# **Research and Applications of Calculators in Mathematical Fields**

# Enhancing Conceptual Understanding in Calculus Using Class Pad 300

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**Abstract:** World over, Computer Algebra Systems (CAS) have greatly influenced mathematics teaching and learning. The last two decades have witnessed extensive research in this area and Mathematics educators have been investigating various ways of integrating CAS with classroom teaching to develop a balanced curriculum, which lays less emphasis on paperpencil techniques and focuses more on understanding concepts. Computer Algebra Systems such as Mathematica, Maple, Derive etc. provide powerful dynamic working environments. However the availability of CAS in the form of handheld calculators such as the Casio Class Pad 300 has brought the power of visualization and exploration right into the hands of the student.

This paper describes a research study conducted with 40 students of year 11 in a traditional teaching environment where the prescribed curriculum emphasizes on mastery of paper-pencil skills and using technology is not a general practice. Two exploratory lab modules in calculus, one based on understanding of limits and the other on application of derivatives to optimization problems, have been discussed. These modules utilize the graphic, numeric and symbolic manipulation capabilities of the Class Pad 300 to facilitate conceptual understanding.

The study revealed that CAS provided opportunities for resequencing concepts and skills thus making it possible to teach concepts and applications before manipulative skills. The easy graphing capability of the Class Pad lead to a 'geometric' approach, which allowed the students to visualize and explore concepts. The study also showed that CAS led to the constructivist approach where the learning environment was transformed from the traditional teacher-centered classroom to a student-centered laboratory where the students discovered mathematical ideas for themselves.

# **1. Introduction**

The past two decades have witnessed extensive research related to the use of Computer Algebra Systems (CAS) in mathematics instruction ([1],[3] – [7]). The primary concern of mathematics educators has been to study the relevance of paper-pencil skills in an environment equipped with CAS and reforming the curriculum so as to make appropriate use of this technology [10]. The mid-1980s witnessed the Calculus Reform Movement when researchers began to focus on the implications of CAS in the calculus classroom. They claimed that CAS could improve conceptual understanding, enable students to explore more complicated problems, reduce the burden of tedious calculations, improve exercise and test questions and overcome limitations imposed by poor algebraic skills [11]. Their graphic, numeric and symbolic manipulation capabilities allow students to visualize and explore concepts, make and test hypotheses and discover mathematics truths for themselves [2]. With CAS now available in handheld form through devices such as the Casio Class Pad 300, the power of visualization and exploration has been brought to the palm of the hand. These devices allow students to explore mathematical concepts graphically, numerically, symbolically and geometrically [9]. This

paper describes a research study in which 40 students of year 11 underwent a CAS enabled calculus course, which integrated the Class Pad 300.

## 2. Educational Setting and Background Knowledge of Students

The research study described in this paper was conducted at the Mathematics Laboratory & Technology Centre based at the senior secondary school where the author is a practicing teacher. The school prescribes the CBSE (Central Board of Secondary Education) curriculum, which is the central board of education in India. In the classroom Mathematics is taught in the traditional manner using chalk and board and using technology is not a general practice. The Mathematics Laboratory & Technology Centre was set up with the primary objective of supplementing classroom teaching with innovative teaching methods and technologies. Under the leadership of the author the Centre conducts various activities, projects and supplementary courses for students from year 6 to year 12 as well as professional development programs for teachers of schools across the country. In most of these courses technology plays a vital role. The Centre is equipped with computer algebra systems such as Mathematica, dynamic geometry software such as Geometer's Sketchpad and graphics calculators (Casio CFX 9850 GB plus). It has recently acquired the Class Pad 300 along with the Class Pad Manager software.

This paper describes two lab modules, which were a part of an introductory calculus course conducted by the Centre. The course was designed to develop conceptual understanding in calculus and the vehicle for exploration was selected as the Class Pad 300. 40 students of year 11 opted for this course (this was in addition to their regular classroom teaching). Though these students had been taught some calculus in the traditional manner they were familiar with the Casio CFX 9850 GB plus graphics calculator from a pre-calculus course in their previous year. The objective of the lab modules described here, was to enable the students to visualize, explore and experiment with the concepts using the Class Pad thus discovering mathematical facts for themselves. Worksheets were specially designed to guide them in their exploration. Each module was conducted in the lab where students worked in pairs. Since only 10 Class Pad sets were available, each pair of students was either given a handset or was allowed access to the Class Pad manager software through a computer. They were instructed to sketch graphs, write observations and perform their calculations on the worksheets. These worksheets along with interview sessions with students provided the data on which this research study has been based.

# **3. Lab Module 1: Exploring the Limit of a Function**

In the prescribed curriculum the emphasis is laid on developing manipulative skills rather than on the conceptual understanding of the limit of a function. Students are expected to gain mastery of applying the appropriate result or method (such as substitution, factorization, rationalization and the standard results  $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ ,  $\lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1$ ,  $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ ,  $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ ) to evaluate the limit of a function. The 40 students who were a part of this research study had acquired some of these manipulative skills in their regular classes. The worksheet for this module was designed so as to enable

them to graphically explore and visualize the concept of the limit of a function before perfecting their paper-pencil skills.

# Aim of Lab Module

- To develop an intuitive understanding of the limit of a function.
- To evaluate the limit of a function graphically, numerically and algebraically using the CAS features of the Class Pad and finally verifying the result by hand.
- Understanding the conditions under which the limit of a function exists.

# Method and Teaching Sequence

The worksheet given to the students required them to evaluate the following limits and <u>sketch the</u> graphs in the space provided.

(i) 
$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$
, (ii) 
$$\lim_{x \to 2} \frac{x^3 - 3x - 2}{x^2 - 4}$$
, (iii) 
$$\lim_{x \to 0} \frac{\sin x}{x}$$
, (iv) 
$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$
 (v) 
$$\lim_{x \to 2} \frac{x}{x - 2}$$
, (vi) 
$$\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$$
, (vii) 
$$\lim_{x \to \infty} \frac{\sin x}{1 + x^2}$$

The following instructions were given to guide the students in their exploration.

Step 1: Enter each function in the Main Application of the Menu.

Step 2: Tap the Graph icon to get the graph screen, select and drag the function to the graph window. Incase the graph is not visible, tap the View window icon and adjust the range on the x and y axes. The range along the *x*-axis may be chosen as a small interval around the point at which the limit is to be evaluated.

Step 3: Use the Trace option to trace a cursor along the graph and observe the value that y approaches as x approaches the required point.

Step 4: Tap the Table Input icon and specify the Start, End and Step values to generate a table of values of the function. Generate various tables each time reducing the step size and taking note of the value approached by the function (that is y-value) as x comes closer to the given point from either side. Step 5: Evaluate the limit of the function by using Class Pad's built-in limit function (lim) from the Action-Calculation menu. (The syntax is lim(function,variable,value).

Step 6: Verify your answers manually in problems (ii), (iv) and (vii).

The solution of problem (i) was demonstrated by the author using the Class Pad Manager software interface on the whiteboard.



**Figure 1:** Screen shots of the Class Pad 300 for visualizing that  $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$ . The graphical output

(i) shows a gap while the tabular representations (ii) and (iii) display 'Error' at x = 2 indicating that the function is not defined at x = 2. Tracing the cursor along the graph and the tables indicate that the function approaches 4 from either side as x approaches 2 and the built-in lim function (iv) confirms that the limit is 4.

#### **Students' Explorations**

The following screen shots throw light on how students developed the concept of limit by solving the problems numerically and graphically.





Figure 2: Screen shots of Class Pad 300 (as used by the students) to evaluate the limits given in the worksheet.

The graphical output for (ii) (Fig 2(i)) indicates that the limit is around 2.23. The exact answer 9/4 is obtained by using the lim function (Fig 2(ii)). Students were asked to evaluate this manually by eliminating the common factor (x - 2) from the numerator and denominator. For (iii) and (iv) the graphical as well as tabular outputs (Fig 2(iii),(iv),(v)) indicate that functions the  $\frac{\sin x}{x}$  and  $\frac{1-\cos x}{x^2}$  are not defined at x = 0 but approach 1 and  $\frac{1}{2}$  respectively as x approaches 0 from either side. When asked to evaluate (iv) manually, students approached the problem in two ways. In the first method they rationalized the numerator by multiplying and dividing the expression by  $(1 + \cos x)$ . Thus  $\lim_{x \to 0} \frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x} = \lim_{x \to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{1}{1+\cos x} = 1 \times \frac{1}{2} = \frac{1}{2}$ . In the second method

they directly replaced  $(1 - \cos x)$  by  $2\sin^2 \frac{x}{2}$ . The manual calculation was  $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{x^2}$ 

$$= 2 \lim_{x \to 0} \frac{\sin^2 x/2}{x^2/4} \times \frac{1}{4} = 2 \times 1 \times \frac{1}{4} = \frac{1}{2}$$
. The graph for (v) (Fig 2(vi)) show two branches of a hyperbola.

The students concluded that the limit of this function does not exist at x = 2, since the function values approach  $+\infty$  and as x approaches 2 from the right, and  $-\infty$  as x approaches 2 from the left. Here the left

hand and right hand limits are not equal. The plot of  $\sin \frac{1}{x}$  (Fig 2(viii)) and table of values (Fig 2 (vii))

indicate quick oscillations near the origin between -1 and 1, which remain even after 'zooming in' around the origin. Students observed that oscillations continue to exist no matter how close one gets to the origin. This helped to highlight another situation in which the limit does not exist. The graph and table for problem (vii) (Fig 2(ix)) clearly indicate that the limit of the function is 1. When asked to work this out manually, students rationalized the numerator before evaluating the limit. Problem (viii) was given to highlight the fact that a function may approach a fixed value as  $x \to \infty$  i.e as x becomes very large. The x-axis range was chosen as 0 to 40 and the function value converges to 0, which is the limit.

The graphical outputs and tabular representations helped to emphasize the fact that the  $\lim_{x \to a} f(x)$ , is not

the value of the function at x = a, but the value f(x) approaches, as x approaches a through values less than a and greater than a. The concepts of *left hand limit* and the *right hand limit* of a function and the idea that the limit of a function at a point exists only when the left hand and right hand limits are equal were reinforced using the above examples. The problems helped to emphasize that the limit of a function may exist at a point even if the function is not defined at that point and also to highlight situations when the limit does not exist.

Students recorded their graphical observations and manual calculations on their worksheets. Based on these worksheets it may be concluded that this module led to a physical or graphical understanding of limits, which is difficult to achieve in a traditional chalk-board class. The graphical, numeric and CAS features of the Class Pad provided the tools for exploration and the 'concepts before skills' approach ensured that the students grasped the concept before trying out the problems by hand. Since the students were made to solve the problems manually after evaluating the limits graphically and numerically there was no compromise on 'by-hand' skills.

# 4. Lab Module 2: Application to Optimization Problems

Optimization problems form an important part of the traditional calculus course. In the typical problems the student is expected to formulate the problem by defining the function f(x) which is to be maximized or minimized, identify its domain, and use the second derivative test to find the value of x which makes the function as large or as small as possible, whichever is appropriate. The student is expected to solve the entire problem manually and is tested on his ability to apply the method to obtain the answer. Before going through this lab module the students had undergone similar modules to understand the concept of the derivative and finding extrema of functions using the first and second derivative tests.

#### Aim of Lab Module

- To formulate the optimization problem by defining the function (and its domain) to be optimized.
- To use Class Pad's CAS features for applying the second derivative test and finding the maximum or minimum value of the function.
- To obtain a graphical or geometrical understanding of the problem (which is completely missing in the traditional curriculum).

## Method and Teaching Sequence

Students were given five problems in a worksheet (only two of which will be discussed here) and were instructed to explore them in the following manner.

Step 1: Read the problem carefully, define the function f(x) to be maximized or minimized and identify its domain. Enter this function in the Main Application of the Class Pad Menu.

Step 2: Use the diff function to find f'(x). The syntax is diff(function,variable).

Step 3: Use the solve function to solve the equation f'(x) = 0 to find the critical points or 'candidates' for maxima or minima. We shall refer to these as  $x_0$ .

Step 4: Use diff function to find f ''(x) and substitute the  $x_0$  values obtained in step 3. If f ''( $x_0$ ) <0 then  $x_0$  is a point of maxima. If f ''( $x_0$ ) >0 then  $x_0$  is a point of minima.

Step 5: Use the fMax or fMin functions to find the maximum or minimum values of f(x). The syntax is fMax(function, variable, variable range).

Step 6: Tap the Graph icon, drag f(x) on to the graph screen and use the View window option to graph f(x). On the same window graph f'(x), identify the critical points of f(x) and the corresponding function values and verify the answer obtained in step 5.

