

A Bold Mathematics Course to Support a New Civil Infrastructure Program

(RMIT's Experience)

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Abstract: We discuss the implementation of a problem-based course supporting a new Civil and Infrastructure program of study at RMIT. Based on the set learning objectives we describe the framework of study implemented and illustrate these with two of the major components of the course: hand-written practice-class exercises and formative WebLearn testing that provides both generic and targetted feedback to support student learning. Course evaluation is discussed from both the students' and the program teams point of view.

1 Introduction

RMIT University has recently restructured its Engineering disciplines into three new schools, each one including a diversity of teaching programs that lead to qualifications in different disciplines. As part of the regular quality assurance process used at RMIT, the Civil Engineering program was earmarked for renewal in 2004. Throughout 2003, monthly meetings were held with all staff—internal and external to the school—likely to be involved in the new program. At these meetings details of course content were mapped out, paying attention to synergies with other courses sharing content, to offer the possibility of reinforcement of concepts. With the intention of encouraging the students' involvement in their learning, these discussions were predicated by the requirement that all courses be “problem-based”.

This provided a special challenge to the mathematics discipline, since the preponderance of all other courses offered to these schools was conventional in nature. That is, the mathematical skills and tools were presented first followed later by particular applications. If the course was to be problem-based, the standard approach had to be changed to put less stress in what the instructor had to do to teach, and instead emphasize what the students had to do to learn. This paper explores the main features that drove the development of the first semester mathematics course, and discusses some of the main features resulting from its first presentation in 2004.

2 The Teaching and Learning Approach

In order to implement the required problem-based course, we focused first on what the students were required to learn. As an initial step, we established the course learning objectives, to determine the skills that the students should exhibit at the end of the course. The intention was to align all the course teaching with the stated learning objectives, making sure that every aspect of the course contributed to addressing the objectives. The aspects considered were:

- Learning material: mainly information to introduce and discuss concepts, present examples, etc. This aspect is mostly related to the presentation and transmission of information.
- Formative activities: activities designed for the learner to learn by doing and solving, such as questions, problems, quizzes and exercises.
- Tests: automated formal testing instruments, to be able to determine whether a learner satisfies the set of objectives under consideration.

In previous work [1], we presented an integrated approach to address different levels of cognitive processes using WebLearn, an online student-centred environment. A brief description of an early version of WebLearn in use at RMIT University in 1999 is available in [3]. We based our approach on the well known Bloom's "Taxonomy of Learning Objectives" ([2]), which identifies six levels within the cognitive domain, from the simple recall or recognition of facts (Knowledge and Comprehension) at the lowest level, up to the highest order (Synthesis and Evaluation). In this way, a set of learning objectives establishes not only what topic or concept is under consideration, but also the required level of learning required on that topic.

Although automated learning environments have been mainly used for drilling activities that address the lower levels of the taxonomy, we have demonstrated that it is possible to induce learning at the higher cognitive levels by designing activities that appropriately address them. The teaching content should be designed and presented in such a way as to make it possible for students to make the higher-level cognitive contributions.

Hence, there is no restriction as to what the learning objectives for a course component may be. Lower levels of the taxonomy are straightforwardly addressed by questions of the simpler type, while higher levels require more carefully aligned teaching material and activities to leave open questions for the students to fill in the gaps by implementation, exploration and extrapolation. However, since these are all student-centred activities, the strategy must include a targeted feedback cycle that ensures proper learning. In a traditional teaching environment this is typically handled by student interactions with a human tutor, but in our case we have been able to furnish the learning environment with the ability to provide targeted feedback.

Since this course was taught for the first time in 2004, most of the questions that have been implemented address the lower levels of the taxonomy. In 2005 we intend to incorporate questions that tease out some of the higher levels of this taxonomy.

3 The Student Cohort

The two most advanced courses at secondary level mathematics in Victoria, Australia are Mathematical Methods and Specialist Mathematics. The main distinction between the two from a tertiary education point of view is that students in specialist mathematics are subject to a deeper exposure of mathematics, as well as studying vectors, mechanics and more extensive calculus and trigonometry than their counterparts in mathematical methods.

