A Global View of Curriculum Issues on Mathematics with Technology
Second Issue

A Panel Discussion

Moderator: *Wei-Chi YANG, USA (wyang@radford.edu)*

Panelists: (in alphabetical order)
Douglas BUTLER, United Kingdom (debutler@argonet.co.uk)
Peter FLYNN, Australia (p.flynn@pgrad.unimelb.edu.au)
Jozef HVORECKY, Slovakia (jhvorecky@cutn.sk)
Masami ISODA, Japan (msisoda@ningen.human.tsukuba.ac.jp)
Hee-chan LEW, South Korea (hclew@knue.ac.kr)
NG, Wee Leng, Singapore (wlng@nie.edu.sg)

Abstract. This is a continuous discussion on what has been said in the first special session (http://epatcm.any2any.net/EP/2003/2003G002/fullpaper.pdf) at ATCM'03. We believe that the issues discussed here are important to many educators around the world. More importantly, we would like participants to bring back their findings from ATCMs and share the discussions here with curriculum decision makers of their respective countries

1. Why is technology not adopted fully in mathematics curriculum in some countries?

We believe that every country's education service realizes the importance of relating technology to the teaching and learning of mathematics in schools and colleges for some simple reasons such as the followings.
(i) A motivational tool for teachers: helps to make the teaching more varied and more enjoyable.
(ii) A motivational tool for students: helps with understanding and retention.
(iii) A serious factor in combating loss of teachers from the profession.
(iv) A serious factor in combating loss of students wishing to study mathematics post age 16 or earlier.

In addition, technology has transformed the workplace; most political and even educational leaders today contend that technology can serve as a catalyst to bring about the change necessary to transform our schools. It does seem not politically correct that any country should or would prohibit the use of technology in mathematics classroom. That technology is not actually used in mathematics classroom in some countries is a different story altogether.

Let us look at Japan and India in which technology is not actually used, at least relatively, in the process of teaching and learning mathematics and yet are able to produce many students which excel in mathematics.

In Japan, most university professors, educators, teachers and parents do not think that using technology to do mathematics is doing mathematics. As a result, technology use in mathematics classroom is few and far between. Japanese schools can well afford many of the ICT tools that are presently available but most teachers choose not to use ICT because they don't think they need them as they feel that mathematics in Japan is not any worse off in not using ICT. However, in all economies, there has been rapid expansion in the proportion of knowledge-based and information-based jobs, now comprising more than 50% of the workforce in many countries, with many such jobs requiring far more than basic arithmetic and measurement to be covered in primary school. Completion of primary education is no longer sufficient to enable today's young people to gain access to these new kinds of work. As a result, a stronger definition of numeracy is needed for the elementary school in order to ensure that young people have a sound foundation in mathematics to prepare them for further study of mathematics and science and related subjects in junior and senior high school. Technology is a necessary tool for developing strong meaning of numeracy. In any educational reform for realizing Education for All, it is equally essential to reconsider what is necessary numeracy in this twenty first century.

In the case of India, it's more an issue of affordability when it comes to use of technology in mathematics classroom, or the lack of it. But again India maintains a very high standard in mathematics among its schools and it is able to produce highly competent IT professionals.

There are at least two learning points here: (a) Technology use in mathematics classroom may be useful but it's not crucial. It might give the students some advantage in learning basic mathematics but the edge over those who do not get to use technology in learning mathematics is likely to level off as they advance into higher mathematics. (b) To motivate countries (from political leaders, educational leaders, academics, teachers, students and parents) to accept technology use as an essential element in a mathematics classroom, they must be convinced of the true benefits that it could bring – that mathematics can be taught and learnt more effectively with the use of technology. For example, government officials or decision makers should provide adequate funding for teachers, professors or researchers to attend international conferences which emphasize the use of technology in teaching and research in mathematics. At the same time, those funded participants should make positive impacts when they return to their posts.