Step 7: After obtaining the solution using Class Pad, solve the problem manually and convince yourself of the answer.

Students were instructed to write their solutions, observations and sketch the graphs on their worksheets. They worked in pairs while exploring the problem on Class Pad and were interviewed as they worked. An analysis of their explorations was done based on their responses as well as the observations recorded by them on the worksheets.

**Optimization Problem 1**: A rectangular sheet of paper has dimensions 15 cm by 8 cm. Square pieces are to be cut from all four corners and the remaining sheet to be folded so as to form an open box. What should be the length of the square to be cut out so that the volume of the box is maximum?

# **Students' Explorations**

32 students were able to define the volume of the box (without any assistance) as V(x) = x(15 - 2x)(8 - 2x) where x is the length of the squares cut from the corners. For the remaining students a figure had to be drawn before they could write the expression. About 12 to 14 students needed help in understanding that the domain of the function is (0,4). Following the steps given above all students were able to obtain the critical points as 5/3 and 6 (Fig 3(i)) but only 34 students figured that x = 6 was unacceptable since it is not within the domain of V(x). All students confirmed the 5/3 is a point of maxima (since V''(5/3) = -52 <0) and that the maximum volume of the box is  $\frac{2450}{27} = 90.74$  cm<sup>3</sup> (this was obtained using the approx function (Fig 3(ii))). Almost all students faced a problem while

selecting the appropriate View window for graphing V(x). After a few trials (keeping in mind that ymax must exceed 90.74) V(x) was plotted (Fig 3(iii)). Most students concluded that V(x) is a cubic function and hence V'(x), a quadratic, must be represented by a parabola. They adjusted the x-range (Fig 3(iv)),used Analysis-> G-Solve->fMax to confirm that the maximum volume is 90.74 cm<sup>3</sup> (Fig 3(v)). After plotting V(x) and V'(x) on the same graph they were able to visualize that the critical points (5/3 and 6) are the roots of V'(x) i.e the x-values where V'(x) cuts the x-axis and that corresponding to x = 5/3 (the only acceptable point) V(x) has the maximum value 90.7.



Figure 3: Screen shots of Class Pad 300 (as used by the students) to explore optimization problem 1.

**Optimization Problem 2**: A wire of length 28 meters is cut into two pieces. One is bent into a square and the other into a circle. How long should the pieces be so that the combined area of the square and circle is minimum?

### **Students' Explorations**

All students approached the problem assuming that the pieces are of length x meters and (28 - x) meters respectively. 28 students assumed that the piece of length x cm is bent into a circle (of radius r) and the other into a square (of side a). The combined area of the two figures, A(x) was calculated as  $\frac{x^2}{4\pi} + \frac{(28-x)^2}{16}$ . (This was done manually as follows:  $2\pi r = x \Rightarrow r = \frac{x}{2\pi}$  and  $4a = 28 - x \Rightarrow a = \frac{28-x}{4}$ . Thus A(x) =  $\pi r^2 + a^2 = \frac{x^2}{4\pi} + \frac{(28-x)^2}{16}$ ). Students figured that the domain of A(x) is [0,28] since x can be equal to 0. The solution obtained by them is shown in the screen shots given below (Fig 4 (i),(ii)). They used the approx function of Class Pad to approximate symbolic solution  $x = \frac{28\pi}{\pi + 4}$  to 12.32 and then used the fMin function to find the minimum value of A(x). Class Pad returned a complicated expression which was simplified to  $\frac{196}{\pi + 4}$  using the simplify function. Students tried various View window options to graph A(x) (Fig 4 (iii)) and its derivative (Fig 4 (iv)). They recognized that the A(x) is a quadratic expression and its derivative is linear (a straight line) which cuts the x axis at 12.32, the only critical point. Finally they concluded that the minimum area (of the square and circle) is 27.44 m<sup>2</sup> and the lengths of the pieces are 12.32 meters and 15.68 meters respectively.



**Figure 4:** Screen shots of Class Pad 300 (as used by the students) to explore optimization problem 2 assuming that the piece of length x meters is bent into a circle.

Alternatively 12 students approached the problem with the assumption that the piece of length x meters is bent into a square (of side a) and the other into a circle (of radius r). The combined area A(x) in this case is  $\frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$ . The value of x which minimizes the combined area was found to be

 $x = \frac{112}{\pi + 4}$  or 15.68 cm (Fig 5). The graphs obtained were similar to the previous case. The students

compared the solutions obtained by the two approaches.

For both optimization problems, while Class Pad took over the calculations the students focused on the application of the problems and understanding them graphically. They related the graphs of the first and second derivatives to the maxima or minima of the original function and the critical points. Following this, they manually verified the Class Pad solutions thus focusing on the paper-pencil skills and techniques such as calculating the derivatives, solving the equations to find the critical points and evaluating the maximum or minimum value of the function. They were required to show their working on their worksheets. It may be pointed out here that in the traditional curriculum the emphasis is on applying techniques and testing 'procedural knowledge'. Almost all the students in the research study had reasonably good 'by-hand'skills.



**Figure 5:** Screen shots of Class Pad 300 (as used by the students) to explore optimization problem 2 assuming that the piece of length x meters is bent into a square.

#### **Students Feedback/Results.**

All 40 students who were a part of the research study were to appear in the traditional examinations at the end of the academic session where their manipulative skills would be tested. It was therefore imperative to monitor the impact of the Class Pad on their paper-pencil skills and to compare their performance with a group of students who had been taught the same topics in a traditional manner without any technology intervention. A group of 35 students (whose performance level was similar to the Class Pad group of 40 students, that is the mean marks of both groups varied between 75% and 85% in all tests and exams conducted in year 10) was selected and a 40 marks test was administered to both groups. All problems were to be solved by hand and access to Class Pad or any other software was not allowed. The results of the test revealed that the Class Pad group had a mean score of 31.48 while the traditional class had a mean score of 30.2. Table 1 provides an analysis of the performance of both groups giving the average scores in each exercise as well as the number and nature of questions posed in each topic. The scores are comparable for both groups for all exercises. The Class Pad group

performed better than the traditional group in exercises 1, 3 and 4 whereas the traditional group performed better in exercise 2. A closer analysis of the answer scripts revealed that the Class Pad group was able to score better in exercises 3 and 4 since they were able to provide better and more accurate graphical interpretations. However in exercise 2 the traditional group seemed to have better computational skills than the Class Pad group. From the results it appears that the Class Pad group was not disadvantaged in comparison with their traditional counterparts as far as computational skills are concerned.

Exercise	Topic	Nature of exercise	Marks	Class	Traditional
		allotted t		Pad	Group
			exercise	Group	
1	Limits	There were 4 problems and			
		students were expected to	8	6.48	6.1
		evaluate the limits using	(2 marks per		
		substitution, factorization,	problem)		
		rationalization and standard			
		results.			
2	Differentiation	There were 4 problems and			
		students were to be tested on	12	8.52	8.8
		using sum, difference,	(3 marks per		
		product, quotient and chain	problem)		
		rules to evaluate derivatives			
		of functions			
3	Application of	This exercise (consisting of	8		
	Derivatives to	2 parts) was based on	(4 marks for		
	Lagranges	verifying LMVT and Rolles	each part with		
	Mean Value	theorem for functions on	1 mark	6.28	6
	Theorem/	given intervals. Students	allotted for		
	Rolles	were also expected to	graphical		
	Theorem	interpret the results	interpretation)		
		graphically.			
4	Application of	Two problems were posed			
	derivatives to	(one on maximizing and the	12		
	optimization	other on minimizing the	(6 marks per		
	problems	objective function). The	problem with	10.2	9.3
		students were required to	one mark		
		solve the entire problem	allotted for		
		manually by finding the	graphical		
		point of maxima (or	interpretation)		
		minima) along with the			
		maximum (or minimum)			
		value of the function.			
	r	Fotal Average Score		31.48	30.2

Table 1: Analysis of performances of the Class Pad and Traditional groups on a calculus test.

# **5.** Concluding Discussion

This paper describes two lab modules in calculus, which were part of a research study to integrate CAS with traditional teaching, tried out with 40 students of year 11. Through regular classroom teaching the students were familiar with some basic concepts on which the labs were based and had acquired some paper – pencil methods. The lab modules were designed so as to enable the students to explore the concepts on their own before gaining mastery of the paper-pencil methods. They were given access to the Casio Class Pad 300 and were allowed to explore the problems in the worksheets using the graphic, numeric and symbolic manipulation capabilities of the device. After the exploration, they were required to record the solutions and verify the same by hand. The students underwent a calculus test in which they were not allowed access to technology and their scores were compared with a group of students of similar ability who were taught the same topics in a traditional manner. Assessment of students' worksheets, interviews conducted at the end of the modules and analysis of test scores helped to corroborate the following published research findings related to the use of CAS for teaching and learning mathematics.

- (a) **Exploration through CAS led to a deeper understanding of concepts.** The Class Pad allowed students to explore the concepts through graphing, creating tables and trying alternative solution methods. Each student made his own observations and recorded them in his worksheet. These modules highlight 'learning by discovery'.
- (b) **Resequencing of concepts and skills.** The Class Pad allowed the students to focus on concepts and applications before practicing the paper-pencil methods. They grasped the mathematical ideas before gaining mastery of the manipulative skills.
- (c) **Geometric approach to learning calculus**. Since the Class Pad can instantly produce accurate graphs of functions the problems in the modules could be approached geometrically. The emphasis was on developing graphical understanding of concepts by plotting graphs, varying the view window, using the TRACE option or playing around with an animation to actually visualize what is happening.
- (d) Balance between conceptual understandings and paper-pencil skills. In each lab module the students were made to verify the Class Pad solution by hand and spend sufficient time on practicing the paper-pencil techniques once the conceptual understanding was achieved. This was done to ensure that there was no compromise on developing the 'procedural knowledge' or 'by hand skill' due to the use of technology.
- (e) **CAS and the Scaffolding Method**. Since the Class Pad could take over lengthy calculations, even students with poor algebraic manipulation skills were able to attempt the problems and grasp the concepts. After understanding the concept, they spent extra time on practicing and improving their 'by hand' techniques. Thus the Class Pad acted as a scaffold for the less able students. This made the subject matter easier and accessible to students of all levels of abilities.
- (f) **Collaborative Learning**. During the lab sessions students primarily worked in pairs and discussed their work. After completing the worksheets discussions between groups was also allowed. Students also grouped together to solve homework assignments and often corrected or assisted each other. In all this the role of the teacher was to facilitate meaningful discussions.

- (g) **Redefining Learning Goals**. This research study corroborates that the learning goals of calculus courses have to be redefined. While traditional calculus courses emphasize manipulative skills and routine problem solving, CAS integrated calculus courses must focus on conceptualization, graphical understanding, applications to non-routine problems and building interconnections between various forms of mathematical representations.
- (h) CAS and Constructivism. Through their graphic, numeric and algebraic manipulation capabilities, CAS provide students with tools to explore mathematical ideas thus lending themselves to the constructivist approach. They enable students to construct their own 'mental models' of mathematical concepts by allowing them to form and test conjectures, discover patterns and generalize important results. In the modules discussed here students were actively engaged in the learning process constructing their own understanding of the concepts, through discussions and explanations facilitated by their teacher.

It may be added here that the traditional curriculum emphasizes on testing students' paper-pencil skills often at the expense of conceptual understanding. This research study suggests that through appropriate integration of CAS technology a balanced approach can be adopted which allows students to focus on concepts without losing out on important paper-pencil skills.

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# Effects of Integrating the Use of Graphic Calculators on Performance in Teaching and Learning of Mathematics from the Cognitive Load Perspective

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**Abstract:** Cognitive load theory assumes that some learning environment impose greater demands than others, consequently impose a higher information processing load on limited cognitive resources in working memory. The theory holds that if an instructional strategy reduces extraneous cognitive load and/or increases germane cognitive load during learning as compared to another instructional strategy, then it will be more efficient in promoting learning, provided that the total cognitive load does not exceed the total mental resources. Based on this premise, three phases of quasi-experimental studies were conducted to investigate the effects of integrating the graphic calculator in mathematics teaching and learning on Form Four Malaysian secondary school students' performance. The findings from this study indicated that integrating the use of graphic calculator can reduce cognitive load and lead to better performance in learning of Straight Lines topic and increase 3-dimensional instructional efficiency index. Thus the graphic calculator strategy is instructionally more efficient than the conventional instructional strategy. Overall, this study has shown promising implications for the potential of the tool in teaching mathematics at Malaysian secondary school level.

