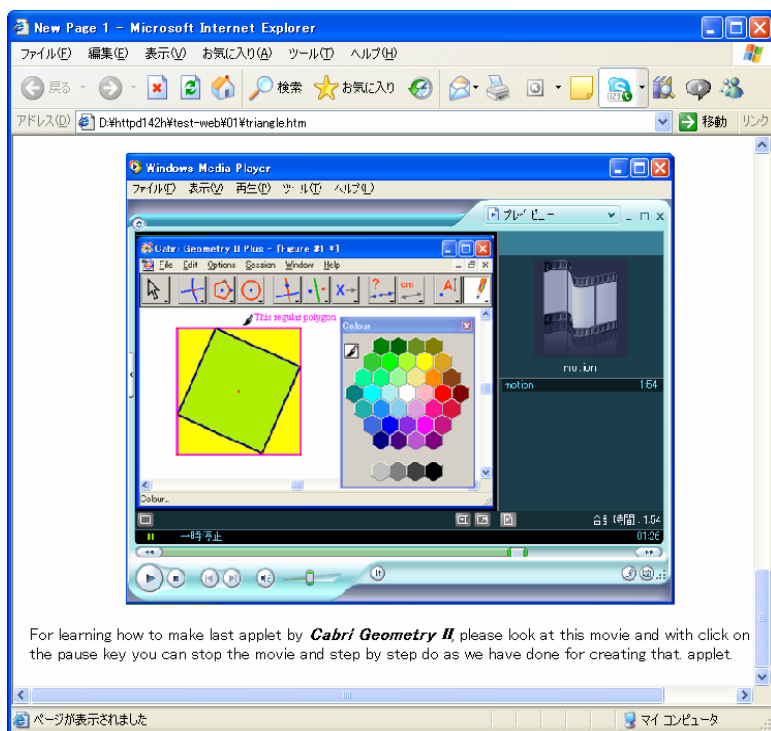


Figure 4.8 A sample of how to use Cabri software by movie streaming

The main component of this website which differs from other sites is the learning software. We tried to teach, how to use the software that we used in making interactive parts in each session. (Figure 4.8) The users not only can use the interactive parts for learning Geometry but also they can also create new tasks provided their school allows for common usage. (Of course some of the software can be downloaded for free on the internet). We provided movie demonstrations of each part allowing the users to pause as needed and learn step by step.

5. Conclusion

Multimedia presentations are advantageous because they can developed students levels of understanding in stead of traditional classroom setting. Typically, students do not expect to actively participate in the lecture. They expect that the teacher will come to class and tell them what they need to know; i.e., what material will be on the exam. The students' role is to copy all of this information into their notebooks and memorize each problem without analyzing or synthesizing the material. We should take a different role, such as that of a coach, or a guide, who leads students in the right direction instead of trying to fill their mind with facts. There are a number of potential advantages of using the computer as a tool for instruction in an educational setting. First, technological tools help to support cognitive processes by reducing the memory load of the student and by encouraging awareness of the problem-solving process. Second, tools can share the cognitive load by reducing the time that students spend on computation. Third, the tools allow students to engage in mathematics that would otherwise be out of reach, thereby stretching students' opportunities. Fourth, tools support logical reasoning and hypothesis testing by allowing students to test conjectures easily (Lajoie, 1993). Taylor (1980) described potential computer roles as tutor, tool, or tutee. In this categorization, the student can be tutored by the computer, the student can use the computer as a tool, or the student can tutor the computer through languages or commands. More recently, Handal and Herrington (2003) described categories of computer-based learning in mathematics, including drills, tutorials, games, simulations, hypermedia, and tools (open-ended learning environments).

The tool-based approach has been shown to be an effective means to use technology to enhance student thinking in mathematics (Lederman & Niess, 2000). A tool is defined as a cultural artifact that “predisposes our mind to perceive the world through the ‘lens’ of the capability of that tool,” making it easier or more productive to perform certain activities (Brouwer, 1996-1997, p. 190).

The purpose of this paper was to develop an interactive web resource for the mathematics curriculum concentrated on Geometry. With this web resource it is expected that the teachers access virtual sections by printing or using simulations and active functions which can be controlled by students. They also can see the results of students' progress at the end of each lesson by checking the answers of the questions or the degree of difficulty of each section. Even more, they can make new simulations by themselves after they read our guide manual about the usage of software used in these resources. These online lectures will continue for other grades. We are going to expand the teacher's abilities in usage of the computer and software.

For future research we hope to include the following: first to conduct some experiments with students by using this method of teaching in classrooms, followed by a questionnaire which analyses the effects of each part of the lecture to be filled out by students, and second we would like to use the students' data in order to further develop the Server side system.

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Using MS Word and Field Codes for Teaching Calculus for the Blind

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Abstract: At Winona State University, we have had a few blind students major in or take courses in the mathematical sciences. This talk relates some of my experiences in teaching calculus to a blind student in a classroom situation, including some of the problems we faced, and the strategies, technological and non-technological, that we used to get around these problems. During lectures, I used MS Word for Windows XP, which made typed transcripts of the lectures immediately available for Braille translation. I also used a standard feature of MS Word called Field Codes that are also available for Macs and with which I could create mathematical expressions that could be rendered in Braille. The talk will feature some examples of field code-generated expressions and the creation of shortcut keys for these expressions.

1. Introduction

I got interested in the problem of making mathematics instruction accessible to blind students after Prof. Joan Francioni of the Computer Science Department at Winona State University asked me in June of 2003 about having a blind student in my calculus class for the coming semester. When Prof. Francioni contacted me, she had been working on a project on the problem of mathematics and science instruction for blind and visually impaired students. She had stated that it may be possible that more blind students would choose to major in the mathematical sciences, in particular in computer science, that produce the technological aids that had become more and more pervasive in everyone's lives especially in the lives of those with impaired or without sight [1]. I had heard of blind students majoring and doing well in the mathematical sciences in other places.

I had never had a blind student in any of my classes, but I gave her request serious thought because I believed that the project was important from the standpoint of making education accessible for all. I strongly believed then and still do that education with a solid foundation in mathematics opens doors of opportunity whereas these doors remain tightly shut when we deny someone the chance to learn mathematics because we did not provide him/her the means to achieve.

2. Discussion of the Logistics and Preparation

My preliminary emails and face-to-face discussions with Prof. Francioni and the student gave me a better idea of the problems that conventional methods of teaching pose for blind students. I also learned some of the possible modifications that I might have to make to the way I taught in order to accommodate the blind student. We discussed the type of aids, technological and otherwise, that were to be available to the student and to myself if and when he was to join my class. During these discussions, everyone agreed that, though I would have to make modifications to my teaching techniques and style, I would have to make them without compromising quality and standards. A few of the suggestions included the following:

- That I should provide the student the outlines of the lectures with the examples, graphs, equations, etc., a day or so prior to each lecture.
- That an electronic transcript of each lecture would have to be prepared either by myself or by somebody else using a font type called the Tiger Font that the Braille machine required (The Braille machine that Winona State currently uses is a Tiger Machine Viewplus Max Embosser). The student also had a piece of software called JAWS that converted typed text into audio. JAWS, however, did not work with mathematical expressions. This is where the Braille notes played a major supplemental role.
- That I had to be very elaborate in my descriptions during lectures.