We believe that it's a matter of time that countries which presently prohibit the use of technology in mathematics classroom be compelled by "external factors" to use technology in some ways. Singapore, for example, launched the first Master Plan on IT in Education in 1997 and invested S$2 billion between 1997 and 2002 mainly because the government of Singapore saw the need to harness technology in the schools to meet the needs in the knowledge-based economy. The motivation did not arise from the education scene per se. Likewise, factors such as the more advanced state of graphics calculators (GC) in some countries have a direct bearing on the decision to allow the use of graphing calculators (GC) in Advanced Level Further Mathematics exams in 2001 and in all Advanced levels mathematics exams come 2006 as well.

From practical point of view, manual calculations are slow and do not allow to perform sufficient amount of calculations such as for realistic modeling or "mathematical experiments" in which students study various variants of solutions of the same problem in their efforts to find the most appropriate one. In the meantime, we agree that there is a decline in algebraic proficiency in all schools. Therefore, it might be necessary for a curriculum decision maker to decide how much hand calculations are required before introducing technology in a classroom.

From historical point of view, new technologies have exerted a considerable influence on the teaching, learning and assessment of mathematics. For example, the introduction of electric calculators in the 1960s changed the manner in which numerical calculations were performed in society and business. A consequence of this technological innovation for mathematics classrooms was the need to reconsider the goals of the mathematics curriculum. Later, when hand-held electronic calculators became affordable to school students, a fundamental re-assessment of curriculum goals, assessment regimes and teaching practices began. This is a process that continues up to the present.

Similarly, newer and more powerful technologies such as hand-held CAS can be used to achieve a better alignment of technology in school mathematics with technology use in business, industry and research. It is very important that technology maintains a central role in the teaching and learning of school mathematics by encouraging students amongst other things to:

- become successful users of mathematics;
- achieve a deeper understanding of mathematical concepts;
- possess more positive attitudes towards mathematics.

Further, the introduction of technologies such as CAS, with its potential to automate most symbolic procedures, can free up curriculum time to allow the teaching and learning of new topics or existing topics in greater depth. This freed up curriculum time comes from a downgrade in the over practicing of routine symbolic manipulation.

2. Should technology become a privilege for some economically better countries? If not, how do we make technology accessible to mathematics communities?

It would be impossible to hold back progress anyway. Those who have it are the experimenters, and as new countries come on board the ideas will be tried and tested, and the technology will have become more affordable over the time. In the present time, technology is a privilege of economically better countries and it could remain so for a prolonged period of time. There are many ways of harnessing technology in teaching. There is a tendency for educators to go for powerful tools but it might be better (for economical reasons) to use simple tools in education. For example there are many Java applets [1], courseware and simple ICT tools suitable for teaching school mathematics that are available in the Internet. Graphing software or even CAS can be downloaded for free [7]

For example, basic software like Excel is capable of producing a wide range of templates that can be used by teachers to demonstrate mathematical concepts or by students to explore mathematical results and properties. At the same time, there is a trend that calculators makers are recognizing the affordability issues for some countries. They are developing 'scientific calculators' which can be purchased at a fraction of the cost of a high-end CAS graphics calculators, and yet they can perform some remarkable numerical computations.

So go for the simpler, cheap or even free tools, even though they might be less ideal, to get started. Countries that are more advanced in using technology in mathematics classroom can reach out to those that are less advanced to share with them how to use ICT tools that they can have their hands on. In the case of Singapore, most schools do not use powerful tools such as CAS as it is considered too expensive. So to meet the requirement to integrate IT in teaching, mathematics teachers use basic software such as PowerPoint and Excel and software and courseware that are more affordable. Training of mathematics teachers in using technology in teaching at the National Institute of Education, Singapore, follows the same guiding principle, which is to use ICT tools that may be less impressive but are readily available.

3. How has new technology changed the way we explore math and ask questions? Should exam be open-ended sometimes?

Let's start with Singapore scenario. In response to the IT initiative, some teachers now design worksheets for use in a computer laboratory setting and explore mathematics with the aid of basic software such as Excel or Dynamic Software such as Geometer's Sketchpad. Given the interactive and dynamic nature of the software, students can then explore the relationship between certain variables and 'discover' the mathematical results. This supports the multimodal approach as well as the constructivist approach to teaching mathematics.

