Exploring the Use of Dynamic Geometry Manipulative Tasks for Assessment

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Abstract: This paper discusses the potential use of dynamic geometry environment to examine quantitatively students' understanding of geometrical knowledge through their performance in manipulative tasks. Most of the studies on the impact of dynamic geometry computer environment tend to focus on students' performance in open-ended explorations or problem solving tasks by means of detailed observations, analysis of individual works and in-depth interviews, aiming to understand the nature of geometric investigations or explorations afforded by the interactivity possible in such environment. A different focus of study is proposed here, which considers dynamic geometry environment as a means to assess students' geometrical knowledge based on an analysis of their responses made quantifiable in specially designed manipulative tasks. These tasks elicit students' responses to mouse dragging of points in pre-constructed dynamic figures. Students' prior knowledge of how to use dynamic geometry software is not assumed. Their responses will be described by and recorded as a prescribed parameter value designed for each task. This will give a set of data describing a range of students' performance on tasks specifically designed to assess students' geometrical conceptions. The design of these manipulative tasks is explained, with some examples from a recent computer-based mathematics contest for Secondary 1 students. Responses to these contest items illustrate a range of outcomes while students grab and drag certain points in attempts to adjust continuously geometric quantities, such as length, angle and area, to the required configurations. Further analysis of the distribution of these responses in some items reveals the nature and varieties of approaches adopted by the students. This will lead to considerations of developing the tasks into diagnostic tests.

Introduction

Dynamic geometry (abbreviated as DG hereafter) brought about mainly by Cabri-geometry and Geometer's Sketchpad, and more recently by C.a.R. (i.e. Compass and Ruler, with its original German version ZuL, Zirkel und Lineal, designed by R. Grothmann) has stimulated a range of inquires both in mathematicians' studies of geometry and mathematics educators' understanding of the teaching and learning of geometry (see, for example, Whiteley, 1999). In essence, these DG programs offer opportunities for the users to manipulate and, precisely speaking, *to act directly on*, geometrical diagrams – particularly by grabbing and dragging certain geometrical objects (e.g. points) with the mouse. These new opportunities of direct access to and interactions with geometric diagrams open a new *field of experimentation* (Balacheff & Kaput, 1996). More importantly, the environment enlivened by such manipulable objects and tools implies a kind of geometric reasoning very different from the traditional kind, mostly deductive in nature, found in school geometry with paper and pencil. As noted in Whiteley (2000), "these dynamic sketches create new answers, new methods or reasoning, and new questions."

One particular aspect of the well acclaimed powerfulness of DG is that in the new experimental field it granted, the geometric *drawings*, as opposed to *figures* by virtue of the distinction made by, for example, Laborde (1993), preserve the (invariant) properties salient to the geometric configurations. Whilst grabbing and dragging the geometric objects (e.g. a point that in turn changes the shape of a triangle), explorations and investigations are made possible by the images on the screen which consist in a whole class of objects sharing the same set of conditions and properties in question. The "dragging" thus facilitates the reasoning process in, for example, helping the user to move backward and forward between particular instances of geometric relations and general theories about invariant relationships. However, all these depend quite a lot on the students' ability in grasping the geometric properties via purposeful manipulation and visualization. As Whiteley (2000) spoke about his reflection on his experiences in both studying and teaching geometry, the whole point about using these DG programs is "learning to 'see' differently and therefore think differently". In their final analysis of what digital technologies (such as DG software) can take from and bring to research in mathematics education, Hoyles and Noss (2003, p.341) asserted that "DG-maths is not the same as maths *per se*, and that – by implication – neither is the knowledge that learners develop." Yet, despite the promising future offered by DG tools, Whiteley (1999, 2000) has already alluded to the present situation in which the use of visual tools and skills of visual reasoning (of which transformational reasoning, Whiteley contends, is of particular importance) are seldom taught but taken for granted. Alerted by this observation, we come to a question that has yet to be asked: How can we be sure that our students have gained a certain level of this ability (of, say, visual investigation and sense-making dragging) before we try engaging them in using this tool for further explorations (in the constructions of geometric knowledge)? In other words, if we believe that such a new kind of reasoning is still geometric in essence and thus crucial in learning and doing mathematics, assessment of how well a student is capable of exploiting this tool in solving problems must be included in our geometry curriculum.