I had no problem with the first item since I always had made a point to write up an outline of the core ideas and examples of a lecture prior to presenting that lecture to my class. However, there was going to be very limited help in typing up the transcripts which meant that there could be substantial delays in getting the Braille transcripts to the student. My solution was to lecture using MS Word for Windows XP on my laptop so that the transcript of each lecture was to be ready as soon as I got done presenting that particular lecture. As such, we did not have to use valuable resources that would have been used up for outside help. I just had to work out how I was to create mathematical expressions efficiently in an MS Word document while I lectured.





3. Mathematical Expressions and Field Codes

3.1 Braille Output and Field Codes

While we were discussing how to resolve the problem of transcribing mathematical expressions into Braille, there might have been mention of the Nemeth Code. *Dotsplus*[®] in conjunction with MathType was already in use in other places [3], but we decided upon what are called "field codes" for three reasons:

- The package of field codes was as a standard feature of MS Word for Windows XP and for Mac OS.
- I already knew how to use field codes for mathematical expressions.
- Field code-generated mathematical expressions such as " $\frac{x^2 + \sqrt{3e^x}}{2x + \sin x}$ " or " $\lim_{x \rightarrow \infty} f(x)$ " got transcribed with *Braille-type* characters as opposed to *embossed* characters when run through the Braille machine (See Table 1). This was very important for the student who was to supplement his notes with the lecture transcripts. Note that all characters including symbols should be in plain type and not in italics.

Table 1. Sample Braille Machine Output Using the Equation Editor and Field Codes

Word (Field Codes)	Field Codes Transcribed by the Braille Machine	Equation Editor Transcribed by the Braille Machine
$\frac{3}{4}$		
$x^2 + e^{2x}$		

Someone had suggested using the Equation Editor, but the expressions from the Equation Editor did not translate well into Braille because the characters in mathematical expressions turn out as embossed rather than in Braille.

Note that symbols such as the horizontal bar between the numerator and the denominator of fractions and "+" as shown in Table 1 end up as embossed symbols even for field codes. But the student was able to figure these out quickly enough, especially when these symbols were put in a specific context.

3.2. Creating Mathematical Expressions with Field Codes

We now take a look at the steps and keystrokes in creating mathematical expressions using field codes in an MS Word document using Windows XP. (I should point out that MS Word for the Mac can create and read the mathematical expressions generated on Windows XP. In fact, I learned about creating equations using field codes on a Mac back in 1992.)

- Within a new MS Word document start by pressing **CTRL+F9** (**Command+F9** on a Mac). Two brackets "{ }" should appear with the cursor between the brackets. The two brackets/braces are not the same as the usual brackets that one gets by using the bracket keys on the keyboard.

Table 2. Examples of Mathematical Expressions with Corresponding Field Code Lines

Mathematical Expression	Field Code
Fraction: $\frac{x}{y}$	{ eq \f(x,y) }
Square Root: \sqrt{x}	{ eq \r(x) }
The n^{th} Root of x : $\sqrt[n]{x}$	{ eq \r(n,x) }
Exponent: x^n	x { eq \s\up6(n) }
Subscript: x_n	x { eq \s\do6(n) }
Subscript and Superscript: x_k^j	x { eq \s(j,k) }
Summation: $\sum_{j=1}^n f(j)$	{ eq \i\su(j=1,n,f(j)) }
Integral: $\int_a^b f(x) dx$	{ eq \i(a,b,f(x) dx) }

- With the cursor in between the two braces, type in "eq \" to get the following: { eq \ }. The symbol "eq" calls for the generation of "equation" or mathematical expression field codes.
- All we need now is to type in the proper command line for each of the different mathematical expressions. Table 2 provides examples of some of these field code-generated expressions.

- Once the full command line has been typed in, press **ALT+F9** to view the expression (**Option+F9** on a Mac). One can toggle between the field code and the math format by pressing **ALT+F9** (**Option+F9** on a Mac).

3.3 Shortcuts and Macros

One can create shortcut keys to speed up the creation of mathematical expressions, especially those that are used most often in class such as the ones given in Table 2. This can be done by going to the item "Macro" under the "Tools" menu in MS Word.

Table 3. A Sample Lecture Using Field Codes

<p>Ex. $y' = (1/2) * \left[\frac{2x+3}{x^2+3x} - \frac{4}{4x+1} \right] * y$</p> <p>Since $y = \sqrt{\frac{x^2+3x}{4x+1}},$</p> <p>$y' = (1/2) * \left[\frac{2x+3}{x^2+3x} - \frac{4}{4x+1} \right] * \sqrt{\frac{x^2+3x}{4x+1}}$</p>
--

3.4 Other Considerations

After deciding to use field codes for mathematical expressions, we had to consider a few other things that affected the spacing of the Braille dots and characters on the Braille transcripts:

- I was to use a fairly large font size of 36.
- I had to put generous spaces between terms. For example, I should write the fraction " x^2+2x+1 " as " $x^2 + 2x + 1$ " where there is clear spacing between terms. One gets exactly the same spacing in the Braille transcript as in the original.
- Exponents and subscripts, respectively, had to be raised or lowered a bit more than usual so that they were easily identifiable as exponents or subscripts in the Braille transcripts. Hence, " x^2 " might appear as " x^2 ".

4. What Was the Student to Do

The student's difficulties when writing mathematical expressions were more complex compared to those that he encountered while reading them. While figuring out how to get around this problem, I felt that the most important thing was that he first be able to get his ideas across regardless of the format. Format is important in mathematics, but I felt that the student should be able to focus more on the mathematics. I did not want to have him have to learn field codes if it was going to take time away and detract him from learning calculus. I decided that he could write mathematical expressions using a format that was similar to what one might see in computer programming languages since, well, the student was a computer science major. For example, he could write " $x^3 + 4*e^{(x^2)}$ " for the expression " $x^3 + 4e^{x^2}$ ", of course with the proper pairs of brackets or parentheses. For the more complicated expressions such as $\frac{x^3 + 4e^{x^2}}{\sqrt{2x + 1}}$, I suggested breaking the whole into pieces such as writing the numerator and the denominator separately when describing the fraction as "The numerator is $x^3 + 4*e^{(x^2)}$ and the denominator is the square root of $(2x + 1)$." (See Table 4).

Table 4. Examples of Student Output from the Student

Mathematical Notation	What He Wrote
$\lim_{x \rightarrow 5} f(x)$	the limit of $f(x)$ as x approaches 5
$\sqrt{x - 3}$	The square root of $(x - 3)$
$\frac{x^2 - 5x + 7e^x}{\sqrt{2x^2 + 7}}$	Fraction Numerator: $x^2 - 5*x + 7*(e^x)$ Denominator: Square root of $(2*(x^2) + 7)$

In the course of the two semesters that he took Calculus I and Calculus II, I asked and received constant feedback from the student to make sure that he kept apace with the class, to find out what worked and what did not, and to make quick changes when things did not work out. He did ask questions more often than everyone else during lectures, but these were primarily about notation. A sample situation could be when I say, "the fraction with the numerator $x^7 + 3x + 4$ and with denominator $\sqrt{x + 2}$," he might ask about the numerator and if the exponent of x was 7 or $7 + 3x + 4$.