### **1. Introduction**

Recently, technology tools are increasingly available to enhance and promote mathematical understanding. Among those, there has been a steady increase in interest in using hand-held technologies, in particular the graphic calculator. Generally, this tool has gained widespread acceptance as a powerful tool for learning mathematics. As this technology is commonplace in classroom, consideration of the extent to which its usage can impact students' understanding of mathematical concepts within particular course content is vital. Kastberg and Leatheam (2005) in reporting research studies on the use of graphic calculator up to this time, argue that the maximum potential for this technology has not been explored. Those studies provide a starting point for effort to be better understanding how to effectively use the technology in the classroom. Thus, further rigorous research is needed. This study directly responds to the need for empirical evidence regarding the effects of integrating the use of graphic calculator in mathematics instruction at the Malaysian secondary school level. Apart from studying the effectiveness of integrating the use of graphic calculator in teaching and learning of mathematics on performance measures, this research

attempts to provide explanation on the benefit of graphic calculator as a tool for learning from the cognitive load perspectives.

# 2. Cognitive Load Theory

More and more applications of cognitive load theory (CLT, Sweller, 1994; 1999) have begun to appear in the field of technology learning environment recently (van Merrienboer & Ayres, 2005; Mayer & Moreno, 2003, Pass et al., 2003). Research within cognitive load perspective is based on the structure of information and the cognitive architecture that enables learners to process that information. Specifically, CLT emphasizes structures that involve interactions between long term memory and short term memory or working memory which play a significant role in learning. One major assumption of the theory is that a learner's working memory has only limited in both capacity and duration. Under some conditions, these limitations will somehow impede learning.

Cognitive load is a construct that represents the load which performing a particular task imposes on the cognitive system (Sweller et al., 1998). CLT researchers have identified three sources of cognitive load during instruction: intrinsic, extraneous and germane cognitive load (e.g. Cooper, 1998; Pass et al., 2003; Sweller et al., 1998). Intrinsic cognitive load is connected with the nature of the material to be learned, extraneous cognitive load has its roots in poorly designed instructional materials, whereas germane cognitive load occurs when free working memory capacity is used for deeper construction and automation of schemata. Intrinsic cognitive load can be reduced.

According to CLT, learning will fail if the total cognitive load exceeds the total mental resources in working memory. With a given intrinsic cognitive load, a well-designed instruction minimizes extraneous cognitive load and optimizes germane cognitive load. This type of instructional design will promote learning efficiently, provided that the total cognitive load does not exceed the total mental resources during learning.

Some researchers have suggested that the use of calculators can reduce cognitive load when students learn to solve mathematics problems (Jones, 1996, Kaput, 1992; Wheatley, 1980). Thus, in this study, it was hypothesized that integrating the use of graphic calculators in teaching and learning of mathematics can reduce cognitive load and lead to better performance in learning. Specifically, this method uses an instructional strategy that minimizes extraneous cognitive load and hence optimizes germane cognitive load.

### **3.** Purpose of the Study

The purpose of this study is to investigate the effects of integrating the use of graphic calculator in mathematics teaching and learning on students' performance for Form Four secondary school students when learning Straight Lines topic. Thus, two types of instructional strategy that is the graphic calculator strategy and the conventional instruction strategy were compared on performance, mental load and instructional efficiency. Three phases of experiments were conducted in this study. Experiment in Phase I was a preliminary study. It was carried out for three weeks. Phase II was part replication of experiment in Phase I. In addition, the possibility that the use of graphic calculators can reduce cognitive load was tested in this phase. Finally, Phase III was conducted to investigate that the effectiveness of using graphic calculator may well depend on different levels of mathematics ability. Both experiments in Phases II and III were carried out for 6 weeks.

# 4. Methodology of the Study

#### Design

The quasi-experimental nonequivalent control-group posttest only design (Cook & Campbell, 1979, Creswell, 2002) was employed. In addition, for Phase III, a 2 x 2 factorial design was integrated in order to investigate two main factors mainly the instructional strategy (graphic calculator (GS) strategy and conventional instruction (CI) strategy) and mathematics ability (low and average). For all phases, the groups that were selected were ensured for their initial equivalence (similar mathematics ability) and classes involved were randomly assigned to GC strategy and CI strategy groups.

#### **Population and Sample**

The target population for this study was Form Four (11<sup>th</sup> grade level) students in National secondary schools in Malaysia whilst the accessible population was Form Four students from one selected school in Selangor and Malacca. Each phase was carried out within one particular school only. A total of 40 students took part in Phase I such that there were 20 students in the GC strategy group and there were 19 students in the CI strategy group. A total of 65 students took part in second phase of the study. The GC strategy group consisted of 33 students while the CI strategy group consisted of 32 students. A total of 77 students took part in the third phase of the study. The average mathematics ability of GC strategy and CI strategy groups consisted of 17 students and 18 students respectively, whereas, the low mathematics ability of GC strategy and CI strategy and CI strategy and CI strategy groups consisted of 20 students and 22 students respectively.

#### **Materials and Instruments**

The instructional materials for Phase I consisted of six sets of lesson plan, whilst for Phases II and III consisted of fifteen sets of lesson plan of teaching and learning of Straight Lines topic. The main feature of the acquisition phase for the GC strategy group was that students used "balanced approach" in learning of Straight Lines topic. Waits and Demana (2000) illustrated that the "balanced approach" is an appropriate use of paper-and-pencil and calculator techniques on regular basis (p.6). Specifically, the TI 83 Plus Graphing Calculator was used in this study. The CI strategy group was also guided by the same instructional format with conventional whole-class instruction without incorporating the use of graphic calculator.

There were two instruments used in this study namely the Straight Lines Achievement Test (SLAT) and the Paas (1992) Mental Effort Rating Scale (PMER). The SLAT had three variations because these instruments were modified based on the results of preceding phases. For Phase I, the SLAT comprised of seven questions based on the Straight Lines topic covered in the experiment. The total test score for the SLAT was 40. The reliability index using Cronbach's alpha coefficient was .57. This index was not an acceptable level based on Nunnally (1978) cut-off point of .70. However, according to Ary, Jacobs and Razavieh (1996), a lower reliability coefficient (in the range of .50 to .60) might be acceptable if the measurement results are to be used in making decisions about a group. Thus, the reliability of SLAT for this phase was reasonably acceptable. For Phase II, the SLAT comprised of 12 questions and the total test score was 60. The computed index of reliability,  $\alpha$ , for the SLAT was determined to be .68. Whereas, for Phase III, the SLAT comprised of 14 questions, the total test score was 75 and the reliability index was .82.

The PMER was used to measure cognitive load by recording the perceived mental effort expended in solving a problem in experiments of Phases II and III. It was a 9-point symmetrical Likert scale measurement on which subject rates their mental effort used in performing a particular learning task. It was introduced by Paas (1992) and Paas and Van Merrienboer (1994). The numerical values and labels assigned to the categories ranged from very, very low mental effort (1) to very, very high mental effort (9). For each question in SLAT of Phases II and III, the PMER was printed at the end of the test paper. After each problem, students were required to indicate the amount of mental effort invested for that particular question by responding to the nine-point symmetrical scale. The computed indices of reliability for PMER in both phases were .87 and .91 respectively.

### 5. Results

The exploratory data analysis was conducted for all the data collected in all phases. The total number of students taking part in Phase I was as follows: GC strategy group consisted of 21 students, whilst CI strategy group consists of 19 students. For Phase II, the GC strategy group consisted of 33 students, whilst the CI strategy group consisted of 32 students. For Phase III, the outliers were taken out. Thus the total number of students taking part in this phase was as follows: group 1 designated of students with average mathematics ability undergoing CI strategy consisted of 15 students; group 2 designated of students with average mathematics ability undergoing GC strategy consisted of 16 students, group 3 designated of students with low mathematics ability undergoing GC strategy consisted of 19 students, and group 4 designated of students with low mathematics ability undergoing GC strategy consisted of 20 students.

Students' performance was measured by the overall test performance, number of problems solved and transfer problems performance. There were two kinds of subjective ratings of mental effort taken during the experiments in Phases II and III. Firstly, the subjective ratings of mental effort were taken during learning in evaluation phase for each lesson. Secondly, it was taken during test phase. The mental effort per problem was obtained by dividing the perceived mental effort by the total number of problems attempted for each evaluation phase during learning and that of the test phase.

Further, the 3-dimensional (3-D) instructional condition efficiency indices were calculated using Tuovinen and Paas (2004) procedure and were taken into the analyses as dependent variables. The three dimensions namely the learning effort, test effort and test performance was taken into account when calculating these indices. In the computational approach, the three sets of data (learning effort, test effort and test performance) were converted to standardized z scores. Then, the 3-D efficiency index was computed using the formula,  $E = (P - E_L - E_T)/\sqrt{3}$ , where P is z score for performance,  $E_L$  is z score for learning effort and  $E_T$  is z score for the test effort (Tuovinen & Paas, 2004). The greatest instructional condition efficiency would be occurred when the performance score was the greatest and the effort scores were the least. On the other hand, the worst instructional efficiency condition would occur when the performance score was the least and the effort scores were the greatest.

For Phases I and II, comparative analyses using independent samples t-tests were used to explained differences exist in means of dependent variables between GC strategy and CI strategy groups. Further, the planned comparisons were conducted in order to ascertain that the means of dependent variables for GC strategy group are significantly higher from those of CI instruction strategy groups. In addition, all data for Phase II were analyzed using a two-way analysis of variance (2-way ANOVA) and followed by planned comparison tests.

#### Phase I Effect of GC Strategy and CI Strategy on Performance

The means, standard deviations of the variables under analysis and the results of the independent samples t-test are provided in Table 1. As can be seen from Table 1, the mean overall test performance of GC strategy group was 16.81 (SD=4.76) and mean overall test performance for CI strategy group was 12.53(SD=4.99). Independent samples t-test results showed that there was a significant difference in mean test performance between GC strategy group and the CI strategy group, t(38)=2.78, p<.05. The magnitude of the differences in the means was large based on Cohen (1988) with eta squared =.17. Further, planned comparison test showed that mean overall test performance of GC strategy group was significantly higher from those of CI strategy group, F(1, 38)= 7.71, p<.05. This finding indicated that the GC strategy group had performed better for test phase than the CI strategy group.

An independent t-test analysis on mean number of problems solved also revealed a significant difference between GC strategy group (M=2.19, SD=1.12) and CI strategy group (M=1.53. SD=.84), t(38)=2.10, p<.05. The magnitude of the differences in the means was moderate based on Cohen (1988) with eta squared =.10. Planned comparison test showed that mean number of problems solved of GC strategy group was significantly higher from those of CI strategy group, F(1, 38)=4.40, p<.05. This finding suggested that the GC strategy group had solved more problems than that of CI strategy group.

For transfer problems performance, the results of independent t-test showed that there was no significant difference in means between the GC strategy group and CI strategy group, t(38)=1.92, p>.05. The effect size was 09(moderate) using eta squared value based on Cohen (1988). Planned comparison tests showed that means of GC strategy group was not significantly higher from those of CI strategy group. This finding suggested that GC strategy group performed as well as the CI strategy group on transfer problems during test phase.

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Performance	Group	Ν	Μ	SD	SEM	t	df	р
Test performance	Experimental	21	16.81	4.76	1.04			
						2.78	38	.008
	Control	19	12.53	4.99	1.15			
No. of problems	Experimental	21	2.19	1.12	.25			
solved	•					2.10	38	.043
	Control	19	1.53	.84	.19			
Transfer problems	Experimental	21	7.14	5.00	1.09			
performance	-					1.92	38	.062
	Control	19	4.32	4.22	.97			

Table 1. Independent samples t-test for overall test performance in Phase I

# Phase II

### Effect of GC Strategy and CI Strategy on Performance

The means, standard deviations of the variables under analysis and the results of the independent samples t-test are provided in Table 2. As can be seen from Table 2, mean overall test performance of the GC strategy group was 24.21 (SD=9.69) and mean overall test performance of CI strategy group was 17.75 (SD=10.54). Independent samples t-test results showed that there was a significant difference in mean overall test performance between GC strategy group and the CI strategy group, t(63)=2.57, p<.05. The magnitude of the differences in the means was moderate based on Cohen (1988) using eta squared =.64. Planned comparison test showed that the mean test performance of GC strategy group was significantly higher from those of CI strategy group, F(1, 63)= 6.60, p<.05. This suggested that the GC strategy group had performed better on overall test performance than the CI strategy group.