Current curricular emphasis as regards geometry is largely connected to the Euclidean tradition. This is reflected by an observation about the geometry curriculum, particularly the assessment items found in the examinations (in Hong Kong and probably various other parts of the world). That is the predominance of computational and proving problems (though the latter being less and less favoured in practice) based on well-remembered geometric properties or theorems. Contrasted with the above-quoted comment of Hoyles and Noss (2003), it is noteworthy that the geometric knowledge being assessed in this new era is not much different from what we had in the past. Even more disturbing is that, as far as these assessment requirements are concerned, questions as to whether DG has been incorporated in the learning process and whether students are prepared (at least partially with the aid of DG) for the visual demands of the future world are left totally unheeded.

To summarize, there seems to be at least two problems of mismatch in current practice of using DG in our teaching and learning of school geometry:

1. When DG claims, as evidenced by various research studies, to support the exploratory process in geometric investigations, we, as bound by the existing curriculum (in many places such as Hong Kong), are concerned with the traditional kind of geometric knowledge.
2. When learning geometry in DG environment calls for various basic skills very different from those required for traditional kind of geometric studies, we do not seem paying sufficient heed to the development of such skills.

These mismatches are well revealed in the domain of assessment. This is where the following pages are situated and it is in the possible use of DG manipulative tasks that we are going to look more carefully into the geometric knowledge and basic skills not very accessible by the teaching and learning practice found in traditional mathematics classrooms. By way of an introduction to such manipulative tasks in general and a few of them in particular details, we try to propose a direction for further inquiries on the above two problems.

Manipulative Tasks in Dynamic Geometry Environment

DG packages do not simply provide powerful tools for students' learning, they are also useful tools for teachers designing a wide range of learning activities. Pre-constructed sketches, sometimes accompanied by teaching ideas, are available on the web for sharing among teachers or supporting curricular and pedagogical innovations. Some sketches can even be easily converted into other formats integrated with web pages and become more accessible. One way of doing so is converting a DG figure into Java applets embedded in html files. Dynamic sketches of this kind can be handled by compatible browsers, without using the original software. (In fact, some developers create directly Java applets or applications from other tools, such as Flash, to provide similar DG figures for specific learning activities.) What students can do with these web-based dynamic figures can vary considerably from one example to another. Even so, some basic elements of interactivity can be assumed in most cases and are crucial for understanding students' learning experience in DG environment, as compared with traditional means.

MathsNet (http://www.mathsnet.net), a UK-based website, is a good source of well-designed and organized interactive geometry learning resources. Within its vast collection of geometry curriculum materials created with different dynamic geometry packages, there are carefully designed tasks to help developing students' concepts based on their manipulation in pre-constructed sketches. Two examples are given below to illustrate some important features of the tasks.

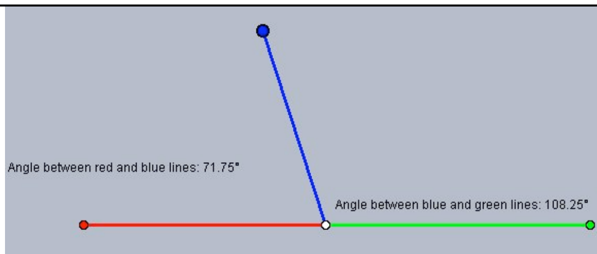
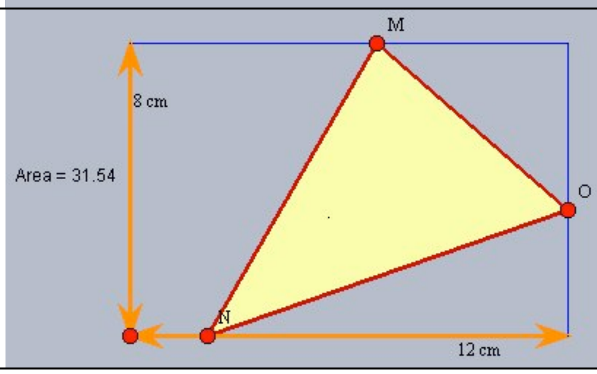
Drag the points to alter the size of the angles. Measurements are given and change continuously when the points are dragged around. Learners are suggested to make two right angles. http://www.mathsnet.net/shape/ks2anga4.html	
The vertices of the triangle can be dragged along independently on 3 segments, which may result in a change of its area, given in the same figure. Learners are suggested to find the largest area possible for the triangle. Answer can be entered in a text box and checked against the correct one. http://www.mathsnet.net/shape/ks4gcsa1.html	

Figure 1 Two examples of activities from MathsNet

These examples differ in complexity and difficulty, but we can identify some essential elements of interactivity. As a dynamic figure, users are able to drag some movable points, which are determining other geometrical objects in the figure. Users could then observe changes in the relation among different parts of the figure and some numerical measurements. Such feedback can help them to make sense of the manipulation in order to perform a specified task.