5. Graphs and Diagrams

The issue of graphing and the sketching of diagrams is the one problem that I think I had not addressed sufficiently, and it was not for lack of trying. This is one of the reasons that motivated me to write this paper and give a talk on this topic, that I feel that I had not completely "enabled" the student. Maybe this is where the reader or audience of this paper can provide suggestions and ideas as to how this problem can be worked out.

From my end, lecturing with graphs and diagrams was not a problem for the student. I just had to be more descriptive in class when working with graphs and figures as illustrated by Figure 1.

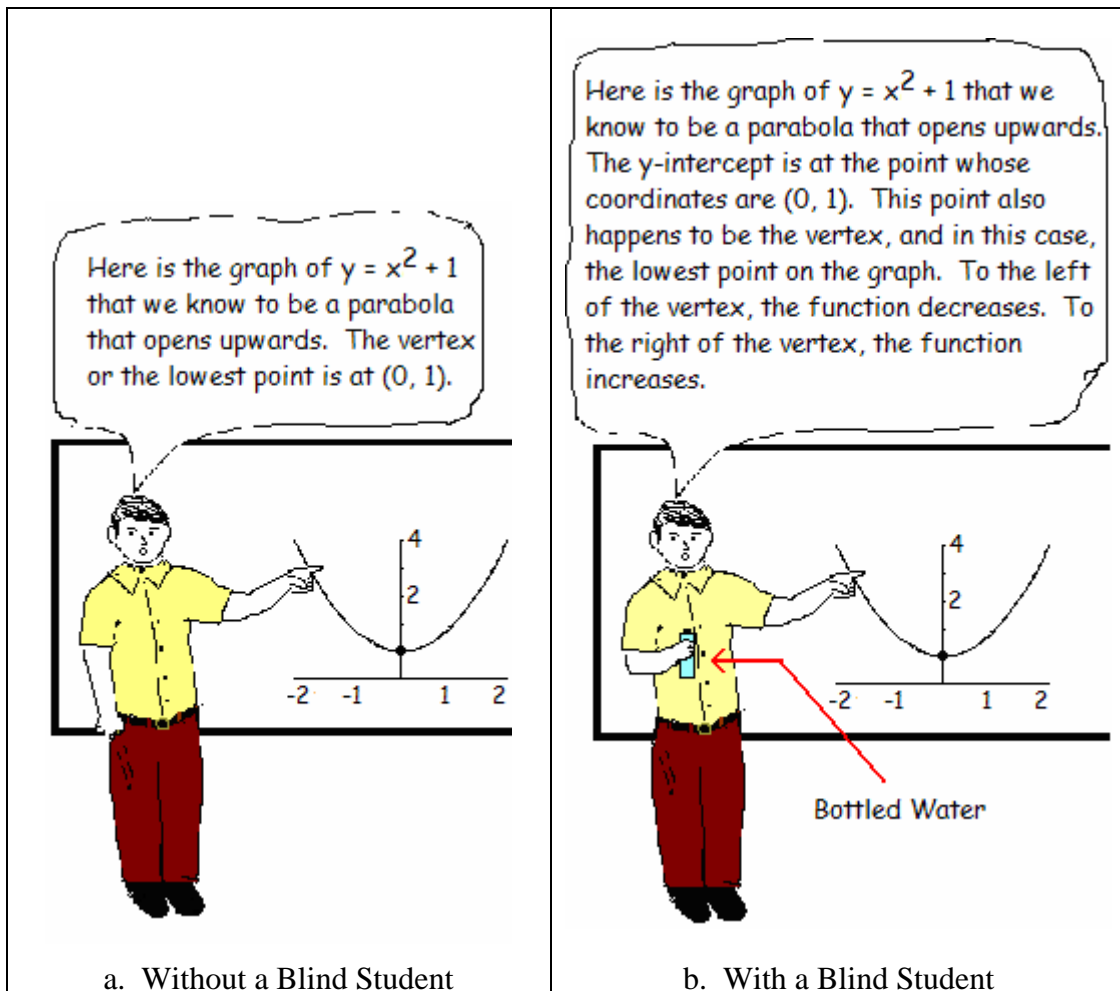


Figure 1. Lecturing In Class

The other students found this as a source of amusement, but they also thought that it was a good thing since, according to them, I often pointed out things that they usually would not have paid much attention to or noticed.

Reading graphs presented only minor problems for the student since he was able to do tactile explorations of the graphs that were rendered as embossed by the Braille machine. The problem when he had to draw graphs and diagrams was more difficult to work around. He had a piece of software that worked with graphs and such that the computer generated varying tones depending upon the positions of points on the graph -- the higher the points on the graph, the higher the pitch. Still, while his results for very simple curves such as parabolas and lines were more than satisfactory, we had mixed results for the more complicated graphs. I am not sure if this is a problem of becoming more adept at matching and identifying subtle changes in sound and pitch to subtle changes in graph properties such as monotonicity and curvature. If it is, I am not sure if this is an ability that can be acquired with repeated use of the software and feedback from an instructor. Even if this had been so, this might have required time that we did not have so that I suggested that he include detailed descriptions of the behavior of the curve in support of his work. Again, the important thing was to have him get his ideas across clearly. What happened was that I often received detailed descriptions of the graphs, descriptions that were more meticulously "drawn" out compared to most of the graphs and descriptions that I got from the other students (See Table 5.).

Table 5: An Example of the Student's Supporting Description of a Curve

In sketching the graph of $f(x) = \frac{x}{x^2 + 1}$ for the interval $[0, \infty)$, he wrote the following:

f increases from $x = 0$ to $x = 1$ where f has a critical point. f is concave downwards on this interval. $f(0) = 0$ and $f(1) = 1/2$

f decreases from $x = 1$ to $x = \text{square root of } 3$. f is still concave downwards on this interval. $f(\text{square root of } 3) = \text{square root of } 3$ all over 4 which is approximately 0.43.

f decreases but is concave upwards for $x > \text{square root of } 3$. Hence, f has a point of inflection at $x = \text{square root of } 3$.

f tends to 0 from the right when x approaches infinity.

6. Final Thoughts

I discovered that teaching a blind student, in many respects, is fundamentally the same as teaching a traditional student. It is true that I had to make changes in my teaching style and techniques, but this is something that I often do when teaching a class of traditional students. The only difference is that the changes I have had to make this time are of a different nature. More importantly, however, an important objective is still to make mathematics accessible even to the most unique of individuals without sacrificing standards by finding tools and ways through which these individuals can participate in the learning experience on an even footing with everybody else.

I believe that having a blind student in class enhanced my teaching because it forced me to search wider afield for a common ground on which the student and I could meet without watering down the level of the course or alienating the other students. On the contrary, I feel that this inadvertently made mathematics in some ways more comprehensible for the traditional students.

Using the laptop allowed for more elaborate and more organized lecture notes that all my students could access. I noticed a similar change in many of the works of the other students who became more thorough in their own descriptions and explanations apart from becoming more organized in their final works. Some even started to use field codes although I did not require it nor did they get extra credit for using field codes.