For number of problems solved, the results of the independent t-test showed that there was no significant difference in means between the GC strategy group (M=2.73, SD=1.96) and the CI strategy group (M=2.22, SD=1.85), t(63)=1.08, p>.05. The effect size was .02 using eta squared value which was small based on Cohen (1988). Planned comparison test showed that mean number of problems solved for test phase of GC strategy group was not significantly higher from those of CI strategy group, F(1, 63) = 1.17, p>.05. This finding suggested that both groups had solved more or less the same number of problems during test phase.

Further, there was a significant difference in mean transfer problems performance between the GC strategy group (M=15.09, SD=5.33) and the CI strategy group (M=8.41, SD=5.87); t(63)=4.81, p<.05. The effect size was .27 using eta squared value which was large based on Cohen (1988). Planned comparison tests showed that means of GC strategy group was significantly higher from those of CI strategy group, F(1, 63)=23.14, p<.05. This suggested that the GC strategy group performed better on transfer problems performance as compared to CI strategy group.

Table 2. Independent samples t-test for performance									
Performance	Group	Ν	Μ	SD	SEM	t	df	р	
Test performance	Experimental	33	24.21	9.69	1.69				
						2.57	63	.012	
	Control	32	17.75	10.54	1.86				
No. of problems	Experimental	33	2.73	1.96	.34				
solved						1.08	63	.285	
	Control	32	2.22	1.85	.33				
Transfer problems	Experimental	33	15.09	5.33	.93				
solved	-					.30	63	.000	
	Control	32	8.41	5.87	1.04				

#### Effect of GC Strategy and CI Strategy on Mental Effort

Table 3 provides the means, standard deviations and analyses of independent samples t-test on mean mental effort per problem during learning and test phase. As can be seen in Table 4, the GC strategy group (M = 2.93, SD=.78) had lower mean mental effort per problem during learning phase than the CI strategy group (M = 4.13, SD=.91). The result of an independent t-test showed there was a significant difference in the mean mental effort per problem, (t(63)=-5.72, p<.05)between the GC strategy and CI strategy group. The effect size was .34 using eta squared value which was large based on Cohen (1988). Planned comparison showed that the mean mental effort for CI strategy group was significantly higher from those of CI strategy group, F(1, 63)=32.72, p<.05.

In addition, it was also found that the GC strategy group (M=5.41, SD=1.45) had lower mean mental effort per problem for test phase than the CI strategy group (M=6.44, SD=1.27). The results of an independent t-test showed that there was a significant difference in the mean mental effort per problem, (t(63)=-3.03, p<.05 between the GC strategy and CI strategy groups. The effect size was .14 using eta squared value which was large based on Cohen (1988). Planned comparison tests showed that the mean mental effort per problem invested during test phase for CI strategy group was significantly higher from that of GC strategy group, F(1, 63)=9.18, p<.05.

1	able 5. Independer	nt samp	dies t-tes	st for n	iental e	HORU		
Variables	Group	Ν	Μ	SD	SEM	t	df	р
Mental effort	Experimental	33	2.93	.78	.14			
(Learning phase)						-5.72	63	.000
	Control	32	4.13	.91	.16			
Mental effort	Experimental	33	5.41	1.45	.25			

#### 2 Indonondant complex t text fo

(Test phase)						-3.03	63	.004
· - ·	Control	32	6.44	1.27	.22			

#### Effect of GC Strategy and CI Strategy on Instructional Efficiency

Table 4 shows the independent samples t-test results for evaluating the hypotheses that the experimental and control groups differ significantly on measures of 3-D instructional condition efficiency index for phase II. The 3D instructional efficiency indices as calculated for the experimental and control groups of experiment in this phase were 0.70 and -0.73 respectively. The results of an independent samples t-test showed that there was a significant difference in mean 3-D instructional condition efficiency index (t(63)=4.46, p<.05) between the GC strategy group and that of CI strategy group. The effect size was .34 using eta squared value which was large based on Cohen (1988). The planned comparison test on mean 3-D instructional condition efficiency index showed that the mean 3-D instructional condition efficiency index significantly higher from that of CI strategy group, F(1, 63)=19.89, p<.05. This suggested that learning by integrating the use of graphic calculator was more efficient than using CI strategy.

#### Table 4. Independent samples t-test for 3-D instructional condition efficiency index

Variables	Group	Ν	Μ	SD	SEM	t	df	р
3-D instructional efficiency	Experimental	33	.70	1.31	.23			
						4.46	63	.000
	Control	32	.73	1.28	.23			

### Phase III

#### Effect of GC Strategy and CI Strategy on Performance

For this phase, students' performance was measured by overall test performance only. The means and standard deviations for overall test performance as a function of the level of mathematics ability and type of instructional strategy are provided in Table 5. The ANOVA performed on the mean overall test performance showed a significant main effect of level of mathematics ability (F(1, 66)=65.23, p<.05) with large effect size (partial eta squared=.50) based on Cohen (1988). Similarly, the main effect of type of instructional strategy also yielded a significant differences ((F(1, 66)=23.82, p<.05) with large effect size (partial eta squared=.27). However, the interaction effect between mathematics ability and instructional strategy did not reach statistical significant (F(1, 66)=.87, p>.05, partial eta squared=.01). About 58% of variance in test performance was predictable from both the independent variables and the interaction.

Table 5. Means and standard deviations for overall test performance as a function o	f
mathematics ability level and instructional strategy type	

mai	nematics admity lev	ei anu msu uc	uonai su alegy lype	5
Mathematic ability	Instructional	Ν	М	SD
	strategy			
Average	CI	15	24.20	8.74
	GC	16	30.38	7.74
	Total	31	27.39	8.69
Low	CI	19	10.11	4.03
	GC	20	19.20	5.26
	Total	39	14.77	6.54
Total	CI	34	16.32	9.58
	GC	36	24.17	8.51
	Total	70	20.36	9.81

Planned comparisons were further conducted to ascertain that the mean of GC strategy group were significantly higher from that of CI strategy group. As can be seen from Table 6, the GC strategy group (M=23.97, SD=9.58) had higher mean test performance than that of the CI strategy group (M=16.32, SD=9.58). The planned comparison showed that the mean test performance for GC strategy was significantly higher from that of CI strategy group, F(1,68)=13.18, p<.05. The results indicated that the GC strategy is significantly better than the CI strategy.

#### Effect of GC Strategy and CI Strategy on Mental Effort

As in Phase II, the subjective ratings of mental effort were also taken during learning in evaluation phase for each lesson and during test phase for this phase. The means, standard deviations for mental effort invested during learning phase as a function of the level of mathematics ability and type of instructional strategy are provided in Table 6. The ANOVA performed on mean amount of mental effort invested during learning phase showed that the main effect of level of mathematics ability (F(1,66)=2.52, p>.05, partial eta squared=.04), and the interaction of mathematics ability level and instructional strategy type (F(1,66)<1, P>.05, partial eta squared< .01) were not significant. However, the main effect of type of instructional strategy (F(1,66)=4.46, p<.05) was significant with small effect size (partial eta squared=.05). About 10.1% of variance in mean amount of mental effort invested was predictable from both the independent variables and the interaction. The results of planned comparison showed that the mental effort invested during learning phase for CI strategy was not significantly higher than that of GC strategy (F(1,55.67)=4.08, p>.05). This suggested that the GC strategy and the CI strategy group had more or less the same amount of mental effort invested during learning phase.

a function o	i mainematics abi	nty ievel and	u msu ucuonai su	alegy type
Mathematic ability	Instructional	N	M	SD
	strategy			
Average	CI	15	4.71	.86
-	GC	16	4.06	.77
	Total	31	4.37	.87
Low	CI	19	4.88	1.31
	GC	20	4.59	.59
	Total	39	4.74	1.01
Total	CI	34	4.81	1.12
	GC	36	4.36	.72
	Total	70	4.58	.96

 Table 6. Means and standard deviations for mean amount of mental effort during learning as a function of mathematics ability level and instructional strategy type

The means and standard deviations for mental effort invested during test phase as a function of the level of mathematics ability and type of instructional strategy, respectively, are provided in Table 7. The ANOVA performed on mean amount of mental effort invested during test phase showed a significant main effect of level of mathematics ability (F(1,66)=15.25, p<.05, partial eta squared=.19). The main effect of type of instructional strategy was also significant (F(1,66)=41.66, p<.05, partial eta squared=.39). In addition, there was also a significant interaction between mathematic ability levels and instructional strategy type (F(1,66)=5.68, p<.05, partial eta squared=.08). About 47.8% of variance in mean amount of mental effort invested was predictable from both the independent variables and the interaction.

Figure 1 depicts the interaction between mathematic ability levels and instructional strategy type. It is observed that as mathematics ability increased, the amount of mental effort invested

during test phase of the GC strategy decreased. For low mathematics ability, this strategy was less beneficial, but, for average mathematics ability group, it led to decrease about 2.16 points (6.69 - 4.53) which is doubled mean amount of mental effort than the low mathematics ability group which reported decreased in mean amount of mental effort of about 7.06 - 6.06 = 1.00 points.

Further planned comparison results showed that the mental effort invested during test phase for CI strategy group was significantly higher than that of GC strategy group, F(1,68)=30.25, p<.05 such that students in GC strategy had invested less mental effort during test phase as compared to that students in CI strategy group. This finding suggested that the GC strategy had invested less mental effort during test phase as compared to the CI strategy.

1	mathematics abin	ity it it and	mon actional off at	cgy type
Mathematic ability	Instructional	Ν	М	SD
	strategy			
Average	CI	15	6.69	.90
	GC	16	4.53	.75
	Total	31	5.57	1.36
Low	CI	19	7.06	1.06
	GC	20	6.06	1.21
	Total	39	6.55	1.23
Total	CI	34	6.89	1.00
	GC	36	5.38	1.28
	Total	70	6.12	1.37

 

 Table 7. Means and standard deviations for mental load during test as a function of mathematics ability level and instructional strategy type





### Effect of GC Strategy and CI Strategy on 3-D Instructional Efficiency Index

The means and standard deviations for 3-D instructional condition efficiency indices as a function of the level of mathematics ability and type of instructional strategy, respectively, are provided in Table 8. The ANOVA performed on the 3-D instructional condition efficiency indices revealed a significant effect of mathematics ability level (F(1,66)=31.59, p<.05, partial eta squared=.32). The main effect of instructional strategy type was also significant (F(1, 66)=33.40, p<.05, partial eta squared=.34). However, the interaction between mathematics ability level and instructional strategy type were not significant (F(1,66)=1.24, p>.05, partial eta squared=.02). About 49.9% of

variance in mean 3-D instructional condition efficiency index was predictable from both the independent variables and the interaction.

The planned comparison test on mean 3-D instructional condition efficiency index showed that the mean 3-D instructional condition efficiency index for GC strategy group was significantly higher from that of CI strategy group, F(1,66)=22.37, p<.05. This finding suggested that GC strategy was more efficient than CI strategy.

Iune				sindles ype
Mathematic	Instructional	Ν	М	SD
ability	strategy			
Average	CI	15	10	1.11
	GC	16	1.57	.94
	Total	31	.76	1.32
Low	CI	19	-1.19	1.15
	GC	20	06	.80
	Total	39	61	1.13
Total	CI	34	70	1.24
	GC	36	.67	1.18
	Total	70	.00	1.39

Table 8. Mean and standard deviation for 3-D instructional condition efficiency indices as a
function of mathematics ability level and instructional strategy type

### 6. Discussion

Past studies on effects of the use of graphic calculators offers different results. Generally the results have favored the use of this technology in mathematics classroom (for example, Acelajado, 2004; Adams, 1997; Connors & Snook, 2001; Graham & Thomas, 2000; Hong *et al.*, 2000; Horton *et al.*, 2004; Quesada & Maxwell, 1994; Ruthven, 1990; Smith & Shotberger, 1997). Those studies reported that use of graphic calculators improved students' mathematics performance.

The findings from this study suggest that integrating the use of graphic calculator can reduce cognitive load and lead to better performance in learning, thus increase instructional efficiency when Form Four students learn Straight Lines topic. In addition, the findings form the second and third phases provide empirical evidence to support the contention by Jones (1996), Kaput (1992) and Wheatley (1980) that the use of calculators can reduce cognitive load and hence facilitate learning.

The findings provide a possible explanation from the cognitive load theory perspectives why GC strategy is more efficient as compared to CI strategy in learning of Straight Lines topic. The GC strategy was found to have beneficial effects such that this strategy can increase germane cognitive load whereby the total amount of cognitive load stays within the limits due to low intrinsic cognitive load or due to low extraneous cognitive load. The use of the graphic calculator freed students' mental resources from the tedious computation, algebraic manipulation and graphing skills and hence enabled them to redirect their attention from irrelevant cognitive processes to relevant germane processes of schema construction. This was evident from the significantly lower levels of mental effort reported which theoretically would indicate a lower cognitive load and the significantly higher performance achieved by the students from the GC strategy group in Phases II and III.