The introduction of GC has resulted in some changes in the nature of the Advanced Level Further Mathematics exam questions set. While no significant changes have been observed in questions on topics such as Mathematical Induction, Summation of series, Roots and Coefficients of Polynomial Equations, Differentiation, Integration, Differential Equations, Reduction Formula, Complex Numbers, and Vectors, marked changes are seen in questions on Curve Sketching as well as Polar Coordinates. The questions now award fewer marks on sketching of curves. For questions on Linear Spaces and Statistics, fewer questions involving computations which can be done on a GC have been set. In other words, attempts have been made to make the questions GC neutral.

For example, before the introduction of GC, a typical question on curve sketching looks like the one below:

The curve C has equation $y = \dfrac{x^2 - 2x - 3}{x + 2}$.

i) Find the equations of the asymptotes of C.
ii) Draw a sketch of C showing the asymptotes and the coordinates of the points of intersection of C with the axes. **GC biased**
iii) On the same diagram draw a sketch of $y = \dfrac{4}{(x+4)^2}$. **GC biased**

iv) Hence show that the equation $x^4 + 6x^3 - 3x^2 - 60x - 56 = 0$ has exactly 2 real roots. **Not GC biased but CAS biased**

After the introduction of GC, the exam questions such as the following have been set:

The curve C has equation $y = \frac{(x-a)(x-b)}{x-c}$, $0 < a < b < c$.

i) Express y in the form $x + P + \frac{Q}{x-c}$, giving the constant P and Q in term of a, b and c.
ii) Find the equations of the asymptotes.
iii) Show that C has two stationary points.
iv) Given that $a + b > c$, sketch C showing the asymptotes and the coordinates of the points of intersection of C with the axes.

For problems on Polar Coordinates, students used to have to solve problem such as the following which is GC biased as GC such as TI83Plus is capable of performing numerical integration.

Find the area of a loop of the curve whose polar equation is $r = a\sin 4\theta$, where a is a positive constant.

Following the introduction of GC, students will now have to solve problems such as the following:

The curve C has polar equation $r\theta = 1$, for $0 \le \theta \le 2\pi$.

i) Use the fact that $\frac{\sin\theta}{\theta}$ tends to 1 as θ tends to 0 to show that the line with Cartesian equation $y = 1$ is an asymptote to C.
ii) Sketch C.

The points P and Q on C correspond to $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ respectively.

iii) Find the area of the sector OPQ, where O is the origin.
iv) Show that the length of the arc PQ is $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{1+\theta^2}}{\theta^2}\, d\theta$.

In the Australian state of Victoria, integrated use of graphics calculators in the teaching, learning and assessment of school mathematics began in the mid 1990's. After an interim period where graphics calculator neutral examinations were conducted, from 2000 onwards, student access to an approved graphics calculator has been assumed for Victorian Certificate of Education (VCE) mathematics examinations in Further Mathematics, Mathematical Methods and Specialist Mathematics. As a result, these examinations have contained questions or question parts that are graphics calculator active in addition to containing questions or question parts that can only be answered using symbolic techniques or cannot be answered with technology use.
The graphics calculator's numerical, graphical and statistical capabilities have offered many new and exciting ways of doing and learning school mathematics. For doing

mathematics, these capabilities have created an increase in the range of feasible methods to solve problems and converted solution methods previously only possible in principle to methods now possible in practice. Figure 1 below illustrates part of an examination question that can now be solved in a variety of ways using the numerical solving and graphical capabilities offered by a graphics calculator.

> Tasmania Jones is redesigning the ride so that the platform moves further up and down each cycle.
> During the first 60 seconds of the redesigned ride, *y* metres, the distance of the platform above the ground *t* seconds after the ride starts, can be modeled by the formula $y(t) = 15 + e^{0.04t}\sin(\pi t)$, $0 < t \le 60$.
> i). According to this model, the platform is exactly 6 metres above the ground for the first time about 58 seconds into the ride. Find this time correct to two decimal places of a second.
> ii). According to this model, how many times is the platform exactly 15 metres above the ground from $t = 40$ to $t = 59$?
> iii). According to this model, find the time from when the ride starts until the platform first reaches 24 metres above the ground. Give your answer correct to the nearest second.