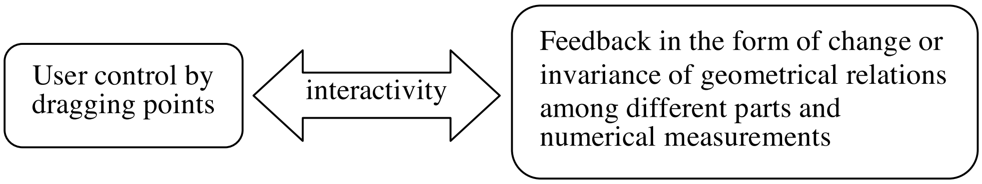


Figure 2 Features of DG Manipulative Tasks

We are particularly interested in such activities presented as tests or exercises, in which the result of students' manipulation may reflect certain geometrical knowledge and understanding. While working with the dynamic figures, they may be directly controlling some geometric quantities, such as length and angle. Sometimes, it can be indirect manipulation that depends on deeper understanding of geometric relations. No matter the complexity, to what extent a student is making an appropriate manipulation could be indicated by some parameters specific to the given geometric situation. These can be critical geometric quantities that students try to control in each task. In some cases, the students are dragging part of a figure to control an angle, distance or length, area of a region in order to give a specific value. On the other hand, the final value of such quantity when the task is completed indicates the correctness of student's action. In some sense, we are making use of some parameters to quantitatively describe students' performance in this kind of DG manipulative tasks.

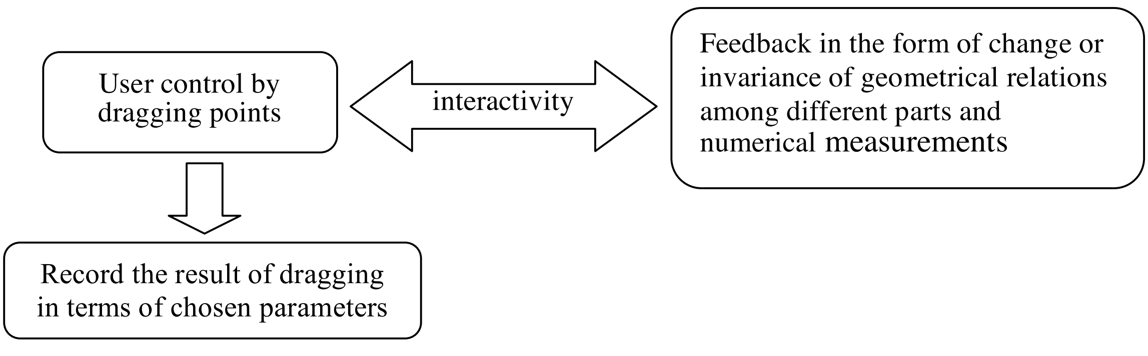


Figure 3 Capturing Students' Work on DG Manipulative Tasks

In a recent local mathematics contest for secondary level year one students, we have introduced some computer-based geometry tasks among other traditional paper-and-pen contest items. In each task, students were given a DG figure, created with C.a.R., and required to manipulate the figure by dragging any movable points in order to produce a specified result. These figures were presented in the form of Java applets (directly converted from the C.a.R. files) and accessed through web browser programs. Working directly with the Java applets, the students were not provided with any original construction tools (though it is possible to include those tools during the conversion process) and there was no assumption of any previous experience in using DG packages.

The following example is an item based on the manipulation of an angle. In figure 4, students are required to rotate the inner polygon for an angle of 160 degrees by dragging its vertex P, which can be freely rotated about the center. The angle of rotation is measured and substituted into a hidden formula. The highlighted value ‘X’ gives the result of the calculation based on the measured value. Without knowing the formula, the students have no idea how the angle is linked to the value of X, but they are asked to report this value on their answer sheet after the required manipulation. These encrypted values therefore tell us how each student has actually rotated the polygon. (There should be some other means to extract these data directly from the program but it is not explored here.)