It is pertinent to note that the argument only holds under certain circumstances namely the sample of students participated and the particular content area learnt in this study. Changing the composition of sample to include higher achievers can lead to a decrease of intrinsic load for this Straight Lines topic. Thus, the findings are only true for that particular sample of students and also apply to the content area of Straight Lines topic for Form Four Malaysian Mathematics syllabu.

It is also pertinent to note that the results of Phase I showed the difference were not significant in several instances important performance variables particularly the transfer problems performance. The findings indicate that the interventions of very brief duration (about three weeks) was not enough to show that the GC strategy is an effective instructional strategy for obtaining schema acquisition. Dunham (2000) noted that a few studies that produced negative results due to treatment of very brief duration such that the learning of graphic calculator may have interfered with learning of content (for example, Giamati, 1991; Upshaw, 1994). However, for Phases II and III, the treatment was conducted for about six weeks and the findings were in favor for GC strategy. More importantly, the GC strategy group performed better on transfer problems performance as compared to the control group that executing the CI strategy. Such findings suggest that the GC strategy group have acquired effective schemas that enabled transfer to be enhanced (Gick & Holyoak, 1983).

The findings of Phases II and III also suggest that the GC strategy group possibly may not have split attention effect with the use of worksheet (for graphic calculator instructions) and the graphic calculator screen. The results showed that if the split attention effect exists, its negative consequences are far outweighed by the reduction in cognitive load. In both phases, students in GC strategy group were found to be sufficiently proficient enough in graphic calculator use because besides having the pre-experiment training of introducing the graphic calculator and learning how to use the graphic calculator, they had longer duration of intervention. Thus, this explanation confirms the results for phase I such that the difference were not significant in the transfer problems performance could be due to any advantage of using graphic calculator was negated by the split attention effect.

Hence, it is pertinent to note that if students who had hardly knew how to use the graphic calculator had been selected, the results might have been different. The negative consequences of the split attention effect might have outweighed the positive effects of cognitive load reduction. On the other hand, the results on performance might have been further magnified if students very proficient with the use of graphic calculator had been selected in this phase.

Another important finding in this study namely Phase III was that both factors mathematics ability and instructional strategy separately influence test performance, mental effort invested during learning and instructional efficiency. However, there was a significant interaction between levels of mathematics ability and types of instructional strategy for amount of mental effort invested during test phase. It was found that as mathematics ability increased, the effectiveness of GC strategy increased. The average mathematics ability group was greatly beneficial from the GC strategy as it led to doubled decrease mean amount of mental effort than that of low mathematics ability group. However, it is pertinent to note that even though there was no significant interaction between ability group of GC strategy had performed better on test performance.

# 7. Conclusion

The findings from this study reaffirm Sweller's (1994, 1999) contention that the limited capacity of working memory is very important consideration when planning instructions. More efficient and effective instructional designs can be developed if the limited capacity of working memory is taken into consideration. In this study, it was found that graphic calculator strategy is instructionally more efficient and thus is superior to conventional instruction strategy. This study shows promising implications for the potential of the tool in teaching mathematics at Malaysian secondary school level.

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# Integrating Calculators into the Singapore's Primary Mathematics Curriculum

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**Abstract:** The use of scientific calculators will be first allowed in Singapore Primary School Leaving Examination (PSLE) for all primary level mathematics subjects from the year 2009 onwards. Following the revised mathematics syllabus and curriculum in 2007, not only is the use of calculators is included in the national examination, all Primary 5 and 6 mathematics teachers will be expected to integrate the use calculator into their mathematics lessons from 2008 onwards. As a result, primary school mathematics teachers are required to be proficient in using the calculator and adept at facilitating pupils' usage of the calculator so as to meet the new assessment requirements. Evidence from literature review and research has showed that calculator is an effective tool for enhancement of mathematical concepts, development of mental arithmetic skills, pattern recognition, mathematical investigation, solving real-life problems and improving problem-solving ability. Yet, many teachers and parents continue to believe that they can bring more impairment than good in the learning of mathematics, therefore their use for instruction should never be encouraged in the primary schools. The purpose of this paper is to review what research says about outcome of calculator use in the learning of primary mathematics. This paper also describes six appropriate calculator activities that can be integrated in the teaching and learning of mathematics at the primary level.

# 1. Introduction

With the launch of the electronic calculator for more than 40 years, there has been increased coverage on the use of handheld technology in enhancing teaching and learning of mathematics (Pomerantz, 1997). In Singapore the use of scientific calculators in national examinations was first allowed in early 1980s for all mathematical subjects offered at the secondary level. The use of calculators will be introduced in the Singapore primary mathematics curriculum in Primary 5 (Grade 5) from year 2008 onwards. With the introduction of calculators, Primary 6 pupils (12 years old) will be allowed to use calculator in the Primary School Leaving Examination (PSLE) for the first time in 2009 to solve mathematical problem in one of the examination papers (Ministry of Education, 2007). The Primary School Leaving Examination (PSLE) is a national examination conducted in Singapore annually. The examination covers topics from whole numbers, fractions, decimals, measurement, data analysis, geometry, speed, ratio, percentage and algebra (for syllabus the Singapore Ministry of Education web details. see site. http://www.moe.gov.sg/cpdd/syllabuses.htm). Clearly, there is an advantage to primary school pupils in having a scientific calculator when attempting problems involving whole numbers, fractions and decimals, as it automates the long and tedious computations when solving real-life word problems which do not have nice numbers. The impact of the decision to integrate the use of calculators in the primary revised mathematics curriculum has been significant: for schools, it means restructuring curricular programmes and refining modes of assessment so that they could incorporate the use of calculators; for teachers, it means acquiring new skills in using the calculators as well as improving classroom pedagogy to include instructions on calculator use and

to harness the power of calculators in teaching; and for pupils, it means acquiring the skills in using a scientific calculator to solve mathematical problems. In fact, calculators, like any other technology, cannot substitute basic understanding and intuitions in mathematics. This idea was also reported in one of the Singapore's Ministry of Education press release statement on the introduction of calculators in Primary 5 - 6 Mathematics:

The introduction of calculators at Primary 5 and Primary 6 aims to enhance the teaching and learning of mathematics at the primary level in two ways. First, calculators facilitate the use of more exploratory approaches in learning mathematical concepts, some of which may require repeated computations, or computations with large numbers or decimals. With a calculator, pupils can perform these tasks and better focus on discovering patterns and making generalisations without worrying about computational accuracy. Second, the use of calculators also enables teachers to use resources from everyday life, such as supermarket advertisements, to set real-life problems with real-life numbers that may be difficult for pupils to work with without a calculator. Pupils would hence be better able to see the connection between mathematics and the world around them. (Ministry of Education, 2007)

Mathematical skills continue to be an important goal in the primary school; much can be done with a calculator to support these skills and to improve the quality of pupil's responses. In my work with both pre-service and practicing teachers in Singapore, I frequently hear the same concerns, particularly from upper primary mathematics school teachers, about incorporating calculators into the mathematics curriculum. Moreover upper primary mathematics teachers may face a number of difficulties when they introduce calculators in their classrooms. As this is the first time Singapore primary mathematics teachers will be using calculators in their teaching, the purpose of this paper is to review what research says about outcome of calculator use in the learning of primary mathematics. This paper also describes six appropriate calculator activities that can be integrated in the teaching and learning of mathematics at the primary level. The activities will assist the mathematics teacher to focus less on the calculator and more directly on the mathematics and concepts so that pupils will see that mathematics has value.

# 2. A Review of Literature on the Use of Calculators

Teachers and parents are concerned that the use of calculator at the primary school level might lead to a dependence on the calculator and a reduction of mental arithmetic and basic computation skills. Teachers and parents are also fretful that pupils will become so dependent upon the use of calculators to the point of not being able to do simple calculations in their daily lives without the aid of a calculator. Their concern was that the only mathematical skill that pupils will acquire upon completion of their mathematics education is button-pushing. Primary school teachers fear that the use of calculators may thwart pupils from learning the basic mathematics that they need later in life. Many teachers are more than concerned at the mere thought of implementing the use of calculator. According to Groves and Stacey (1998), they could not get any evidence that the third and fourth grade pupils became dependent on calculators at the expense of their mental computation ability. In fact, they indicated that compared to the non-users, the calculator users in their study performed better overall and were able to make appropriate choices of calculating devices. Furthermore, they indicated that it was feasible to use calculators to assist young pupils to develop number sense and mental computation strategies even before they were taught the formal algorithm.

Smith (1997) conducted a meta-analysis that extended the results of Hembree and Dessart (1986) meta-analysis. Smith analyzed twenty-four research studies conducted from 1984 through 1995, asking questions about attitude and achievement as a result of student use of calculators. As in the Hembree and Dessart study, test results of pupils using calculators were compared to those of students not using calculators. Smith's study showed that the calculator had a positive effect on increasing conceptual knowledge. This effect was evident through all grades and statistically significant for pupils in third grade, seventh through tenth grades, and twelfth grade. Smith also found that calculator usage had a positive effect on students in both problem solving and computation. Smith concluded that the calculator improved mathematical computation and did not hinder the development of pencil-and-paper skills.

Another issue raised was related to pupils' mathematical problem-solving ability. Hembree and Dessart's meta-analysis (1986) showed that using a calculator in problem solving created a computational advantage and more often resulted in selection of a proper approach to a solution. Moreover, calculator use produced a greater positive effect size for high- and low-ability students than average-ability students. Studies have shown that appropriate use of calculators enhances young children's ability to learn basic facts (Suydam, 1987) and that those who use calculators frequently exhibit more advanced concept development and problem solving skills than those who do not use calculators (Hembree & Dessart, 1992). When calculators are incorporated into the learning process, achievement in problem solving increases, and more solution methods and strategies are utilized. Moreover, the calculator makes exploration of hypotheses feasible, and is useful in developing counting, computation, estimation and other mathematical skills (Suydam, 1985). In the same vein, Dick (1992) stated that calculators could lead to improved problem solving as they free more time for classroom lessons, provide more tools for problem solving, and change students' perception of problem solving as they are freed from the burden of computation to concentrate on formulating and analyzing the solution. It was supported by Campbell & Stewart (1993) who has shown that appropriate use of calculators resulted in greater persistence in problem solving. This could be explained by the fact that since calculator use allow more time for them to explore, pupils could solve enough problems to discover and observe patterns which are not seen when computations are done by tedious paper and pencil methods. Furthermore, Waits and Demana (2000) also recommended that "teaching problem solving should use paper-and pencil and then support the results using the technology, or vise versa; and use manipulative and paper-andpencil techniques during the initial concept development and use calculators in extension and generalizing phases" (p.59).

Teachers think that since calculators do all the work, pupils will be less motivated and challenged. This is not the case as Hembree and Dessart (1986) found that students who use calculators exhibit greater self confidence and that calculator use generates more enthusiasm about mathematics. Many teachers believe that mathematics is and should be hard work, which normally are associated with manual computations and manipulations. Calculators eliminate much of that work, making them appear nothing but a "crutch" for students who are too "lazy" to perform the assigned mathematical tasks. The truth is calculators are simply tools to help pupils solve problems. They do not do the work for pupils. It is still up to the pupil to read the problem, understand what is asked, determine the solution, and decide whether the answer makes sense. The use of calculators simply allows teachers and pupils to spend more time on the non-computational parts of the problem-solving process (Campbell & Stewart, 1993).

Ellington (2003) use the method of meta-analysis to combine the findings of 54 research studies carried out between January 1983 and March 2002 and determine the effects of calculators on pupils' achievement. Each of the studies compared the outcomes of experiments in which the

treatment group used calculators while a control group received equivalent method of mathematical instruction with no access to calculator. Results revealed that pupils' operational skills and problem-solving skills improved when calculators were an integral part of testing and instruction. At the other end of the spectrum, Golden (2000) found that teachers' practices of frequently using calculators for mathematics instruction reduced students ability to do well on computational problems at the year end tests where calculators were not allowed. In a local study, Toh (2006) conducted a use of calculator over two weeks. It was found that there was no difference in basic skills and problem solving skills between the calculator and non-calculator groups. In fact, the National Research Council's publication, Adding it Up (2001), indicated that calculator use was more controversial in mathematics lessons in primary levels than the use of manipulative materials. They stated that "...persistent concerns have been expressed [by mathematics teachers] that an extensive use of calculators in mathematics instruction interferes with students' mastery of basic skills and the understanding they need for more advanced mathematics (p. 254)". From the TIMSS results it is clear that mathematical competence at the grades K–6 level does not require calculators. Two of the highest-achieving countries at the fourth- and eighth-grade levels, Singapore and Japan, use calculators sparingly in primary schools.