Figure 1: VCAA 2002 Mathematical Methods Examination 2, Question 4 (b).

Another feature of graphics calculator active questions, illustrated in Figure 2 below, is the increased range of mathematical functions that can be used to situate such questions.

> A particle travels in a straight line with velocity *v* m/s at time *t* s. The acceleration of the particle, *a* m/s^2, is given by $a = -2 + \sqrt{v^2 + 5}$.
> Find, correct to two significant figures, the time taken in seconds for the speed of the particle to increase from 3 m/s to 10 m/s.

Figure 2: VCAA 2002 Specialist Mathematics Examination 1, Part II, Question 6.

The solution integral $t = \int_3^{10} \frac{1}{-2 + \sqrt{v^2 + 5}}\, dv$ can be calculated correct to two significant figures using the numerical integration capability of a graphics calculator. Note that this question retains its conceptual difficulty as students need to be able to construct the correct solution integral from the differential equation and realise that constant acceleration formulae considerations cannot be applied to this question. Note also that the integration techniques required to solve this question analytically are outside the range of techniques learnt by this cohort of students.

Accordingly, teachers now have to ensure that those who own a GC are not greatly advantaged. While the use of GC does not really affect teachers' setting of questions on Algebra and Mechanics, its impact on setting of questions on Analysis and Statistics seems quite obvious; questions set on the latter topics now have to test students more on their understanding of concepts than numerical or symbolic computations and manipulations.

We agree that it is not time to have examinations that require candidates to use Windows or Apple-based systems. Power supply issues, crashes, and the problem of availability all at once, make this concept a non-starter.

Less powerful hand-held systems are available now, but still the cost of providing everyone with one can be prohibitive without State intervention - the machines would be needed not only for the examination but also for the course. Some governments (for example, Austria) are well advanced with supplying and using hand-held CAS tools in schools.

As far as "exploring" mathematics during the course, there are many exciting new possibilities using dynamic software, either teacher-driven or pupil-driven. There are many new ways to discover the subject, and to use visual images to aid retention.

A contentious area that requires more research occurs with the assessment of mathematical knowledge in timed public examinations used for university entrance purposes. Such examinations have traditionally tested symbolic procedures completed with pen and paper (a technology itself). Allowing CAS into such examinations will require changes to the nature of some of the examination questions set. CAS can change the mathematical knowledge tested by a question, devalue what is tested and sever important mathematical connections by 'gobbling' up intermediate steps and results (see [2]). Changing the types of examination questions we set in CAS-permitted examinations will evolve slowly over time. This corresponds to conservative and pragmatic policy approaches held by examination boards in Australia currently in their infancy of CAS implementation.

If we have to have timed public examinations for university entrance purposes at all, then examination questions where CAS use is allowed can be more open-ended, reflecting the types of tasks and assessing the valued mathematical knowledge that will become more prevalent in mathematics classrooms. Yes, open-ended problems can become a part of exams, at least for future teachers of Mathematics. On the other hand, most problems should be "classical" ones – it would be strange to turn all education upside down. A limited proportion (say, one problem out of five) could help a lot in selecting best students and giving them a chance to demonstrate their abilities.

4. With new diverse technological application available, combining CAS, dynamic geometry, spreadsheet, graphics and etc, what is the impact on mathematics curriculum?

New technologies such as hand-held computer algebra systems and graphics calculators and dynamic geometry software have extended the range of mathematical activity that we can engage students in. While the range of mathematical activity is extended, a high priority for the international mathematics education community is to continue developing and trialling mathematical tasks that encourage students to develop and use valued higher order mathematical thinking and reasoning. For example, the developments of dynamic geometry activities and investigations that encourage conjecture through experimentation and visualisation have rejuvenated the teaching and learning of geometry, particularly in comparison to previous static teaching and learning environments. There is still much research to be undertaken in the continued development of suitable mathematical tasks for CAS permitted teaching and learning environments. In particular, what is the role of pen and paper techniques in future CAS permitted curriculum?