The set-up I had for teaching calculus to blind students was adequate with room for improvement, especially on the matter of the student having to sketch graphs and diagrams. As such, I feel that I had not thoroughly helped the student. This will probably be not the last time that this will be an issue at Winona State. This past spring alone at Winona State, one of our faculty members encountered this same problem when a visually impaired student took her statistics class. I am not sure how this problem can be resolved. But I will be on a constant lookout for ways of providing a bridge to help blind students achieve as others would.

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Graphic Tool for Communication with Visually Impaired Persons

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Abstract: Graphic content is very useful and convenient for effective communication, including that of mathematical concepts. However, this proves to be a barrier for visually impaired people. Our software is a tool for communicating such graphic content between sighted and visually impaired persons. The targets of our system are two-dimensional graphic documents, and the objects used are applied in Japanese high school mathematics. A visually impaired person generally does not understand complicated graphics and the cross relationships of graphical contents. Thus, our main objective is to simplify the graphic content and the information they convey. Our system, which makes use of a tactile display, is primarily designed for communication between sighted and blind persons. However, this system may provide the answer to the question, "How can we create or convey graphic content without visual information?" thereby making it useful for all visually impaired persons.

1. Introduction

We often use electronic information on a daily basis, and these communications are very convenient for visually impaired persons if they are text-based. For example, if we send an e-mail with no image files attached, special provisions are not required for visually impaired persons. However, communication of mathematical content consists of a few graphics, which create barriers particularly for a blind person. By designing communication software for communication that includes some mathematical graphic content, we attempt to remove these barriers.

Recently, we have seen many software and hardware breakthroughs aimed at assisting visually impaired persons, and there has been a great deal of researches improving these technologies ([1], [2], [3]). M. Kobayashi created the MIMIZU System (see [4] for example) comprising a tactile display and a commercially available ultra-sonic pen. The user draws some lines on the tactile pin display with the ultra-sonic pen. This tactile display is also commercialized at this time. Then, anyone can use that system with ease. A complete description of how to design the MIMIZU System is on Kobayashi's web page (<http://www.cs.k.tsukuba-tech.ac.jp/labo/koba/research/dv-1.php>, last access 2007.9.10).

Our system runs on the same hardware as the MIMIZU system. The previous version of our system ([5]) provided a graphic editor for visually impaired persons (primarily for the blind persons). With the help of simple graphics, the user can select graphic object that appear on the tactile display. Verbal explanations are available for the graphic contents. Using this system, a

blind user can draw mathematical graphic objects. However, on practical use of this system, we encountered a problem. The system was not convenient for real-time communication. For example, when a user began working with using our system, it was necessary to explain how to use it. In the case of a blind user, occasionally, we were unable to give clear instructions, making it necessary to take his hand and lead his fingers to the graphic contents on the tactile display. It is clear that the absence of an efficient user interface to express the graphic contents is the reason for this obstacle. In trying to express the graphic contents verbally, we faced the same difficulties that arise in text-based communication. Therefore, for a person to understand how to use this system, some communication functions are necessary at the initial stage itself.

Our new system consists of two sub-systems; one has the tactile display, and the other uses only a standard monitor display. In this system, we have several communication functions. We cannot display some graphic contents as visible images because the tactile display is very coarse, the tactile information is local, and the user cannot understand the cross relationships. Then the two sub-systems have common graphic content parameters, which are used to draw same shapes in an x-y plane. On the tactile display of our system, there are usually small number (not more than 3) of objects, and the user selects one of them as a main object. When there are several graphic objects (more than 3), the user selects some of them to be displayed on the tactile display. The sub-systems also share information on what objects the user selects as main and display. This allows two users to understand each other's intentions, using some additional communication functions.

2. Outline of the System

This system consists of two sub-systems. One is a graphic editor for visually impaired persons (mainly for the blind), and the second is for sighted persons. With the system for visually impaired persons, a user can edit graphic objects using the tactile display (Dot View, commercialized by KGS Corp), and an ultra sonic pen (Pegasus Technologies) (Figure 1). The other system does not require special hardware, except for a standard personal computer. These sub-systems exchange information using a local area network.

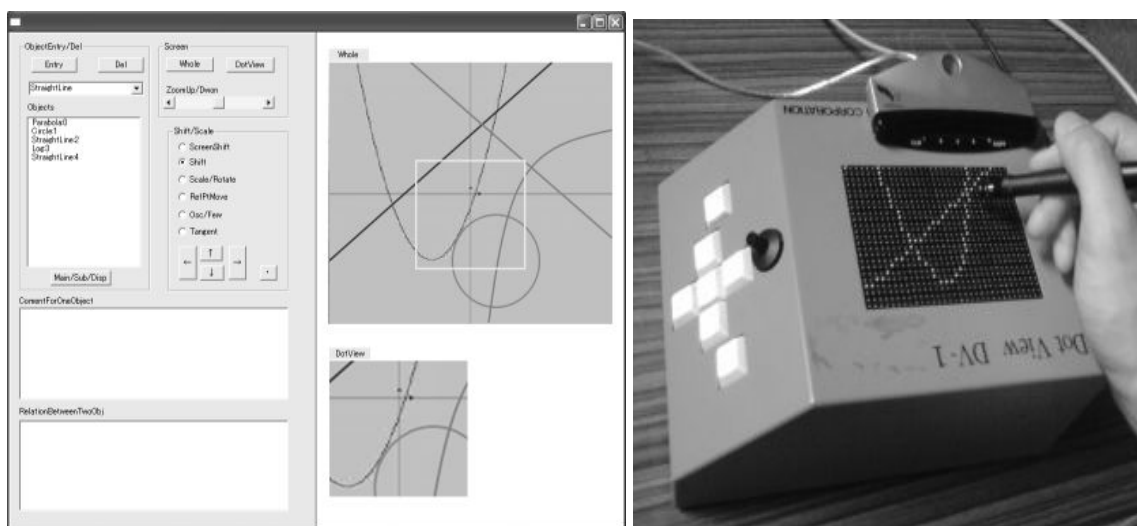


Figure 1 Two Subsystems

2.1 Target Objects

Our system is a graphic communication tool used in high school mathematics. In this model, we restrict the targets to objects in a two-dimensional coordinate system. The types of target objects

are straight lines, parabola curves, trigonometric functions (sin, cos and tan), exponential and logarithmic functions (exp and log), hyperbolic functions (sinh and cosh), and basic figures (triangles, squares, circles, and ellipses). For each object type, we form sufficient parameters to determine each object uniquely, and express these in an XML format. A detailed explanation will appear in section 3.

2.2 The System with a Tactile Display and an Ultrasonic Pen

For this subsystem, we use the following hardware:

- (1) A tactile pin display (Dot View by KGS).
 - 24(vertical) x 32(horizontal) =768 pins.
 - Pitch between two pins = 3mm.
 - One joystick and seven keys.
 - RS232C connection with PC.
- (2) An ultra sonic pen (Pegasus Technology).
 - Area size = A4 paper size.
 - Resolution = 100 dpi.
 - USB connection with PC.

Each sub-system is connected to a common PC, and the user uses a standard keyboard along with the hardware (1) and (2). Our main functions of these sub-systems are as follows:

- (a) Edit graphic contents.
- (b) Save and Load.
- (c) Communicate with the user who uses the other system.