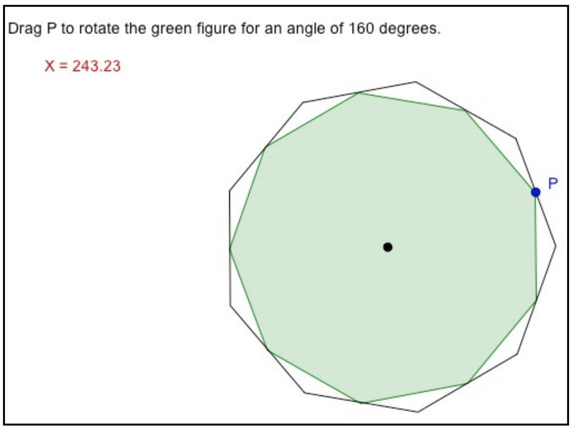


Figure 4 Test Item 1: Initial state of the dynamic figure. Students can drag P to rotate the polygon.

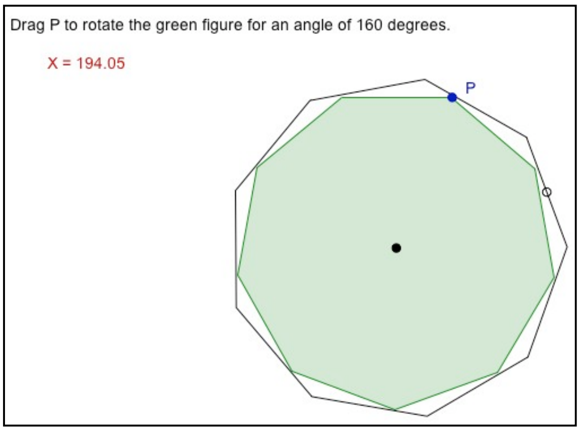


Figure 5 Test Item 1: When P is dragged, the value of X is updated continuously. There is a hidden formula that generates this value based on the angle of rotation.

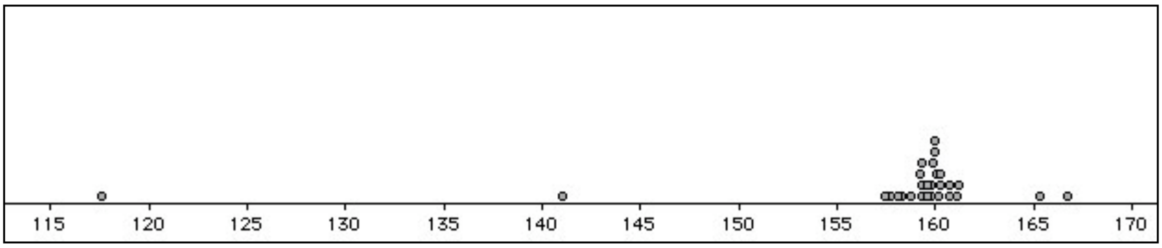


Figure 6 The dot plot of the results from the task in Test Item 1. It shows the angles the students had actually rotated based on the values of X reported.

This is not merely a process of finding who correctly perform the task. From the overall distribution of the results collected, we obtain information about variation of their performance. For example, the results in the dot plot (Figure 6) show the majority who gave roughly the required angle as well as some obvious deviations. They differed not only in the accuracy in controlling the angle, but probably their conception of angle of rotation as well. The task can be modified in different ways so that the notion of angle is linked to other geometric objects. In this example, their knowledge or simply perception about regular polygons (for example, a perception about the angle subtended by the sides at its 'centre', which seems to be fundamental but probably not emphasized as much as such traditional properties as angles' sum of interior or exterior angles) should be a critical factor in handling the task. It is worthwhile to further explore how and what we could assess our students about their geometric knowledge and ability with this kind of tasks.

Analysis of Students' Responses

In this section, we provide some details of students' responses to two similar items in the contest, together with our analysis of the data. Each of these items requires students to drag a point D on a given line in order to make the area of triangle DBC equal to that of another fixed triangle ABC. Thus in both items, the students are looking at two triangles sharing a common side BC. Figures 7 and 8 show the dynamic figures for these two items with their respective initial positions of D. The only movable point is D which can be dragged along the given fixed line. While dragging along, the students in each case are required to copy the value of X on the answer sheet once they think the areas are equal. Without notice to the students, this value X is the disguised form of the parameter which, as explained in the above section, is a measure (in terms of the area units built-into the DG software) of the difference between the areas of the triangles – namely, the area of triangle ABC subtracted from that of triangle DBC.