The current mathematics curriculum is still under the shadow of Math Reform in the 1960's and the mathematical concepts and results we teach are discovered between 300 and 2000 years ago. With the advent of technology it is a good time to consider what mathematics we should teach in schools.

The outcome of the calculus reform in the US is that we should teach calculus graphically, numerically, algebraically and symbolically. Software such as CASIO ClassPad (see [3]) or TI-InterActive (see [4]) that combines text editing, graphing, CAS, spreadsheet, web browser, is poised to support such a change. Similarly, with technology, we should be able to teach school mathematics algebraically and geometrically.

Example. (J. Stewart (see [5]) problem 44, page 119)
The *Figure 3*-A shrinking circle shows a fixed circle C_1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point (0, *r*), Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the *x*-axis. What happens to R as C shrinks, that is, as $r \to 0^+$?

■

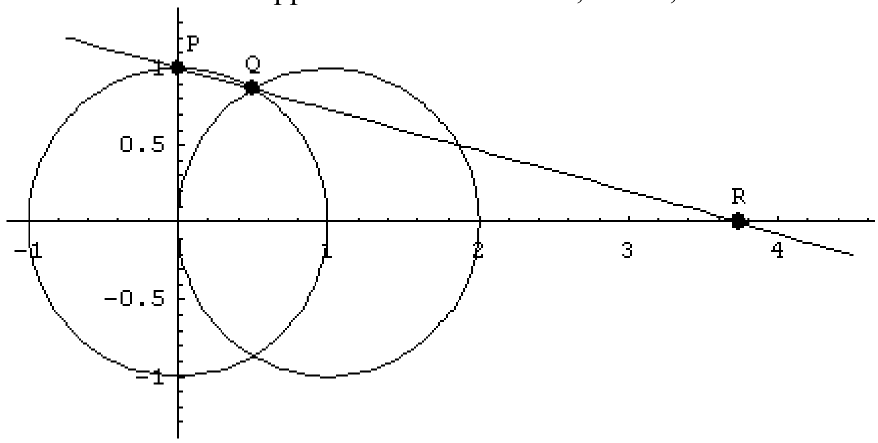


Figure 3-A shrinking circle.

This is a problem which one can use CAS enabled software to find the answer to be 4 and also one can incorporate geometry software such as Geometer SketchPad (GSP) (see [6]) to give intuitive conjecture by using an animation. Interestingly, we can use Casio ClassPad ([3]) to solve the problem with the help of its CAS capability and make conjecture by animating the geometric diagram as one would do with GSP.

CAS can allow more in-depth study of many mathematical models and hence assist in fostering students' abilities to reason mathematically, explain results and make/test conjectures. Here we illustrate briefly how CAS can be used to study projectile motion in the presence of air resistance. Senior mathematics courses in the Australian state of Victoria treat two-dimensional projectile motion without consideration of the effects of air resistance i.e. under the influence of gravity in a vacuum because of the extra algebraic complications introduced when air resistance is considered. Even the simplest mathematical model, which assumes a drag force proportional to the projectile's velocity, requires complicated algebra to solve the equations of motion derived from Newton's second law and numerical methods to find a projectile's horizontal range and time of flight. However, the numerical, graphical and symbolic features of CAS permit this mathematical model to be studied in some depth thus providing a more realistic description of the features of air-resisted motion such as the asymmetry of a projectile's path.

Consider the motion under gravity of a projectile of mass m in the XY plane fired from an origin O with an initial velocity v m/s at an angle θ degrees to the horizontal where $0 < \theta < 90^o$. Let (x, y) be the projectile's position after t seconds of flight and g represent the acceleration due to gravity in m/s^2. Starting with Newton's second law of motion, Table 1 below summarizes the main functions representing a projectile's motion in the presence and absence of air resistance obtained using the symbolic features of CAS to solve the differential and other equations.