Although calculators have their value in the learning mathematics for the upper primary levels, we should not advocate using them merely because they are popular. Instead, teachers need to establish thoughtful rationales for deciding how and when to use calculators in their classrooms. Therefore schools should strongly encourage the use of calculators in all aspects of mathematical instruction including the development of mathematical concepts and the acquisition of computational skills. Moreover, from the review above it appears that calculator is an effective tool for enhancement of mathematical concepts, development of mental arithmetic skills, pattern recognition, mathematical investigation, solving real-life problems and improving problem-solving ability.

# 3. Sample Calculator Activities for Upper Primary School Pupils

In the Singapore Revised Mathematics Syllabus (Ministry of Education, 2007), the conceptualization of the mathematics curriculum is based on a framework where active learning via mathematical problem solving is the main focus of teaching and learning. One of the main emphases of the primary level mathematics curriculum has been the acquisition and application of mathematical concepts and skills. While the revised curriculum continues to emphasise this, there is now an even greater focus on the development of pupils' abilities to conjecture, discover, reason and communicate mathematics with the aid of calculator. Guidance for teachers must demonstrate how mental facility can be developed alongside calculator use. The appropriate use of calculators in the classroom is the key factor. Since calculator is an effective tool for enhancement of mathematical concepts, development of mental arithmetic skills, pattern recognition, mathematical investigation, solving real-life problems and improving problem-solving ability, it will be useful to provide six such appropriate calculator activities for primary mathematics teachers to integrate in their mathematics lessons. The following section describes six such appropriate calculator activities that can be integrated in the teaching and learning of mathematics at the primary level.

#### **Enhancement of Mathematical Concepts**

In Activity 1 below, the concept of place value is reinforced. Pupils are asked to use a calculator to represent a given number in different ways. For pupils to get the sum in part (a), they have to identify the place value of the digit 6 in 162 541 and regroup 162 541, for example, as 152

 $541 + 10\ 000\ \text{or}\ 172\ 541 - 10\ 000\ \text{and}\ \text{so}\ \text{on.}\ \text{Compare this activity to the standard textbook}$  problem of asking the pupils to find the value of the digit 6 in 162 541. Activity 1 is more openend item and it provides opportunities for the pupils to conjecture, discover and reason. Pupils will not be turned off by the tedious calculations. Similarly for part (c), teachers could observe whether pupils show evidence of understanding the distributive law, for example, by expressing 768 x 9 as 9 x (758 +10) or 7 x (778 -10).

Activity 1: Broken Key on Calculator (Primary 5)

You are given a calculator to do some addition and subtraction sums. However, the key '6' on the calculator is broken. If you have to do the following sum using this calculator, how can it be done? (a)  $162\ 541 + 44\ 458 =$ \_\_\_\_\_\_(b)  $239\ 765 - 18976 =$ \_\_\_\_\_\_\_(c)  $768\ x\ 9 \qquad =$ \_\_\_\_\_\_\_(d)  $54\ 657\ \div 8 \qquad =$ \_\_\_\_\_\_\_

#### **Development of Mental Arithmetic Skills**

Sometimes teachers unduly pressurize the child to remember the rules of placing decimal point in multiplication and division computation sums. However, pupils may forget or confuse easily as they have no understanding of why these rules work and it is not meaningful to them to commit to memory. Pupils should be strongly encouraged to use their understanding of the quantities and of the operations to elucidate through the placement of the decimal point. Therefore, the aim of Activity 2 is to enhance the pupils' estimation skills where pupils need to place decimal points in the products and quotients. Pupils should be encouraged to interpret the first problem as "This is about 1 times of 40, so the answer is about 40." The last division problem is demanding to estimate. It requires 15.679 to be thought of as "about 156 tenths" and then this is to be shared among 70 children, so each child will receive at least two tenth.

#### Activity 2: Decimal points missing (Primary 5)

John forgot to place the decimal point in the answer of each calculation. Describe how you can use estimation to place the decimal point correctly.

(a) 0.95 x 43 = 4 085
(b) 35.4 x 17 = 6 018
(c) 7651 x 0.0083 = 635 033
(d) 37.986 ÷ 0.004 = 44 965
(e) 15.6192 ÷ 69.33659 = 225266342

#### **Pattern Recognition**

Number sense involves the flexibility in thinking about numbers that emerges with the ability to relate, compose and decompose numbers (NCTM, 2000). Pupils can use calculator as a tool to explore numbers in the ways that contribute to the development of this flexibility. In the Principle and Standards for School Mathematics (NCTM, 2000), teachers may assume that pupils should know that "mathematics involves examining patterns and noting regularities" (p.262), that "statements need to be supported or rejected by evidence", and that "assertions should always have

reasons" (p.56). In order for the pupils to examine patterns and note regularities in a set of numbers, the decimal-expansion activity involves examining repeating patterns in decimal expansions of fractions with prime denominators (see Activity 3). Activity 3 involves examining patterns in fraction and decimal sequences and it creates opportunity for pupils to explore and appreciate recurring decimals. Performing the computations for activity 3 would be tedious and complex and would render the activity inaccessible to the vast majority of primary 5 pupils; therefore, the use of calculator is necessary. In Activity 3, pupils will observe that the digits in the tenth place and hundredth place of each decimal are repeated and the repeated digits are the products of the numerator and the number 9.

Activity 3: Dec	imal Expansion (Prima	<u>ry 5)</u>		
Use your calcul	ator to express the foll	owing fractions as dec	imals:	
(a) $\frac{1}{11} =$	(b) $\frac{2}{11} =$	(c) $\frac{3}{11} =$	(d) $\frac{4}{11} =$	(e) $\frac{5}{11} =$
(f) $\frac{6}{11} =$ There is a repeat	(g) $\frac{7}{11}$ = ting pattern. What is it	(h) $\frac{8}{11} =$	(i) $\frac{9}{11} =$	(j) $\frac{10}{11} =$

#### **Mathematical Investigation**

Activity 4: Th	ree-Digit Numbers (Primary 5)		
Peter has a tric	ck he does with numbers.		
Here it is. Che	oose a three-digit number.		
For example,	987		
Step 1:	Write down all the numbers that may be formed by changing the positions of the digits, 987, 879, 789, 978, 798, 897.		
Step 2:	Add them: $987 + 879 + 789 + 978 + 798 + 897 = 5328$		
Step 3	Find the sum of the digits in the original number: $9 + 8 + 7 = 24$		
Step 4:	Divide the total by the sum of the original digits: $\frac{5328}{24} = 222$		
Repeat steps 1	, 2, 3 and 4 using other three-digit numbers.		
Peter says that every time he does this trick the final answer is always <u>222</u> . Do you agree with him?			
Investigate Pe	ter's trick. How do you think it works? Write down any observations and results		

With the use of calculators, teachers in investigative classrooms no longer spend a great deal of time transmitting information via talking or reading and waiting for pupils to complete their long calculations by paper and pencil. Instead, the investigative problems if given on a regular basis would instill in pupils that understanding and explanation are critical aspects of mathematics. In Activity 4, when pupils investigate Peter's trick, they will usually begin with some specific examples or special cases (specialisation) before they make an attempt to generalise. Along the

way, the pupils may arrive at certain conjectures which may be false. For example, Peter's trick will not work for three-digit numbers which are repeated. After working through a number of different three-digit numbers, pupils may begin to notice certain features in their solutions. They may articulate these common features and make conjectures to try and explain for them. Therefore, it appears that mathematical investigation would engage the pupils in mathematical thinking: specialisation, generalizing, conjecturing and verifying (Manson et al, 1982). Pupils may not succeed in conjecturing, verifying or generalisation but as long as they examine specific examples or specific cases (specialisation) using calculators with the intention of formulating and justifying conjectures so as to generalise, then pupils are doing mathematical investigation. This task also familiarises with the pupils the usage of operational keys in the calculators. In addition, pupils of wider range of abilities can work on the same task (see Activity 4) using calculators.

#### Solving Real-Life Problems

The use of calculators allows realistic data to be used as problem contexts, problems whose solutions are within the conceptual grasp of pupils but whose computational demands are not. The use of realistic data is motivational and helps pupils to see connections between school mathematics and the mathematics used in the world. Activity 5 is a mathematical task that includes real-life data taken from authentic situations. It also provides an opportunity for pupils to use realistic numbers and experience using large numbers and decimals in authentic situations. The rate of exchange, transaction and sale mentioned in the trip from US to Singapore are real-life situations. By and large, primary school teachers would not give such problems to their pupils as they are worried that the computations will involve products and quotients with too many decimal places. Using calculator in this Activity 5 will reduce the impacts of poor computation skills or anxiety about computations as well as avoid messy calculations. Pupils will experience the benefit of calculator as an everyday tool.

Activity 5: Rate and Percentage (Primary 6)

Peter who is living in United State, would like to visit Singapore for one week during the "Great Singapore Sale".

- (a) Before coming to Singapore, he bought some Singapore dollars. The rate of exchange between US dollars (US\$) and Singapore dollars (S\$) was US\$1 = S\$1.52. He also had to pay the bank 1.5% commission for the money. He bought S\$3500 from his bank. Calculate the total amount, in US dollars, he paid to the bank.
- (b) Peter planned to stay at a hotel in Orchard Road. On the internet, he found a hotel that offered a week's stay for \$1300+++.

This meant that to a basic cost of \$1300 he would have to add a service charge of 10% and CESS of 1% of the basic cost.

In addition, he would have to pay a Good and Services Tax (GST) of 7% on the total, that is the sum of the basic cost, service charge and CESS.

- (i) Calculate the amount Peter would have to pay for hotel accommodation.
- (ii) Express the increase in cost due to service charge, CESS and GST as a percentage of the basic cost of the hotel stay.
- (c) While in Singapore, he bought a digital camera for \$725.

This price included the 7% GST. Calculate, correct to the nearest ten cents, the GST paid for the camera.

#### **Improving Problem-Solving Ability**

#### Activity 6: Square Numbers (Primary 5 and 6)

Mary uses a piggy coin box to save money. She deposits 1 cent on the first day, 3 cents on the third day, 5 cents on the fifth day, 7 cents on the seventh day, 9 cents on the ninth day and so on.

(a) How much money will Mary have saved at the end of the 89th day?

- (b) How much money will Mary have saved at the end of the 365th day?
- (c) How long does Mary need to save at least \$400?

In Singapore the primary purpose of teaching mathematics is to enable pupils to solve problems and it is therefore crucial that pupils learn to use calculator at each stage of the problem solving process in order to fully harness its capabilities. Having access to a calculator permits pupils to study various cases of a problem situation in a way that is both swift and precise. It also provides a means for pupils to identify patterns and relationships between variables, information from which they may generate possible solution methods and strategies to solve the problem. In Activity 6, the calculator also enables pupils to check and correct any computation errors with considerable ease. Activity 6 allows the pupils to use calculator to compute the total amount saved. Precious time that has been formerly spent on tedious paper-and-pencil calculations can now be passed on to the development of problem-solving strategies and thinking skills. The problem-solving strategies use in this problem could be "tabulation", "simplify the problem" and "look for patterns".

These six activities exemplify how calculators assist upper primary school pupils to explore various types of mathematical tasks. This is only possible when the mathematical tasks that teachers use in their classrooms go beyond computations and rote algorithms. The six activities are just first step towards making the integration of calculators in the classroom a meaningful one where emphasis is on the process (reasoning and thinking) rather than the product (final answer).

## 4. Conclusions

Calculators can reduce the time spent in performing tedious calculations and illustrating concepts. However, the use of calculators at the primary level should be restricted and controlled. There is a need to strike a balance between basic numeracy skills, conceptual understanding and problem solving. It is heartening to note that only one of the PSLE examination papers in Singapore is allowed to use calculator in year 2009. This was also reported in one of the Singapore's newspaper, Straits Times, about the use of calculators for the year 2009 PSLE mathematics paper:

The value of basic numeracy skills like mental calculation and estimation will continue to be amphasized and numils will be tested on these in Pener 1 of the new even. (Liew 2007, p.112)

emphasized, and pupils will be tested on these in Paper 1 of the new exam. (Liaw, 2007, p.H12).

The changes that school leaders, curriculum specialists, teachers and pupils need to manage for successful integration of calculators into the primary mathematics curriculum clearly bring a number of challenges along with them. Ultimately, the decision to use calculators in the mathematics lessons is up to the teacher. It is hoped that teachers will bear in mind the appropriate use of calculators by relating it to their pedagogical goals and their pupils' abilities. Finally, calculator is indeed an effective tool for enhancement of mathematical concept by enabling the pupils to perform calculations with speed and accuracy, so that concepts rather than computations become the focus of the pupils' attention. Calculators certainly have their value in the mathematics classroom.