We assume that a user of this sub-system is visually impaired (may be blind), therefore, the system must be controllable without visual information. A blind user can edit graphic objects as follows: First, the user inputs a graphic object on the tactile display by selecting an object type and pushing the corresponding key, causing an object with default parameters to appear on the tactile display. The user can then change its shape or position using a joystick and keys attached to the tactile display. During the edit, verbal explanations for one graphic object and for the relationship between two objects are available.

The display area is very coarse, and cannot accommodate many objects. Therefore, for simplicity, the user selects just a few objects for display. Other objects are not for display; however, they remain in the graphic document. The user also selects a main object that is the target for edit functions (changing its shape and position) and explanations. The user can also obtain explanations on the relationships between the main object and the other object, which the user selects from among the objects for display. There are two categories of graphic objects, function graph and basic figure. Table 1 shows the list of edit functions for each category. In the beginning, an object has its default figure, and the user changes its shape and position using these functions.

Function Graph			Basic Figure		
Edit Type	Parameters	Details	Edit Type	Parameters	Details
Shift	(X,Y) (a vector)	$f(x) \Rightarrow f(x-X)+Y$	Shift	(X,Y) (a vector)	$(x,y) \Rightarrow (x+X,y+Y)$
Oscilation scale up/down	A (a real num.)	$f(x) \Rightarrow Af(x)$	Scale up/down	A (a real num.)	$(x,y) \Rightarrow (Ax, Ay)$
Cycle scale up/down	A (a real num.)	$f(x) \Rightarrow f(Ax)$	Rotate	B (angle)	$(x,y) \Rightarrow (x \cos B - y \sin B, x \sin B + y \cos B)$

Table 1 Edit Functions

In the details column, under the basic figure category of the table, (x, y) represents the relative coordinates of any point, with respect to some reference points for each figure.

In a mathematical graphic document, a graphic object is sometimes tangent to some other object. Despite the importance in mathematical properties, it is almost impossible to create it using a joystick without certain special functions. The function touch move (touch shift, and touch scale up and down) changes the shape or position of some target object (called the touch target) with respect to one curve touching to another curve (called the touch base). The following table shows the list of candidates for touch target or touch base.

	Object Type
Touch Target	Circle, StLine(straight line), ParabolaCrv
Touch Base	Polynomial, Trigonometric, Exponential, Hyperbolic curve

Table 2 Touch Target and Touch Base

Only touch shift works for a straight line while the function touch scale up and down cannot change the parameter of the straight line.

2.3 The System for a Sighted User

One may be skeptic about whether a blind person needs graphic information. The answer may be “no” if they are not communicating with sighted person. However, sighted persons often use mathematical concepts and properties, and some of these are closely related to graphic concepts.

Moreover, many sighted persons understand these concepts with respect to their graphic contents. Thus, for communication with sighted persons, a blind person must use some graphic objects, even if he (or she) can understand all mathematical concepts and properties without any graphic concepts. Thus, we can say that all persons need graphic objects; however, they are convenient only for sighted people.

From this point of view, there are two barriers of communication with graphic content. Graphic content is not easy for visually impaired persons to understand, and it is difficult for sighted persons, to convey the important information in the graphic content, as the information is subconsciously apparent. To remove these barriers, we added the following functions beside the basic functions (a), (b), and (c) in this sub-system.

(c) Check and Control the area of the tactile pin display.

(d) Check and Control the selected objects (main and display objects).

The area of the tactile pin display is coarse (24 x 32 pins); therefore, when a user checks for details in an object, the area is often very small. When he (or she) checks another part or makes a global check, the size and position of the display area often change accordingly. As a result, there are two display areas in the system for sighted persons, one provides global view of graphic contents, and the other replicates the images on the tactile display of the other system.

3. XML Formats

The system document, including graphic content, is stored as an XML file. In this document, there is sufficient information to construct the shapes and positions of the objects. For communication between the two sub-systems, we use an XML format to express the graphic document. As mentioned in section 2, the information on the state of a tactile display is shared information between both the sub-systems. This is also represented in the XML-document.

3.1 XML Format for Graphic Document

All graphic contents of a document are represented by curves determined by certain parameters. Table 3 provides a list of the elements and the names attributed to them. Each attribute has its value type [(i), (r), or (v)]. (i) stands for integer values, (r) for real values, and (v) indicates two-dimensional vectors.

For a graphic representation of a polynomial function (a straight line and a parabola curve), the general expression of the function is $f(x) = c_0 + c_1x + c_2x^2$, and the attributes Coeff0, Coeff1 and Coeff2 represent c_0, c_1, c_2 . For trigonometric, exponential and hyperbolic functions, we consider a generalized function $sg(c_0 + c_1x) + t$. Then, for each base function $g(x)$, the attributes Shift, Scale, Coeff0 and Coeff1 define the function. According to the expression rule, we can express the function as $sg(c_0 + c_1x + c_2x^2) + t$. We can display this function if the XML source file includes this document. However, we only have functions, (x,y)-shift, oscillation scale up and down and cycle scale up and down. Therefore, this system cannot change the default function $g(x)$ to $g(c_0 + c_1x + c_2x^2)$.

Each element always has an attribute Id, which is a serial number for the graphic objects. The system identifies an object using this attribute. A circle is defined by its Center and Radius. An ellipse is defined by its Center, Radius, Radius2 and Angle (see Figure 2). The vertices of the corresponding number define a polygon. Every attribute has its default value, that is, every object has its default shape. We can define their default values using an XML file. Figure 2 is an example of a XML source code for a graphic document and its figure.

Curve	Element Names	Attribution name (value type)
Common	*****	Id (i)
Polynomial	StLine, Parabola	Coeff0 (r), Coeff1 (r), Coeff2 (r)
Trigonometric	SinCurve, CosCurve TanCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Hyperbolic	SinhCurve, CoshCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Exponential	ExpCurve, LogCurve	Shift (r), Scale (r), Coeff0 (r), Coeff1 (r)
Circle	Circle, Ellipse	Center (v), Radius (r), Radius2 (r), Angle (r)
Polygon	Triangle, Square	Pt1 (v), Pt2(v), Pt3(v), Pt4(v)

Table 3 Element Names and Attribution Names

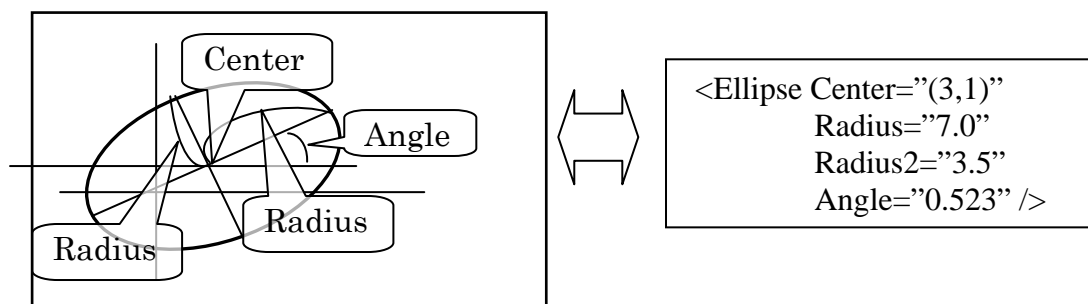


Figure 2 Ellipse and its Xml Expression

An XML sub-element Relation may belong to any XML element corresponding to a graphic object. This sub-element has an attribute SubId and text document. The text document explains some relationship between the graphic object and the other graphic object corresponding to the id number SubId. This sub-element is important to understand the document, and we explain their details in section 4.