Projectile Motion (Without and With Air Resistance)
Neglecting Air Resistance Equations of Motion:
Horizontal: From $m\ddot{x} = 0$, $x = vt\cos\theta$ [1]
Vertical: From $m\ddot{y} = -mg$, $y = vt\sin\theta - \frac{1}{2}gt^2$ [2]
Cartesian Equation of the Trajectory (A Parabola):

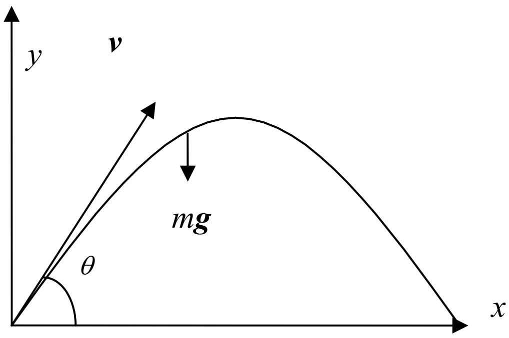
$$y(x) = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$ [3]

Maximum Height Reached:

$$H_{\max} = \frac{v^2\sin^2\theta}{2g}$$ [4] occurs at $$t_{\max} = \frac{v\sin\theta}{g}$$ [5]

Range and Maximum Range:

$$R = \frac{v^2\sin 2\theta}{g}$$ [6] and $$R_{\max} = \frac{v^2}{g}$$ (for $\theta = 45^o$) [7]



Air Resistance Proportional to Velocity
Equations of Motion:
Horizontal:

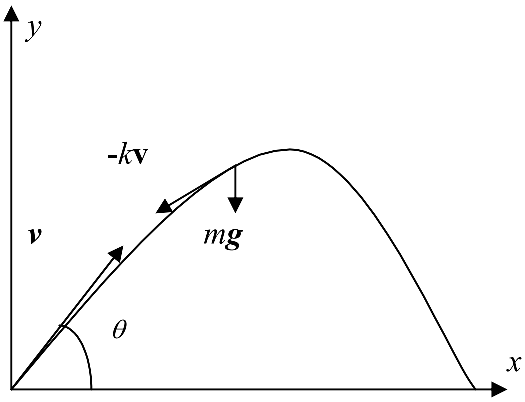
$m\ddot{x} = -k\dot{x}$ where $k > 0$, $x(0) = 0$ and $\dot{x}(0) = v\cos\theta$

$$x = \frac{mv\cos\theta}{k}\left(1 - e^{-kt/m}\right)$$ [8]

Vertical:

$m\ddot{y} = -mg - k\dot{y}$ where $k > 0$, $y(0) = 0$ and $\dot{y}(0) = v\sin\theta$

$$y = \left(\frac{m^2g}{k^2} + \frac{mv\sin\theta}{k}\right)\left(1 - e^{-kt/m}\right) - \frac{mgt}{k}$$ [9]



Cartesian Equation of the Trajectory

$$y_{AR}(x) = \left(\frac{mg}{kv\cos\theta} + \tan\theta\right)x - \frac{m^2g}{k^2}In\left(\frac{mv\cos\theta}{mv\cos\theta - kx}\right)$$ [10]

Maximum Height Reached:

$$H_{AR} = \frac{mv\sin\theta}{k} - \frac{m^2g}{k^2}In\left(1 + \frac{kv\sin\theta}{mg}\right)$$ [11], for $$t_{AR} = \frac{m}{k}In\left(1 + \frac{kv\sin\theta}{mg}\right)$$ [12]

Students can investigate key features of both models (with and without air resistance), develop understanding of the factors that influence projectile motion and compare and contrast both projectile models through the testing of conjectures. In particular, students can comment on the general shapes of the trajectories, determine the effect of air resistance on the maximum height reached, the maximum range attained and the time of flight. Students can deduce that increasing the air resistance results in a number of changes in the path, all of which are consistent with the common-sense appreciation of the effect of air resistance:

- A reduction in a projectile's maximum height (by comparative graphical and/or algebraic analysis of results 3 and 10);
- A reduction in the time taken to reach the maximum height (by comparative graphical and/or algebraic analysis of results 5 and 12);

- A reduction in a projectile's horizontal range (through graphical/tabular comparative analysis of results 3 and 10);
- An increase in the angle of a projectile's impact (by graphical and algebraic analysis of results 3 and 10).