The requirement for selection into the Civil and Infrastructure program at RMIT only requires a reasonable performance in the secondary course mathematical methods. Nevertheless, the first intake of students in the new program consisted of approximately 30% of students who had successfully passed the more advanced specialist mathematics course. A number of other students had attempted the specialist mathematics course, without great success, and the strategy had to contemplate this significant disparity within the student cohort.

4 The Mathematics Course Structure

4.1 Teaching outline

This course constitutes one quarter of a full-time students' semester's load. In 2004 it consisted of four face-to-face lectures (to cover the information aspect of the course), one one-hour weekly practice class and one two-hourly fortnightly Maple practice class. Supporting these activities were a number of formative WebLearn tests that provided both generic and specific feedback targeting students' understanding of the concepts being covered. During the lectures, material that had previously been posted as lecture notes to the BlackBoard site, were discussed, supported by several demonstrations of topical material using Maple. The weekly practice classes consisted of a small number of exercises to be completed within the hour and handed in for marking and assessment that contributed to the final grade. Group work was encouraged in these sessions, as was the opportunity to discuss the problem solving with staff in attendance. One group Maple assignment contributed to the final grade together with a three-hour final examination.

4.2 Teaching material

The problem-based first semester mathematics course that was offered for the first time in semester one 2004 consisted primarily of vectors and differential equations. Motivating examples for the vector material were obtained from the well-regarded engineering text [4].

4.2.1 Supporting practice class and WebLearn activities

The WebLearn system was introduced to the students in week 1 via two diagnostic testing activities. The first activity used multiple choice questions to self-test students recall of certain secondary level mathematics. The second was a fun quiz using prime numbers, which required an answer to be entered into the text box provided. Each question in the first test was simply marked as correct or incorrect, whereas the second provided specific feedback in relation to the question asked. For example, one question asked for the next prime number following a displayed randomly generated number. If the student's response was not a prime number they were informed as such and its factors were presented to the student, if it was a prime number but not the next then they were also informed of this and the correct next prime number provided.

Two examples follow describing how the learning objectives have been addressed within the course content. The first is drawn from the introductory vector material, the second from the differential equations material.

Vector material

A specified learning objective was that students should be able to determine the vector representation joining two points in three dimensions. Upon presenting the underlying theory in class, the students are provided with two supporting activities, one in the weekly practice session and the other as a WebLearn activity. A related practice class activity used exercise 2–98 in [4], which asks for vector representations of the force in each of the cables appearing in Figure (1).

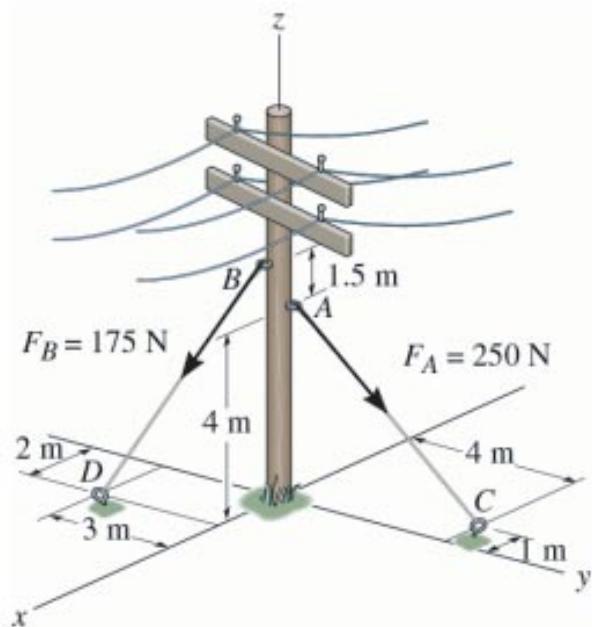


Figure 1: Practice class example for vectors.

To help the student prepare for this activity, a related formative WebLearn quiz presented the student with two randomly generated points in three dimensions and asked for the vector joining them, see Figure (2).