3.3 XML Expression for Tactile Display State

For the functions (c) and (d) in subsection 2.2 we express the following information by XML text document.

- (1) Area of tactile display.
- (2) The main object and the objects for display.
- (3) How to edit the document.

Table 4 shows a list of XML elements and their attributes that express this information. The tactile display has a pin display area (24 x 32), and this corresponds to rectangle area in a coordinate plane and two vector attributions stand for two corners ((left, bottom) and (right, top)) of this rectangle. Two elements, SelectedObj and EditState, are sub-elements of DvArea, and DisplayObj is a sub-element of SelectedObj. The attributes MainId, SubId, and Id are the ID numbers of the corresponding graphic objects.

An object type (n) implies that the value is a string, selected among prepared strings. The value of Target is selected from among {Object, Screen, and Touch}, and that of EditType from {Shift, Rot_Scl, Osc_Frq, TchShift, and TchScale}.

	Element Name	Attribution	Sub Element
(1)	DvArea	LeftBottom (v) RightTop (v)	SelectedObj
(2)	SelectedObj	MainId, SubId (i)	DisplayObj
	DisplayObj	Id (i)	
(3)	EditState	Target (n) EditType (n)	

Table 4 Xml Expression for State

```
<DvArea LeftBottom="(0,0)"
        RightTop="(1.0,1.0)">
  <SelectObj mainId="1" >
    <DisplayObj id="2"/>
  </SelectObj>
  <EditState Target="Object"
            EditType="Shift" />
</DvArea>
```

Figure 3 Sample State for Display

4. Information Sharing

There are two barriers in the graphic communication between sighted and visually impaired persons. One barrier is difficulty for a visually impaired person to get adequate graphic information from graphic content, and the other is, for sighted person, difficulty to be conscious on essential tips of information. Two sub-systems exchange information of curve shapes and positions, cross relationships and states of the tactile display. The system displays the shapes and positions in the tactile display and several explanation functions are available, these will remove the first barrier. It is also very important how the visually impaired user use these functions. Thus, we add the function to exchange the information of the states of the tactile display to remove the second barrier for sighted persons when they communicate with visually impaired persons. In this section, we will explain the information for removing the second barrier.

The XML element Relation is a sub-element of an XML element corresponding to some graphic object. A content of this element is string: an explanation about relations between the main and the

other (user selected) object. Voice explanations are available in the sub-system with a tactile display using this information. Some of these contents, “a positional relation of reference points of both objects” and “the number of cross points and tangent points”, are given by the system automatically”. We can also input any text document as a content of “Relation” in both sub systems. The Figure 4 is a part of a sample XML document. In the “Circle” element, there are two “Relation” elements. The first one is standard one, which the system makes automatically. The second one is specialized for the situation.

The information of the state of a tactile display consists of the area rectangle, the object selection and the edit state. The attributes Target and EditType in the Xml Element EditState explain how the user controls the system. Table 4 shows the candidates for these attributes.

```

< Circle id="1" Center="(0.0,0.0)" Radius="2.0" >
  <Relation refId="2" >
    Two Cross Points.
  </Relation>
  <Relation refId="2">
    Straight line through the center.
    Another straight line is tangent to this circle,
    And two straight lines cross at right angle
    on tangent point.
  </Relation>
</Circle>
    
```

Figure 4 Sample Xml Expression for “Relation”

<i>Target</i>		<i>Edit Type</i>	
Object	Edit the main object.	Shift	(x,y)-shift
Screen	Change the area rectangle.	Rot_Scl	Rotate or Scale up/down
Touch	Edit the main object touching to other object.	Ocs_Frq	Scale up or down for Oscillation or Frequency
		TchShift	Shift touching to other object
		TchScale	Scale up/down under touching

Table 4' Move Targets and Edit Types

5. Extension of Object Type

The target of our system is graphic content in Japanese high school mathematics. This does not cover all situations even in the high school mathematical education. For example, we cannot create a graph of $\sin(x) + e^x$ with this system. Thus, in the near future, we will have to extend the system for new object types. For the extension, we have only to prepare the following functions (1) ~ (5) according to a new object type, if the object is a curve parameterized by one real number.

- (1) Get a beginning parameter and an ending parameter.
- (2) Get a curve point corresponding to a parameter.
- (3) Get a tangent vector corresponding to a parameter.
- (4) Get the parameter of the nearest curve point from given point.
- (5) Get main reference point and sub reference points and their total number.

In the case where the curve is a function graph, (2) ~ (4) are replaced by

- (6) Get a function value corresponding to a parameter.
- (7) Get a differential coefficient corresponding to a parameter.

The system calls (1) ~ (5) in many situations. We list below some of them

- (a) Draw a curve on displays of two subsystems.
- (b) Find a nearest curve from a point.
- (c) Touch move (for a function graph as a “Touch Base”).

The function “Touch move” (see Table 1), for example, a starting tangent point is set using (4) then the position or shape of “Touch Target” are determined using (2) and (3). Then, next position and shape are calculated using (2) ~ (4) according to the type of “Touch Move”. Thus, general graph of a function can be “Touch Base”.

6. Evaluations

Voice explanations are main functions for removing barriers in communication with graphic contents. Especially, we expect that the explanations for the cross relationships are useful for visually impaired person to understand graphic objects. Then our evaluation test is checking their abilities. There are twelve test subjects and all of them are sighted (Table 5).

	age	sex	Comment
1	57	male	Math. Teacher
2	38	female	Math. Teacher
3	23	male	Univ. Student(M2)
4	22	male	Univ. Student(M1)
5	21	male	Univ. Student(B4)
6	21	male	Univ. Student(B4)
7	21	male	Univ. Student(B4)
8	23	male	Univ. Student(M2)
9	24	male	Univ. Student(M1)
10	58	male	Office worker
11	58	female	House wife
12	28	female	Office worker

Table 5 Test subjects

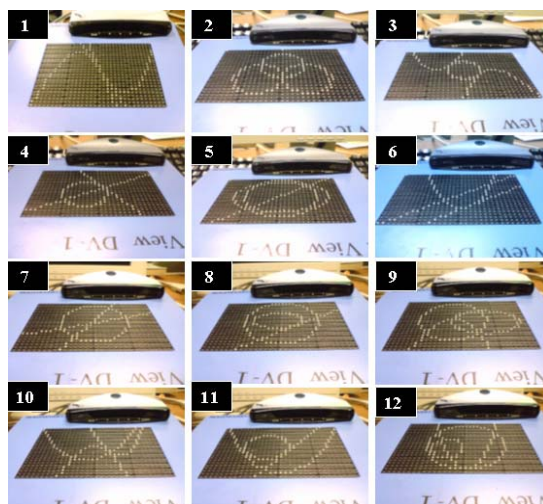


Figure 5 Graphic Documents

There are twelve graphic documents (Figure 5) and we use one document for one test. The tests are divided into 3 categories according to the explanation attached to the document. For the document of first category, there is no relational explanation, only the names of curves are given. For the second, positional relation and number of cross points are attached. For the third, we made an adequate explanation for each pair of objects (Table 6). One test subject takes six tests (two tests for each category) .