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# Calculators in the Mathematics Classroom: A Longitudinal Study

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**Abstract:** A key variable in the use of calculators in the learning of mathematics is the teacher. In turn there are many factors that influence whether an individual teacher uses the calculator, and if they do then how they use it. This study reports on a ten-year longitudinal survey data into the use of calculators in the upper secondary school. It presents the pattern of calculator use, some possible reasons for this pattern, and obstacles to increased use. In addition the relationship between calculators and national assessment and equity are examined. Results show that many teachers see benefits in using calculators in mathematics teaching although a sizeable minority are opposed to their use. Further, there is a continuing need for professional development that specifically addresses how to integrate calculators into mathematics teaching in a manner that focuses on the mathematics.

# 1. Background

In recent years research on calculator use in the learning of school mathematics has tended to move from an emphasis on student learning to the influence of the teacher (see e.g., [1]), recognising their key role in the use of calculators. In turn, there are many factors that impinge on a teacher's use of calculators, as with any technology. Among these, primary influences are teacher affective variables (such as beliefs and attitudes), their thinking about, and perceptions of, the nature of mathematical knowledge and how it should be learned, and their mathematical and pedagogical content knowledge. Other influences that need to be taken into account are social relations, institutional standards, tools, and tasks used (see [2] and [3]), attitude to, and beliefs about the technology, as well as teacher confidence and ability to use it to teach mathematics.

There has been less emphasis on research into the decision process that teachers engage in when deciding whether to use calculators, and if so, how and when to use them in learning. These decisions may be dependent on a number of factors. One of these, described in [4] is the concept of teachers' *pedagogical technology knowledge* (PTK). This has been presented as a useful way to think about what teachers need to know in order to teach well with technology. PTK includes not simply being a proficient user of the technology, but more importantly, understanding the principles and techniques required to teach *mathematics* through the technology. This necessitates a change of mindset on the part of teachers, a shift of focus to a broader perspective of the implications of the technology for the learning of the mathematics. Developing PTK requires attending to the teacher's perspective on mathematics and technology, the relationship between the two, their use of technology and their personal instrumentation of it (see [5]). Developing PTK involves the teacher

in the transformation of the technological tool into an instrument and differentiation of qualitatively diverse ways of employing technological tools in teaching mathematics (such as direct procedural calculation, computational check), or building conceptual knowledge of mathematics (see [6]). Teachers also need to consider the kind of tasks that they set students when using technology in order to assist students to develop simultaneously the calculator and by-hand techniques and theory (see [3]) needed to make progress in mathematics. Hong and Thomas's (see [7]) model of the influences on PTK suggests that teacher confidence in using technology is a key driver of the growth of PTK.

Orchestrating and directing all of the influences mentioned above in such a way that good classroom learning eventuates can be very challenging. Following investigation of the didactic contract of a teacher using a graphic calculator (GC) during function and limit concept lessons, delos Santos and Thomas (see [8]) concluded that the teacher has to be open to new approaches, be willing to work around constraints, be open to personal learning, and be able to reflect "on the tension created between valuing a formal, primarily algebraic approach to mathematics and an investigative style of teaching" with the GC (see [8], p. 357). Orchestrating the integration of technology in teaching involves many aspects. One described in [9] (p. 329) is the need for resequencing as teachers have "to integrate graphic calculators... and to organise, when it is possible, backward and forward motions between calculators, theoretical results and calculus by hand." Another part of this challenge involves the influence of the teacher's practice on their students. For example, Kendal and Stacey (see [10]) have shown that, in an introductory calculus course using calculators, the teachers' *privileging* of certain approaches differentially affected their students' learning.

It was against this background of integration of calculators in teaching that the current research project took place. It sought to ascertain current practice in the use of calculators in the upper secondary mathematics classroom, along with teacher and other factors that might be influencing it, especially the role of external assessment.

# 2. Method

Longitudinal studies, where at least two sets of data are collected from the same population over an extended time span, are relatively rare in mathematics education research. This ten-year longitudinal study, with a population of all secondary mathematics teachers in New Zealand, began in 1995, when a questionnaire on calculator and computer use was mailed to every secondary school in New Zealand. Replies were received from 90 of the 336 schools (26.8%), a reasonable response rate for a postal survey, and from 339 teachers in the schools. Some of the results of this survey were published at the time (see [11]) and were used to form a baseline comparison. This original survey was followed by a second in 2005 in order to gain longitudinal data on how the situation might have changed in schools over this period. Since 1995 teaching has become an even more stressful and demanding profession in many ways, particularly in terms of demands on time, and so teachers are more reluctant than ever to spend their valuable time filling in forms or research questionnaires. However, we had learned lessons from 1995 and stamped, addressed envelopes were enclosed for all the schools. Also the posted questionnaire was followed up several weeks later with a faxed copy. Using this approach we achieved a very good 57.4% response, from 193 of the 336 secondary schools in the country. We also received completed questionnaires from a total of 465 teachers in these 336 schools, as well as the school information. While the questionnaires sent out in the two years were not identical, due to changes in emphasis, they did have a number of questions in common. On both occasions they used both closed and open questions to provide valuable data on calculator issues such as: the number of calculators in each school; the level of access to the calculators; the pattern of use in mathematics teaching; and teachers' perceived obstacles to calculator use (Figure 2.1 has a selection of questions from the second survey). This data enables us to come to some conclusions about the changing nature of calculator use in the learning of mathematics in New Zealand secondary schools. In addition to the survey a group of 32 volunteer teachers, who volunteered via their survey response, were interviewed about their views on technology, and some of their lessons using technology were observed.

Q8	Approximately how many of each of these calculator	Casio		
	types does your mathematics department own?	Texas Instruments		
		Sharp		
		Other		
Q9	Approximately what % of your school's senior	Year 12	%	
	mathematics students own their own calculators?	Year 13	%	
Q15	Would you like to use graphic calculators more often in	Yes	1	
	your mathematics lessons?	No	2	
016	If you arguered yes to question 15, what do you see as	Lack of confidence		
Q10	obstacles to your use of them? Please rank in order any of these which apply (ie 1 for biggest obstacle, 2 for the	Lack of PD		
		Calculator availability		
	next, etc.).	School policy		
	Other	Government policy		
		Other		
Q18	With which years do you regularly use graphic calculators as an	integral part of mathematics less	ons.	
Q19	Year 12 Year 13 Calculus Year 13 Statistics & Do your students use calculators in their mathematics lessons on	Modelling None D ly when directed by you?		
	Yes No Sometimes Depends on			
Q20	What kinds of calculators do your students use in their mathematical	tics lessons ?		
Q22	Scientific Graphic Computer Algebra System Please give the main advantage or benefit you have found, or fee in mathematics lessons.	(CAS) None of these el to be true, of using technology		

Figure 2.1 Sample questions from the 2005 survey (Some formatting changed)

# 3. Results and Discussion

The longitudinal study used the baseline data on calculator use in schools gathered by Thomas in 1995 (see [11]) in order to make comparisons with the current position and look for trends. In 1995 there was an average of 22.6 calculators (52% Casio) owned by mathematics departments and 96% of Year 12 (age 17 years) and 97% of Year 13 (age 18 years) mathematics students owned their own calculators. In 2005 the average number of calculators owned by a mathematics department was 45.7 (of which 68.6% were Casio, 14.4% Texas Instruments, and 15.4% Sharp). In Year 12, 86.4% and Year 13, 87.9% owned their own calculator, which interestingly represented a drop on the 1995 figures. It was noteworthy that in 2005 the calculator types owned were: scientific 76.1%, graphic calculator (GC) 27.1% and CAS 0.2%. The low CAS use is no doubt a reflection of the fact that they were not allowed in external assessment at the time of the survey.

From the survey for all secondary school teachers and heads of departments (HOD's), 75.5% of respondents who teach year 12 classes said that they sometimes used GC's in their lessons. However when they were asked as to whether they regularly used them, this number dropped to just under half (49.4%). Among teachers of year 13 calculus, 91.8% sometimes use GC's, while 75.4% regularly use them. Among teachers of year 13 statistics, 79.4% sometimes use GC, while 66.7% regularly use them. During 1995 mathematics lessons 75.8% of Year 12 and 62.5% of Year 13 regularly used calculators as an integral part of the lessons with 69.8% using them at least once a week, 14.2% at least once a month, and 9.2% at least once a term. These figures represent a drop in regular use of calculators in years 12 and 13, especially in year 13 calculus lessons. This is surprising since this course contains a lot of graphical work on functions that would appear to lend itself to GC work. In 1995 6.2% of Year 12 and 5.0% of Year 13 used the calculator only when directed by the teacher compared with a total of 10.2% in 2005. Thus the majority of teachers surveyed said that students were not using calculators in their lessons only when directed by them, indicating most students use calculators when they decided to, without the direction of a teacher. The question of whether it is better for students to own their own technology or for the school to provide it was specifically addressed in the questionnaire. 66.0% of the teachers agreed that student ownership was the best situation, with only 14.8% disagreeing. The two clear benefits from students having their own technology are improved access and lowering of the pressure on already over-committed department and school budgets. The questionnaire revealed that only 10.3% of mathematics departments have a technology budget, and the average size of these is NZ\$2762.50 per year.

There has been quite a lot said, often in the media by parents and others, about the possible negative effects of calculator use in the mathematics classroom and we wanted to know what the opinion of the teachers was on this subject. The responses to the question of whether calculators 'may be' (1995) or 'are often' (2005) detrimental to students' mathematical understanding are given in Table 3.1. The summary shows that in 1995 24.8% of teachers agreed that calculators may be detrimental, and in 2005 26.6% thought that they often are. In the same period the number disagreeing dropped from 60.2% to 47.1% ( $\chi^2$ =13.7, p<0.001). It seems that what has happened in the intervening years has done nothing to alleviate the perception of a significant minority of teachers that calculators may be more damaging than useful to student understanding. In fact, on the basis of this question, there is some evidence that the situation has changed so that fewer teachers are convinced that calculators are never detrimental.

	1995 Response % ( <i>N</i> =339)	2005 % Response % (N=464)
Strongly agree	4.7	5.0
Agree	20.1	21.6
Neutral	14.2	18.8
Disagree	35.1	33.1
Strongly disagree	25.1	14.0
No response	0.9	7.5

Table 3.1 Teachers' Views on Whether Calculators May be Detrimental to Understanding

However, when the teachers were asked whether they agreed with the statement that 'calculators' (1995) or 'technology' (2005) are/is of little benefit in mathematics teaching, we see from Table 3.2 that there was a large majority disagreeing; 87.3% in 1995 and 75.6% in 2005,

although this too has fallen. Since the calculator is the technology most often used in classrooms, one possible explanation for why many teachers see the calculator as of value but why some also think that it can be detrimental is that it depends on the way in which it is used in teaching. This would agree with the argument that whether the calculator is beneficial or harmful to learning depends on how it is used. Hong, Thomas and Kiernan (see [12]) found that weaker students can become dependant on the calculator to the point where there mathematics is weaker when they don't have access to one, and similarly [13] reports that some students who were not motivated by technology nevertheless became dependent on it.

	1995 Response % (N=339)	2005 Response % (N=464)
Strongly agree	3.2	2.4
Agree	3.8	5.6
Neutral	4.7	9.7
Disagree	34.2	37.7
Strongly disagree	53.1	37.9
No response	1.0	6.7

Table 3.2 Teachers' Views on Whether Calculators are of Little Benefit in Teaching

The teachers were also asked whether they would like to use a calculator (1995) or graphic calculator (2005) more often, and 19% (1995) and 56.7% (2005) respectively said yes, a large increase over the ten years. Those who answered yes were asked to rank a number of obstacles, or add their own. Table 3.3 shows the results of these responses. It is clear that the major obstacle is still a lack of available calculators, but there has also been an increase in the need for professional development and greater teacher confidence. Given that 86% or more of students own their own calculator this is surprising. It may be that the lack of GC's is what the teachers are talking about since only 27.1% own these.

Obstacles	% of 1995 Teachers ( <i>N</i> =64)		% of 2005 Teachers ( <i>N</i> =257)	
	First mentioned	Mentioned	First mentioned	Mentioned
Calculator availability	76.6	81.3	52.5	71.6
Lack of PD	4.6	12.5	19.1	48.2
Lack of confidence	4.7	10.9	13.6	42.4
Government policy	1.6	9.4	1.9	6.2
School policy	3.1	10.9	0	5.1

**Table 3.3** A Summary of Obstacles to Using the Calculator More in 1995 and 2005

In 2005 these obstacles were also examined along gender lines to see if there were any differences (see Table 3.4). The results show that while females appeared a little less confident than males, this was not significant ( $\chi^2$ =2.27, n.s.), and there were no other gender differences.