Specific feedback for an incorrect answer is provided by the system, identifying any incorrect component and specifying the correct one. Furthermore, generic feedback in the form of a precise reference to where the material is discussed in the notes and a similar worked example are provided, see Figure (3).

In all the WebLearn activities both forms of feedback were used where appropriate, and were presented to the student as a WWW page using the WebEQ applet to render any necessary mathematical parts of the response in acceptable typeset form.

There were a total of nine questions in this formative quiz. The illustrative performance of one student on the complete WebLearn quiz is provided in Figure (4). This particular student made four attempts at the formative quiz, improving from two correct answers to five at the fourth attempt. Other examples of student performance in this and other quizzes will be presented at the conference.

Let A be the point $(-1.6, -7.3, -3.3)$ and B be the point $(-2.3, 1.9, 2.7)$.

The directed line segment joining the point A to the point B (in this direction) is given by the vector whose

first (or i th) component is

second (or j th) component is

third (or k th) component is

Your answer should be accurate to at least five significant digits.

Figure 2: Supporting WebLearn question for vectors.

Wrong response:

You should check the last example in Section 1.2.3 of the notes:
Review of Vectors

Please attempt this quiz again.

Your response was:

1.9 and 2.2 and -1.1



The student answered the question incorrectly

The following message was generated

Response was incorrect

Your answer has the correct first component but the second and third components should be 2.4 not 2.2 and 1.1 not -1.1

Figure 3: Supporting WebLearn feedback for vectors.

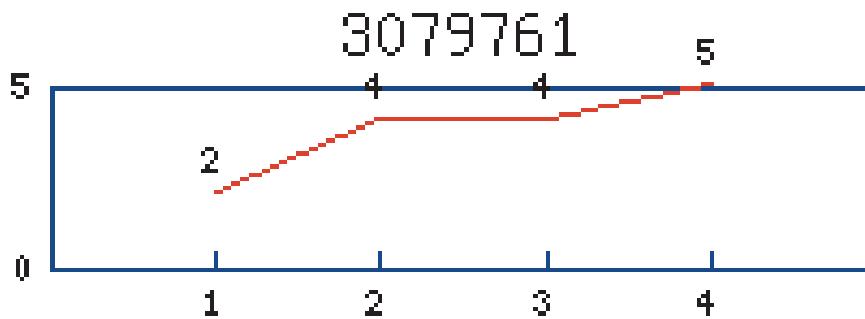


Figure 4: Example WebLearn activity for vectors.

Differential equations material

One learning objective in the differential equation material is for students to be able to determine the general solution of a second order differential equation in order to prepare them for formulation and solving of oscillation problems such as the forced vibration of a stretched cable. Upon presenting the underlying theory in class, the students are provided with two supporting activities, one in the weekly practice session and the other as a WebLearn activity. A sample practice class activity is the following.

- (a) Determine the solution of the Initial Value Problem (an under-damped system)

$$\ddot{x} + 4\dot{x} + 13x = 0 \quad \text{with} \quad x(0) = 0 \quad \text{and} \quad \dot{x}(0) = 9$$

- (b) Sketch, on the same axes, the graph of your solution $x(t)$, the function $3e^{-2t}$ and the function $-3e^{-2t}$. For general Initial Conditions, what would you expect to be the number of times the solution will pass through the equilibrium position $x = 0$?

To help the student prepare for this activity, a related formative WebLearn quiz presented the student with a randomly generated constant coefficient second order differential equation and asked for two linearly independent solutions, see Figure (5).

Specific feedback for an incorrect answer is provided, specifying whether both expressions are appropriate linearly independent solutions—providing a correct representation for those that may be incorrect. Furthermore, generic feedback that provides the related auxiliary equation, identifies its correct roots and displays one form of a possible solution is provided, see Figure (6).

A precise reference to where the material is discussed in the notes and a similar worked example has also been provided.

Determine the two different (linearly independent) solutions of the differential equation

$$\frac{d^2y}{dx^2} + 81y = 0$$

Use x as the independent variable.

The first of the two independent solutions is

The second of the two independent solutions is

Figure 5: Supporting WebLearn question for differential equations.