Object Pair	Explanation
Parabola 1 and Straight Line	A parabola curve with convex upward is tangent to a straight line. The tangent point is in the right area of the peak point.
Parabola 2 and Straight Line	A parabola curve with convex downward is tangent to a straight line. The tangent point is in the left area of the peak point.
Two Parabola Curves	A parabola curve tangent to the other parabola curve. The tangent point is a tangent point to a straight line. Two peak points are point symmetry with respect to the tangent point.

Table 6 Explanation for Document 1

We gave a test according to the following steps.

1. Wear an eye mask and touch the document in a tactile display with above explanation.
2. Remove the objects from the display, and take off an eye mask. After that, the subject draws a picture of the graphic document.
3. We store the total time, and the object evaluate his (or her) picture as a point (1 ~ 5), comparing with a correct figure.

Figures 6 and 7 show the results of the evaluation test. These are graphs of average values and standard deviation of test times and self-evaluation points for each test category. The standard deviation of the third category is very small.

Let $V_T(k)$ and $V_p(k)$ ($k = 1, 2, 3$) denote the unbiased variance of the test time and self-evaluation point for k 'th category. The statistic lows of the fractions $\frac{V_T(j)}{V_T(k)}$ and $\frac{V_p(j)}{V_p(k)}$ are F-distribution with 23 and 23 degrees of freedom if we assume that the population variances of two categories are same and that these values are stochastically independent. In this experience we obtain $\frac{V_T(1)}{V_T(3)} = 2.86$, $\frac{V_T(2)}{V_T(3)} = 3.33$, $\frac{V_p(1)}{V_p(3)} = 4.07$, then by the F-test with the risk rate 0.01, we obtain statistic judgments: the population variance of the category 3 is less than that of the categories 1 and 2 for the data of test time, is less than that of category 1 for the data of self-evaluation point. This implies stable and comfortable use of the system.

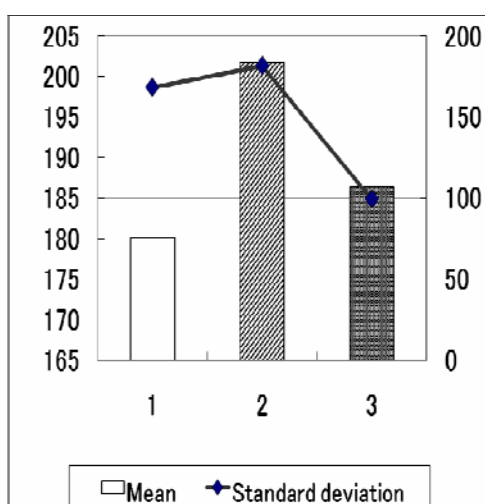


Figure 6 Result(time)

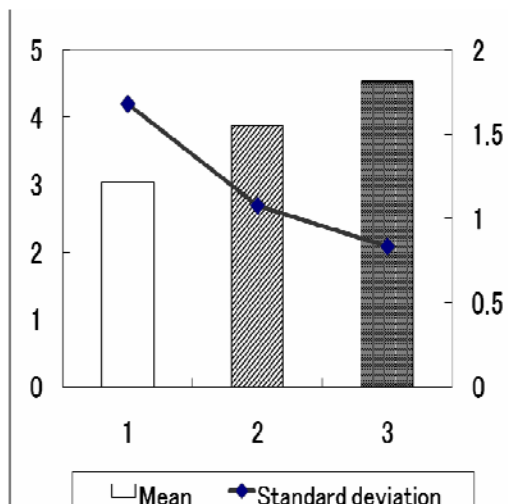


Figure 7 Result(self evaluation)

7. Conclusions

We have designed a communication system for a graphic object. This system enables us mutual communication with graphic objects between sighted and visually impaired persons. Concerning about the LAN connection between two sub systems, it is not difficult to increase the number of subsystems. Then, this system will be useful for e-learning including some visually impaired persons, if we can solve the following problems

1. Improve the user interface and the expression rule for graphic objects.
2. Automatic creation of voice explanations.
3. Connection with other documents or databases.

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Beyond the Web-Based Cognitive and Interactive Metacognition Interface within Teaching Similar Triangles

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Abstract: Most of good performances of teaching and learning outcomes are based on the suitable information management and communication during the teaching and learning processes. Every educational communication is not a simple thing which usually involves encoding and decoding information from teaching and learning interface. The interface maintains information which includes different views in teaching, learning, developing, and administering to transmit and receive criteria information and knowledge during teaching and learning processes. However the transmitters and receivers are not always skillful in expression and comprehension or not always energetic enough to send or receive the information exactly. Consequently, the losing information or misconception happened commonly during each information communication. Similarly, while instructor and learner communicate the concepts via the web pages in the web-based educational system, the event of losing information occurs in the same time. In other words, the communication interface may not always maintain a good or suitable expression in each concept or knowledge and the learners may not always skillful enough to comprehend and receive the expressions via the interface. Then the web-based teaching and learning communicated interface are not always suitable for supporting, diagnosing and monitoring in teaching and learning misconceptions. This paper presents a **Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)** to support learners and instructors to percept, monitor and communicate the critical information and metacognition within teaching similar triangles in web-based educational system. Teaching scenario and empirical evaluation (N= 105, Grade 8, 14~15 years old, from three classrooms) have been done to express multi-stage supporting and multi-level detections of learner's misconception to detect and guide the metacognition of learners to show the WBCIMI is available.

1. Introduction

Metacognition is both as the abilities of individuals' knowledge which can monitor, control,

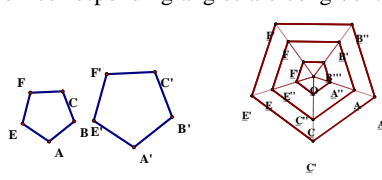
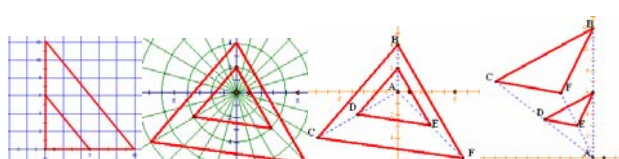
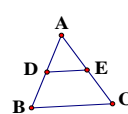
reflect, and understand their own learning processes sensibly and as the resources which can be used and applied to enhance the learning outcomes. Flavell (1976) proposed the concept of metacognition of learners' cognitive processes and regulation of learning processes. The metacognitive knowledge interplay with individual characteristics, task properties and available strategies (Flavell, 1979) in learning situations and the metacognitive skills focus on learners' self-regulatory activities. Schraw & Dennison (1994) defined the metacognition is the abilities of learners which reflect, understand, and control their own learning. However the assessment of metacognitive skills through self-reports was problematic because it appeared that learners had poor insight into their own behaviors (Nisbett & Wilson, 1977; Prins, Busato, Elshout, & Hamaker, 1998; Veenman, Prins, Verheij, 2003, & Veenman, Prins, & Elshout, 2006). Accordingly, mental image became a black box that is not the real subjects can be insight and exploration, whereas a durable improvement of more specific cognitive abilities through training and supporting are an essential issue during teaching and learning processes. Owing to the metacognitive skills are teachable and supportable (Brown & Palincsar, 1989; Schraw, 1999; Veenman, Elshout, & Busato, 1994) then instructors and learners need the efficient ways to communicate their main ideas more transparently and directly. In this paper we present an interface, named **Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)** to implement teaching and learning interface which integrates metacognition knowledge detecting and metacognition strategies supporting. The precision behaviors of **WBCIMI** are identified by well defined teaching programs, real teaching/learning actions and the detectable/reachable interactively information processes between instructors and learners. While learning is happen, the **WBCIMI** tries to maintain and transform the learning processes into a visible and manipulated objects for detecting, reasoning, training, and supporting learning processes. Furthermore, the **WBCIMI** build a specific knowledge framework to characterize metacognition knowledge, strategies to detect and map these cognition behavior, and then to support the guidance for learners to learn more fluently and correctly.