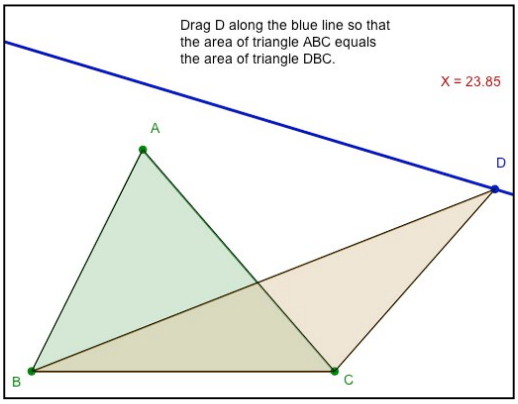


Figure 7 Test Item 2

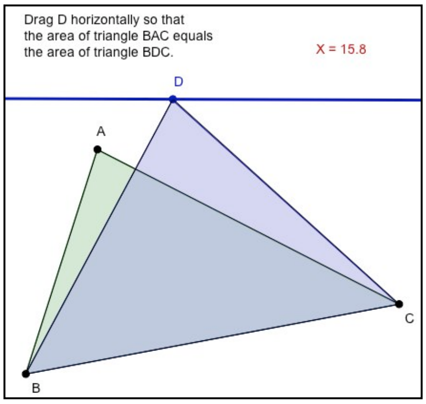


Figure 8 Test Item 3

The students' responses are shown by the dot plots in Figures 9 and 10 below. Noteworthy is that the values revealed here are not exactly the same set of values of X given by the students, but the corresponding parameters that originally, as explained above, capture a crucial geometric measure in the problem situation. Each dot now represents the actual difference (in default sq. units) in the areas of the triangles when the student thinks that they are equal. So, the value zero along the axis in each of the dot plots corresponds to the "correct" answer. We may compare the patterns in both charts to look for interesting features. (A few outliers, far beyond the displayed range, in the second chart are removed to facilitate comparison in the selected range, although they may also be interesting for other reasons.) The distribution of the data set for Test Item 2 is distinguishable from a simple spread around the zero value for there are data points gathering near –1 (Figure 9). While the distribution of students' responses in Test Item 3 (Figure 10) has a more uniform spread across the range, it demonstrates a very distinct shape from the previous one for Test Item 2. With Test Item 2, data points gathering near –1 (Figure 9) suggest that some choices from students could not be simply explained as random errors in estimations.

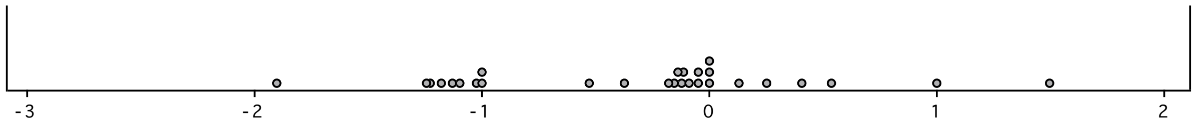


Figure 9 Distribution of students' responses in Test Item 2

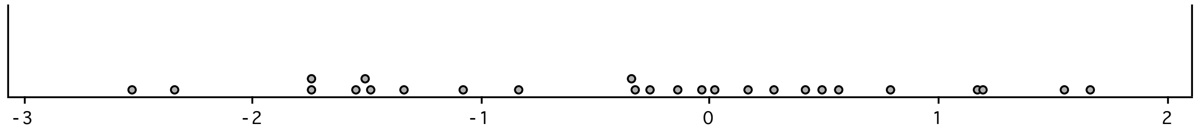
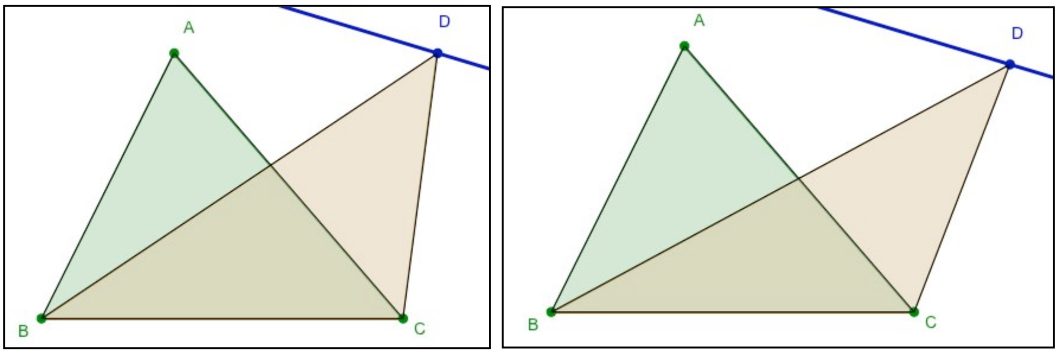