Table 3.4 A Summary by Gender of 2005 Obstacles to Using the Calculator Mo
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	First Mentioned (%)		Mentioned (%)	
Obstacles	Male	Female	Male	Female
Calculator availability	30.2	31.0	42.2	40.5
Lack of PD	12.1	10.0	29.7	26.2

Lack of confidence	6.9	8.6	21.6	28.1
Government policy	1.7	0.5	9.1	3.8
School policy	0.0	0.0	7.3	2.9

The subject of sufficient resources was not raised here but was asked in a separate question, namely whether they agreed with the statement that a major obstacle to teachers using 'calculators or computers' (1995) or 'technology' (2005) is a 'lack of good ideas which work in the classroom' (1995) or 'classroom resources' (2005). While these questions are not precisely parallel they do show that the 41.0% agreeing in 1995 had increased significantly ( $\chi^2$ =76.5, p<0.0001) to 71.1% in 2005, with a corresponding drop in those who disagreed from 32.2% to just 11.0%. Clearly the ten years have seen an even greater need for classroom resources with good ideas for teachers to use when teaching with technology. This is a surprising result given the large increase in calculator use around the world and the consequent increase in available resources, and may be, in spite of the internet, the result of poor communication of ideas. Whatever the reason it is something that educators need to be aware of and try to address.

Two of the open questions on the teacher questionnaire (Figure 2.1, Q's 22, 23) asked what the teachers perceived as the primary advantages and disadvantages of technology use, in order to try and get an idea of the motivation behind its use. There was a wide variety of responses to Q22 about the advantages of technology use and a summary of the number of occurrences of particular points mentioned is given in Table 3.5. Among these, improved efficiency of calculation (quicker calculations) was regularly mentioned, as was the benefit of visual explanation. Some teachers felt that students gained confidence through the use of technology, as they were able to check their solutions, spend less time on trivial manipulation, and eliminate careless errors, with calculators widely believed to provide 'efficient and accurate calculations and predictions'. Motivation was seen as another advantage with a response that technology, in the form of graphics calculators or computers 'can hook students' interest'. However, according to [4] these are the kinds of advantages seen by teachers who are new to technology use, and who have not made great progress in its implementation, or in personal instrumentation of the tools (see [5]). Those who have better PTK tend to perceive the mathematical benefits more. However, in this survey opinion was split about whether the use of technology aids understanding of mathematical concepts, and as Table 3.5 shows, understanding was only mentioned 37 times. One teacher had apparently moved to this point, saying that technology "allows [the] class to concentrate on [the] application of Maths techniques etc, rather than calculations, graph drawing, etc", while another responded that "traditional skills and techniques are being lost". It was also mentioned that technology use can prepare students for how the real world uses mathematics.

Advantage	Frequency mentioned	% ( <i>N</i> =257)
More efficient, quicker	149	32.0
Visualisation/Visual display	42	9.0
Student motivation/interest	39	8.4
Aids understanding	37	8.0
Improves confidence	14	3.0
Fewer errors in calculation	7	1.5

Table 3.5 Distribution of Types of Advantages Mentioned for Technology Use

In the follow-up interviews with 32 teachers we asked why they used technology, and their answers were wide-ranging. It should be remembered that these were teachers who had used the technology and were confident enough to allow observation of their lessons by researchers. It was noticeable here that these teachers make more mention of ideas such as 'clarifies the concept', 'see the concept is really helpful', 'Technology is really important for multiple representations', 'understanding the concepts better', and 'Allows students to investigate'. It appears that their increased confidence with the technology has enabled them to reach a level where they can think about the mathematics more.

In Q23 of the survey all the teachers were asked about the main disadvantages of technology use and Table 3.6 summarises these perceived disadvantages. Interestingly, although, as we saw above, some said that the use of technology aided understanding, others said that it did the opposite. A common concern was that teachers thought that students are not gaining a full understanding of topics, and were instead relying on their calculators to tell them the answer. Also mentioned was how students are more likely to accept answers without considering how reasonable they are. One teacher said that graphical calculators "encourage kids to take short cuts, especially in algebra. Real algebra skills are lacking as a result" and 31 teachers mentioned that students often become very dependent on the calculator. This impedance of understanding was closely linked to a dependence on technology by many respondents. Some said they felt that the benefits of technology are small and often exaggerated, and that the technology should only used to support the primary content being taught. Some teachers also thought that technology is sometimes not appropriate, depending on what is being taught, and that teachers should not force the subject to fit the technology. Some believed that students take advantage of lessons including technology, saying, for example, that it is "seen as an easy period by students". The depth of feeling some have on this topic can be seen in the comment of one teacher who said that "NCEA [the assessment regime-see below] encourages [us] to teach students to get answers only (working is not marked) to questions they do not understand by learning which buttons to press, on a piece of technology that nobody outside a classroom uses, and which will be out of date within 3 years".

Disadvantage	Frequency	Mentioned (%)
Equipment – availability/quality/functionality/cost	93	20.0
Impedes learning/understanding	78	16.8
Dependence on calculator	58	12.5
Lack of confidence/knowledge-teachers or students	33	7.1
Time constraints	24	5.2
Distraction	11	2.4

Several teachers complained that an excessive amount of time is wasted when technology fails, and that sometimes not much learning takes place when students are distracted with some of the other things that technology can do. Varying standards of competence also cause difficulties in the classroom, with some students being highly skilled, while others are not. In summary, in spite of some comments above, based on the survey, we can infer that the teachers generally believe that there are benefits in using calculators in mathematics teaching.

#### 3.1 Issues of assessment and equity

Since 1995 a new national assessment system has been introduced in New Zealand, called the National Certificate of Educational Achievement (NCEA). While a few schools opt for the International Cambridge examinations or the International Baccalaureate for their final year students, the vast majority of students sit NCEA internal assessments and external examinations at ages 16 (Level 1), 17 (Level 2) and 18 (Level 3). Each level of NCEA is divided into a number of standards, and clear guidance is given in the notes accompanying all of these that the use of appropriate technology is expected. Heads of departments were asked (section A of the questionnaire) what their departments do to implement technology in NCEA levels 2 and 3 mathematics teaching. The majority of responses focused on the kinds of technologies they use, with 46.9% identifying graphic calculator use in NCEA. It was clear from the survey responses that while teachers support the use of technology in many of the NCEA achievement standards, they do not believe that *all* of these should be supported with technology. One teacher wrote that:

We are getting a mixed message from the NCEA examiners. The standard says 'appropriate technology' should be used but the Merit and Excellence questions are often designed to require algebraic manipulation, so we generally teach algebraic techniques for solving equations, knowing that weak girls will depend more on their calculators than strong ones.

One reason for the ambivalence is that some teachers are not aware that technology is promoted for all standards. For example, many of the teachers (44.7%, mean agreement score 2.71 out of 5) disagreed when asked if technology use is expected in all NCEA standards, with only 26.3% agreeing or strongly agreeing. When asked whether "NCEA has too much emphasis on technology", only 11.7 % either agreed or strongly agreed, although those who disagreed or strongly disagreed did not reach a majority (44.2%), with 36.9% giving a 'neutral' response (mean agreement score 2.59 out of 5). One teacher mentioned that "NCEA encourages [us] to teach students to get answers only (working is not marked) to questions they do not understand." When asked the reverse question of whether they believe that "NCEA has too little emphasis on technology", 43.3% either disagreed or strongly disagreed while a small number (6.2%) agreed (mean agreement score 2.52 out of 5). Hence the survey seems to indicate that the teachers believe that the NCEA assessment regime has the right level of emphasis on technology. This agreed with the list of achievement standards where technology was used, with the greatest use clearly in statistics and modelling.

When teachers were asked how NCEA had affected their teaching with technology, there were mostly positive responses about the change: "...it has been a positive thing... we've been able to write our own standards and activities using the basis of what we want to do...doing (NCEA) has increased our use of technology in terms of teaching... not that boring monotonous low skill stuff."; "NCEA has been really positive for technology in mathematics because...it says, students will use appropriate technology ... if they're gonna do NCEA, they must use appropriate technology, and school's been really supportive and provided the money."; and "The beauty of NCEA is that you can now teach things properly. We use it [technology] a lot. It has increased our workload a lot, but it is far more valuable in terms of long term gain for students."

A recent innovation in New Zealand is the possible introduction of computer algebra system calculators (CAS) in schools. There has been a lot of international research in recent years on the perceived benefits of using these calculators in the mathematics classroom (see e.g., [3], [10], [14], and [15]). From our survey we knew that only 1.8% of the teachers used CAS with their classes, but we wanted to know whether teachers are in favour of their use in examinations, since this raises the

problem of how to set questions that are both equitable and still test the required knowledge. Some research on this has been conducted (see [12]) and showed that, while it is possible to set examination questions that are equitable, there are considerations in terms of the possible disadvantages to weaker students of using CAS calculators. The responses to the question whether 'All types of calculators should be allowed in examinations' showed that while 21.7% are in favour of this move, there is a sizeable majority of 60.5% who disagree. Currently GC's are allowed in the examinations. Since the New Zealand Ministry of Education is moving towards allowing CAS in examinations from 2011/12 it seems that there is work to do to provide the professional development that will convince many teachers of the value of this.

In the 32 teacher interviews, most said that they use GC's for internal assessment, and that their students also use GC's for external examinations. One of the questions in the interviews with the 32 teachers using technology asked: 'Are there any equity or cultural issues you can see with the use of technology?' It was clear from the responses that there was just one issue that the teachers could see, namely the inequity arising from the fact that some students could not afford their own calculators, and schools were often not able to purchase them either, and this was seen as crucial during examinations. Typical responses were: "There are certainly equity issues among students that come from poorer homes where they can't afford them. I think that is probably going to be very, very difficult for schools in the lower decile areas."; "A lot of our students will come from even low decile areas... when you are asking for another \$75,...for a graphic calculator, it's just nah, it's not gonna happen."; "...to be fair, I think the exam has to be designed in a way in which they can still test the manual understanding so that the students can only really rely on the graphics calculator to a certain extent"; and "I worry about the results indicating that that kid knows more in that assessment than a kid without a graphic calculator. When in fact the other kid may know more about maths and have a better understanding but they've run out of time and they've never had a chance to show what they know". To remedy this problem, two of the teachers said that they loan the school's calculators to the students during examination periods. One teacher said that, in order for the students to be able to use calculators in externals, the school buys in bulk and sells them to the students at a price cheaper than the retail price. However, another teacher said that they stopped loaning their calculators for the reason that some of the calculators were not returned to the school.

# 4. Conclusion

In summary this study has shown that the number of calculators in schools increased during the period 1995 to 2005, but only 27% are GC's. Teachers claim that more are needed, even though around 86% of students have a calculator and regular use by teachers has fallen. The evidence is that teachers are generally still in favour of the use of calculators in the learning of mathematics, and see that there are benefits to doing so. However, a significant minority of teachers (27%) think that using calculators can be detrimental to student understanding of mathematics, depending on how they are used. Their fear is that students will become dependent on them and lose their by-hand skills. In spite of this a majority (56.7%) of teachers would like to use calculators more often in their teaching, and prefer the students to have their own, although there are equity issues over cost associated with this for schools in poorer areas. The major obstacles to such increased use are availability of calculators, relevant professional development, suitable classroom resources and teacher confidence. The lack of resources has increased over ten years and this study has shown a continuing need for high-quality classroom-based resources and the corresponding professional development to make good use of them. This professional development should specifically address

the integration of technology into mathematics teaching in a manner that develops teachers' PTK by focussing on the mathematics more than the technology. Another concern with regard to increased use of calculators is that HOD's were concerned about teachers' knowledge of the technology, their confidence in using it for teaching, and the possibility of teacher resistance to its use in teaching.

Generally the teachers in the study were happy with the level of emphasis on calculators in the national assessment system (NCEA) and were positive about it. However, it should be noted that while the NCEA says that technology use is *expected*, this is not enforced in any way and neither are questions requiring calculators set in examinations. When it comes to moving beyond the status quo, whereby GC's are allowed in examinations, 60.5% of teachers were opposed to the use of all calculators in examinations, presumably referring to those with a CAS facility. The issue of equity of access to calculators (probably GC's) was of special concern with regard to examinations due to the relatively high cost and hence affordability.

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