2. Knowledge Framework

Cornoldi and Vianello (1992) suggested that metacognition knowledge can be characterized with reference to a series of aspects such as its level of specificity, it include the range of application, the ease degree of access, whether it can be verbalized, visualized, and then manipulated. In **WBCIMI**, the knowledge of similar triangle is a framework that represents a set of concepts within similar triangle and a set of linkages between those concepts and solutions of problems. The concept framework refers to the objectives, and subordinate objectives to the learning objects and the relationships between concepts from one to another. And the represented framework of concepts

defines what can be expressed in the objectives represented, how it will be constructed and comprehended by learners. Table 1 illustrates the concept framework of similar triangles.

Table 1: Concepts framework of similar triangle

Teaching Stages	Learning concepts
<p>Prior knowledge stage</p>	<p>To describe, identify, and comprehend the rates, ratios, and proportionality.</p> <p>To confirm, rebuild or recover the concepts of corresponding angle, vertices and side.</p> <p>The sum of all three angles of a triangle is 180°.</p> <p>Equality of Cross Products: For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.</p> <p>To recall similar polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are proportional.</p> <p>Similarity means that:</p> <p>(i) The two figures have congruent angles</p> <p>(ii) The sides are in proportion.</p> 
<p>Parallel lines intersect proportional segments</p>	<p>If two lines are parallel and they are intersected by another two lines then two sides of the two intersected lines separates these sides into segments of proportional lengths.</p> <p>CorollaryI: If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.</p> <p>CorollaryII: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.</p>
<p>Define similarity</p>	<p>The mathematical definition for similar triangles is that they both have corresponding angles that are equal, while the lengths of the corresponding sides are in proportion.</p> <p>Similar triangles have the same shape, but the size may be different.</p> <p>"\sim" is "similar to". Examples $\triangle ABC \sim \triangle DEF$ (i.e. $\triangle ABC$ is similar to $\triangle DEF$)</p>
<p>Plot the similar triangle</p>	<p>Plot the similar triangles in different ways. An enlargement must have a center of enlargement and an enlargement factor.</p> 
<p>Parallel Lines interested the proportional segments of the triangle</p>	<p>To explain, proof, illustrate, and apply the follows:</p> 

	$\text{If } \overline{DE} // \overline{BC} \Rightarrow \frac{\overline{AD}}{\overline{DB}} = \frac{\overline{AE}}{\overline{EC}} \text{ (up vs. down)}$ $\Rightarrow \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{DE}}{\overline{BC}} = \frac{\overline{AE}}{\overline{AC}} \text{ (up vs. all)}$ $\Rightarrow \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AC}} \text{ (down vs. all)}$
<p>Similarity prosperities</p>	<p>(i) AA (angle-angle similarity) or AAA (3 angles): If two corresponding angles of one triangle are congruent then the two triangles are similar. The sum of all three angles of a triangle is 180°. Therefore if two angles of two triangles are congruent, the third is automatically congruent. Therefore the sufficient condition requires only two angles to be congruent. In other words, if you know two of the angles of two triangles are congruent, then the third angles of the two triangles are congruent, and then two triangles are similar.</p> <p>(ii) SAS (side-included angle-side similarity): This one says that if you have two sides that are in proportion (that is, they reduce to the same fraction) and the corresponding included angles are congruent, then the triangles are similar.</p> <p>(iii) SSS (side-side-side similarity): This one says that if the three sides are in proportion (they reduce to the same fraction), then the triangles are similar.</p>
<p>Proportional related properties</p>	<p>If two triangles are similar then the proportions of corresponding sides will equal the proportions of corresponding perimeters, corresponding angle bisectors, corresponding medians, and corresponding heights.</p>
<p>Parallel Lines and Proportional Parts of Triangle</p>	<p>Triangle Proportionality: If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.</p> <p>Theorem A: If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.</p> <p>Theorem B: A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is one-half the length of the third side.</p>

3. Cognitive pool

Owing to learning behavior is influenced by learners' viewpoints and believes, by the way in which learners interpret learning concepts and objectives by learners themselves. Accordingly, the WBCIMI tried to maintain and integrate the knowledge, misconception and metacognitive knowledge/strategies as the cognitive pool of similar triangles in order to set the related concepts and as knowledge base to detect the conceptions or misconception from learners themselves transparently and interactively. In other words, WBCIMI tried to figure out what learner thought about a specific problem via the selected terminology by learner themselves. After the terminology

was selected by learner, then the system followed the factors to see the knowledge, metacognition or misconceptions that learners might have, and to direct learner to essential learning objects or hints which are the necessary concepts of similarities for learners to go further more confidently. The cognitive pool includes five components that are terminology, knowledge/concept hierarchy, problem spaces, common misconception, and linkages. The five components of cognitive pool illustrate in the following: 1) Terminology is a set of the properties, concepts, and postulates of similar triangles. 2) Knowledge/concept hierarchy is the overall knowledge and concepts in teaching/learning objectives. 3) Problem spaces are based in part on the concepts of similar triangle that is the fundamental process for attainment the solutions for difficult problems. Problem spaces were based on the three-level abstract/separate/deliberate control structure. And the problem spaces were separated by different subordinate objectives of similar triangles. 4) Common misconception is the faults, difficulties, or misconception from learners that was found by experience instructors. 5) Linkages are the relationships among the selected terminology, meant concepts and suitable directions to solve these misconception or problems. In other word, the linkages are the relationships among the terminology, knowledge/concept hierarchy, specific problem, common misconception, and guidance of the solutions

4. Web-Based Cognitive and Interactive Metacognition Interface (WBCIMI)

Initial metacognition detection: The purposes of web-based cognitive and interactive metacognition interface are transplanting problems with the related concepts and terminology to the web sites to enhance the accessibility, usability, comprehension, and application of similar triangles. However, the misconception and metacongition knowledge are not visible features for the system to acquire and enhance learning/teaching efficiency during learning/teaching processes. Furthermore, learning is a complex task; several different aspects must be taken into consideration. The WBCIMI built ways to visualize and manipulate misconceptions and metacognition knowledge during learning similar triangles. Figure 1 illustrates the structure of the initial metacognition detection with terminology selection.