Figure 10 Distribution of students' responses in Test Item 3

Figure 11 shows two special positions of D in Item 2, which correspond to the case of equal area (zero difference) and another one of a difference of 1 sq. unit. It is interesting to know how the students approach the task and work on it according to their geometrical understanding. The numerical data surely cannot provide answers to these questions but may at least give us some hints on further investigations. We may wonder if the students are making wild guess or guided by some conception about area of triangles or a particular geometric configuration. For example, in this item, did some students intend to make DC parallel to AB, believing that the areas concerned would then be equal or perhaps feeling that the figure was more symmetrical this way?



Point D is in the position that makes the triangles equal in area. (AD is parallel to BC.)

Another position of D which makes triangle DBC smaller than triangle ABC by 1 sq. unit. (DC looks parallel to AB.)

Figure 11 Two special positions of P in Test Item 2

Comparison of both sets of data suggests that some students could adjust the area very accurately when the base was horizontal. Perhaps it was more difficult to make AD parallel to BC when they were inclined. As mentioned above, some students might be easily distracted in Item 2 by a special position at which DC seemed to be parallel to AB. Among these students, some made similar choice in the second case but others managed to give rather accurate answers there (see the scatter plot in Figure 12). It is therefore worthwhile to investigate to what extent such visual clue is affecting their judgment on the screen or in fact interfering with their decisions based on certain geometrical conceptions.

Two groups of students are highlighted. They responded similarly in Item 2, but quite differently in Item 3.

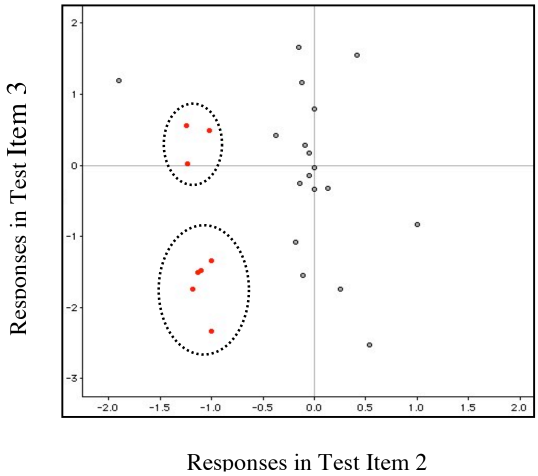


Figure 12 Scatter Plot of the Responses in Test Items 2 and 3

Discussion

Dynamic geometry is now believed to be beneficial to geometry learning, but the experience with the investigative or exploratory kind of learning activities seems to be very different from the experience with deductive geometry in traditional classroom. The latter mainly deals with propositions about static diagrams whereas the former allows direct manipulations on geometric objects which, with their dynamic (and multi-representational) character, in turn support a richer understanding of the geometrical properties in relation to an accessible class of geometric objects. This is not to deny that (successful) students of geometry may have the same ‘dynamic’ kind of understanding (cf. Whiteley’s (2000) personal reflection as a geometer). But most of us in the older generations did not have the chance to enjoy such a rich understanding in geometry. In most cases, the geometry problems in the examinations demand us to remember a full set of theorems and properties, and to make use of them in calculations and proofs. Nevertheless, when we ask the students to find the magnitude of a certain angle in the middle of a given geometric setup of, say a couple of circles and a few lines, what are we assessing? It does not seem to match squarely the objectives of most curricular reforms found in many countries. (For example, in the case of Hong Kong curriculum, there are new learning targets such as “to explore visualize geometric properties of 2-dimensional and 3-dimensional objects intuitively” (CDC 1999, p.10).) Asking the students to prove something does not necessarily probe anything more than the same old kind of propositional knowledge and procedural knowledge, especially when the question types are rather standard.