4.2.2 General comments about WebLearn feedback

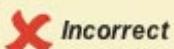
When implementing feedback for incorrect answers to a WebLearn quiz, several levels of increasing sophistication are possible. Some questions may effectively incorporate several of these features.

1. A quiz question can simply be marked correct or incorrect.
2. A quiz question, when marked incorrect, supplies the correct answer.
3. A quiz question, when marked incorrect (or even at the pre-submission stage), supplies a link to a hint as to how it may be solved.
4. A quiz question, when marked incorrect, supplies a link to a similar worked example.
5. A quiz question, when marked incorrect, supplies a link to theory supporting the learning objective.
6. A quiz question, when marked incorrect, provides an explanation as to why the answer doesn't satisfy the conditions supplied.
7. A quiz question, when marked incorrect, supplies information as to a possible source of the error and the correct answer is supplied.

4.2.3 Supporting maple project

During the first few Maple laboratory sessions, students were provided with several Maple worksheets that introduced them to the package, and a range of standard commands that they would need in the related group project. The main outcome of the project was to provide a complete Maple solution of the motion of a cricket ball travelling in a medium both with and without an air resistance term, modelled in terms of the cricket ball's speed. The governing differential equations for the project were developed in class and a number of the weekly practice sessions explored the mathematical description of the problem. This mastery of the underlying mathematics is felt to be an essential requirement before students should attempt a Maple solution.

Your response was:
 $\cos(9^*x)$ and $\cos(9^*x)$



You have provided the same expression $\cos(9^*x)$ for both parts of the answer. This expression is part of the general solution, but the complete solution requires the second linearly independent solution $\sin(9^*x)$.

The auxilliary equation is

$$m^2 + 81 = 0 \text{ so that } m = \pm i(9)$$

Therefore, as these roots are complex conjugates, the two independent solutions of the given differential equation are

$$\sin((9)x) \text{ and } \cos((9)x).$$

Figure 6: Supporting WebLearn feedback for differential equations.

The major outcome of the project was to generate a graph of trajectories for a range of different air resistances, suitably colour-coded to identify important features, such as distinguishing between the cricket ball's upward and downward motion. Assessment of each group's submission required a clear literal description of their solution strategy, and a physical explanation of the reason why the curves appeared as they did. A qualitative explanation of the effect of the various air resistance models on the resulting range was also required. The suggestion that the motion of the cricket ball could be presented as an animation was taken on board by one or two groups and attracted bonus marks. Each group uploaded their project to the lecturer's drop box in WebLearn.

5 Course Evaluation

During the semester a Student Staff Consultative Committee meeting requested more worked examples in class during which they could "see" the lecturer's solution strategies being developed in real time. From this meeting and later informal discussions with the students it was clear that they perceived this course "not to be mathematics". The approach adopted in this course, to present the problem first and develop the theory as required was a complete departure from traditional teaching, and totally alien to the vast majority of students. Consequently, the intention next year will be to devote some time explaining the problem-based approach more carefully to establish the rationale of this new approach. The results obtained in this new mathematics course were reviewed at the end-of-year Student Progress Committee and were found more than acceptable. A course evaluation feedback form was distributed at the completion of the course and a complete analysis is currently being completed. Despite the new problem-based approach, and the level and difficulty of the material, students reported a liking for the course, felt that they had learned useful material, but stated a clear desire to have a supporting text

book. This is consistent with their existing perception of what a mathematics course should be like. The majority of students did not like having to learn Maple. One comment by a very mathematically capable student was to the effect that they “knew how to solve the problems by hand, so why bother with a package that does the same thing”. This was despite the complexity inherent in some of the set problems. The use of supporting technology was not the issue here since during the practice classes almost all the students reached automatically for their graphics calculator when asked to do reasonably sophisticated calculations. Nevertheless, there was a dedicated band of students who used Maple extensively during the course, which they had installed on their own laptops. On their own initiative they had begun to incorporate its use into the physics course. It was observed that this same group would often use Maple, rather than their calculator, in the scheduled practice sessions.

References

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