The determination of the effects of projection velocity, angle of projection, amount of air resistance and projectile mass on its motion can be animated using TI-Interactive. This computer software package allows students to explore, visualize and test conjectures by allowing parameters such as v, k, θ and m to be varied using "slider" controls. For example, students can vary the value of k with a slider and confirm that as k approaches zero, the projectile's trajectory becomes more parabolic and its horizontal range increases. Superimposing both models on the same set of axes and using a "slider" to vary k illustrates that for particular k values (different air resistances), both trajectories closely resemble parabolic shapes at the commencement of motion. However, as x increases, $y_{AR}(x)$ decreases more rapidly than $y(x)$, indicating that, for low air resistance or short distances, the no air resistance model provides a good approximation to the linear air resistance model. Through analysis of result 11, the effect of mass on the trajectory in an air-resisted medium can also be animated with TI-Interactive. Students can use a "slider" to control the value of m and verify through graphical animation that as the mass of the projectile increases, air resistance has less effect on its path.
Students can also discover that the air-resisted model's trajectory approaches, asymptotically, the vertical line $x = (mv\cos\theta)/k$ and that the terminal velocity (unless it hits the ground) is $-mg/k$. CAS can also support the algebra and calculus required to demonstrate algebraically what is seen in the animation of the two projectile motions that $y(x) > y_{AR}(x)$, $H > H_{AR}$ and $R > R_{AR}$ for all x with $k > 0$. This investigation is rich in conjectures that can be made using graphical animation and then supported algebraically with the symbolic features of CAS.

In summary, this activity allows students to become better users of mathematics and increases the congruence between real mathematics and school mathematics. As demonstrated, this can be achieved by providing opportunities to study more realistic physical situations effectively with mathematics and technology rather than through a combination of no technology and by-hand symbolic algebra. The study of projectile motion with CAS promotes a less procedural view of mathematics by decreasing the time and effort required in the calculation phase of problem solving and allowing greater time to be spent studying the important features of the projectiles' motion. This refocusing of student time (less on procedural algebraic techniques and more on reasoning/interpretation of results through visualization for example) should assist in the achievement of deeper student learning and support the algebraic skills of students so that they can have more chance to reason mathematically and experience the effects of changing parameters in mathematical models generally.
In conclusion, by incorporating dynamic geometry into the existing CAS mathematics curriculum, it will not only spark new development in software and hardware development but it also will move the mathematics reform to a next level.

Most dynamic mathematical software is heading in the similar direction: 2D and 3D, CAS, Spreadsheet, Geometry and Word processor built in or readily accessible. There are on-going training issues to address, so that teachers can become comfortable with the new tools. The inevitable dilemma about what to miss out because of these new tools is not easy to address. If you are not careful, pupils will be expected to understand things at too superficial a level.

References

[1] The International Education Software Inc. website: 'http://www.ies.co.jp/math/java/index.html'.
[2] K. Stacey "Using Computer Algebra Systems in Secondary School Mathematics: Issues of Curriculum, Assessment and Teaching', *Proceedings of the Eighth Asian Technology Conference in Mathematics*, published by ATCM, Inc. ISBN: 957-41-1528-3.
[3] CASIO ClassPad web site: 'http://classpad.net/'.
[4] TI-InterActive website: 'http://www.ettc.net/techfellow/ti.htm'.
[5] J. Stewart, *Calculus Concepts and Contexts,* 2nd edition, published by Brooks/Cole, ISBN: 0-534-37718-1.
[6] Robert Kreczner, *AN INTERESTING PROBLEM FROM CALCULUS AND MUCH MORE* ,'http://archives.math.utk.edu/ICTCM/EP-8/C11/pdf/paper.pdf'.
[7] MuPad website: 'http://www.sciface.com/news.shtml'.
[8] P. Flynn, Victorian Curriculum and Assessment Authority (2002). *VCE Mathematical Methods: Written Examination 2*. Victorian Curriculum and Assessment Authority. Melbourne.
[9] P. Flynn, Victorian Curriculum and Assessment Authority (2002). *VCE Specialist Mathematics: Written Examination 1*. Victorian Curriculum and Assessment Authority. Melbourne