Authentic and extended tasks that require more of an investigative process appear to be one possible solution to the mismatch. But this kind of assessment demands a lot more both on the part of students (their investment of time and effort) and teachers (their marking load, to say the least). There is also the problem of reliability inherent in this kind of assessment for we know all too well that the marking of students’ work in such tasks is loaded with various subjective elements, even if there are a variety of scoring rubrics existing in the literature and practices. Furthermore, given that we are advancing ourselves in a technological age, how can we have 'assessment tasks' that are more relevant to the educational objectives accessible and amenable to the DG environment? First of all, the kind of manipulative tasks as described above can provide simple quantitative results. Although these measures surely cannot capture the many facets of students' geometric understanding, their quantitative character allows a convenient and reliable record of students' performance in a geometric situation. In relation to assessment, the observations about DG manipulative tasks made out of the above analysis have cast even more light to our interpretation of such quantitative measures.

Firstly, a carefully designed parameter which is to be reported by a student after his/her working on a manipulative task may be an indicator of how well he/she has learnt certain concepts (e.g. angle of rotation of a specific magnitude in the context of rotating a polygon, as exemplified in the first example above). It is not uncommon to have a student who can recite the statement that angles meeting at a point give a sum of 360° but cannot understand the meaning of an angle in context. Angle of rotation is but one of the instances. As long as we are interested in students' understanding of an angle measure in action, as distinguished from an angle measure as little more than a number (though with a very small circle at its top right-hand corner), the quantitative measurement we gathered out of the above example suggests a more valid assessment score than those resulted in traditional kind of computational or even proof problems in school geometry. Secondly, we suggest that this quantitative measurement – in some geometric situations at least – may well be used for diagnostic purposes. The examples of triangles of equal areas in the above analysis illustrates how the values reported by the students in some cases may reasonably cluster around the "correct" value, and in some others, may possibly reflect different conceptions of the geometric situation.

However, before we are getting more convinced of the potential use of the manipulative tasks discussed above for assessment, we should give deeper and more careful thoughts to the question of *what* is to be assessed. Whether a student can reach a certain geometric result seems to be a natural answer. As in the traditional type of geometry classes, teachers are concerned with students' ability to compute a certain geometric measure such as an angle or to produce (*re*-produce?) a proof for a certain geometric statement by using a set of well-learnt geometric properties. With such an orientation, construction tasks are far less common. If, fortunate enough, they are included, Test Items 2 and 3, for example, may be conceived as tasks that call for the knowledge *that* equal base and equal height should give equal area, and the knowledge of *how* to use a line parallel to the base to locate vertices at constant heights. However, if these are precisely (and only) what we are looking for, one can easily notice that there is no point putting the tasks in this context of manipulation with imprecise measurements and constructions. Why do we not provide traditional construction tools or, if technologically-inclined, a geometric construction software that, to say the least, can readily construct a parallel line through the point A (in Figures 7 and 8)?

We should thus clarify ourselves that in the above analysis and exposition, we are trying to pursue the question of what is to be assessed from a different perspective. To put it shortly, we are more concerned with whether students are able to make use of the dynamic geometric tool in achieving certain purposes (i.e. to act purposefully in this new environment) than with whether students are able to reach certain correct results. This echoes the problems of mismatch we have posed above. If we believe that dynamic geometry environments are supportive and beneficial to students' learning of geometry, we should try to ensure that our students are able to make use of this dynamic geometric tool in the first place. (And this is desirably to be done when the students are in their lower levels of study.) Otherwise, further geometric studies mediated by this dynamic environment may not be as fruitful as expected. For instance, as Arzarello et al. (2002, cited in Jones, 2002, p.19) pointed out, experimenting on the geometric objects by exploiting the dragging facility is not as obvious and natural as one may wish.

Except for an opportunity to see whether students have the capacity to make use of such DG tools, we are also aware of the situation that most of our students, and perhaps teachers as well, are driven by the assessment. This backwash effect accounts partly for the above-mentioned problems of mismatch. If assessment tasks are prompting for exploratory and investigative skills in using the visually based environment granted by a DG tool, there should be a better chance for the students and teachers to pay careful attention to the development of such skills.

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