# Initial Integration of CAS Calculators in Teaching: One Teacher's Approach

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**Abstract:** The use of computer algebra system (CAS) calculators in the learning of school mathematics is still relatively in its infancy. While there is enthusiasm in many quarters for the possibilities that the technology opens up, there is still much research to do on the possible influences of CAS calculators on curriculum content, pedagogy, and student instrumentation of the tool. This research addresses one aspect of this, namely the role of the teacher, and how the CAS may influence their pedagogical approach. It involved a case study investigation of how a secondary school teacher with no previous experience of a CAS calculator, neither personally nor in their teaching, began the process of integrating it into her teaching of year 13 (students aged 18 years) algebra and calculus. During the implementation the teacher kept a diary of her lessons, we sat in on some of these and videoed her teaching, and she completed a brief questionnaire on the experience afterwards. Using the data collected we describe some of the issues that arose in terms of the didactic contract she constructed in the classroom, and the qualitative ways in which she employed the CAS as a teaching tool. We also address the variety of interactions of students with mathematical representations that her approach provided. The results show that the teacher became very comfortable with using the technology, embraced the use of the inter-representational thinking, attempting to use it to teach conceptually, and was enthusiastic to expand further her use of the CAS in her teaching. While this teacher would readily admit that she has much more to learn about implementing CAS calculators in her teaching, there is evidence that, with the right professional development and support, beginning the process of using CAS in a conceptual manner need not be overly daunting for teachers.

## 1. Background

One of the key factors in the use of computer algebra system (CAS) calculators in the learning of school mathematics (or indeed of tertiary mathematics) is the role of the teacher. Their attitude to the technology, their confidence and ability in using it to teach mathematics and their perspective of the mathematics being taught are all important in obtaining a successful outcome (see [3]). In this paper we consider the emerging use of CAS calculators by an experienced teacher, and analyse the approach used.

According to Brousseau [1], a key factor in the dynamics of learning in the secondary classroom is a recognition that the teacher-student relationship has *reciprocal obligations*, and hence in this respect it resembles a social contract, which he calls the *didactic contract.* This usually unwritten contract, which may be explicit or implicit, formal or informal, defines the roles of both teacher and student in the classroom. The contract begins as a dynamically changing entity (see [5]) as teacher and students react to or accept what happens in the classroom, and eventually it becomes binding. The teacher, explains Brousseau [1], is a prime mover in the development of the contract, with the role of creating an appropriate environment for the acquisition of knowledge.

However, even stable didactic contracts can be subverted or severely challenged by the presence of a third entity, namely technology, in the classroom (see [16]). The technology, in whatever form, introduces new relationships with both teacher and student and alters the existing teacher-student contract. The teacher's relationship with the technology will be based on a range of factors, including her/his view of the mathematics (see [2]), and her/his attitude to the technology, including their perspective on its usefulness. Doerr and Zangor [4] have categorized six different ways in which a calculator may be a useful tool, namely: property investigation; computational; transformational; data collection and analysis; visualizing; and checking. They also add a fuller explanation of the role as a transformational tool, describing its use to: develop visual parameter matching strategies to find equations that fit data sets; find appropriate views of the graph and determine the nature of the function; link the visual representation to the physical phenomena; and solve equations. Thomas and Hong (see [13]) have also described some categories of student CAS use that they identified: direct, straightforward procedures; direct complex procedures; checking procedural by-hand work; procedures within a complex process; and investigating conceptual ideas. Not unexpectedly they report finding little of the last kind of activity, and a lot of the first three. Teachers using CAS in the classroom have to be both aware of the possibilities provided by the technology, and confident in each of the roles they decide to implement.

Teachers also need to recognise that there are decisions they make that influence the students' relationship with the technology. Goos, Galbraith, Renshaw and Geiger [5] describe a hierarchy of student-technology interactions, beginning with the student as subservient to the technology, progressing to the technology as a replacement for pen and paper, or as a partner in explorations, and finally ending with technology as an extension of oneself, fully integrated into mathematical working. However, following their work with CAS, in a later paper they also report (see [6]) some student resistance to movement through this hierarchy, noting that they don't like just pushing buttons, and often wonder what have they actually learned from the CAS. This situation can be exacerbated by an emphasis on button pushing, since then as Tall (see [12], p. 35), explains "students learn what they do. If they press buttons, they learn about button-pressing sequences. What is therefore important is to build a sense of meaning through reflection on the underlying mathematics." These are other issues that the teacher has to address in negotiating an acceptable didactic contract.

Based on the analysis of Rabardel (see [9] and [10]) one of the key determinants of successful use of CAS in learning is actions and decisions that students make to transform the CAS tool or artefact into an instrument, by adapting it to a particular task. This process constitutes *instrumental genesis*, whereby a student, through use and application involving action schemes, considers what the tool can do and how it might do it. Rabardel and Samurcay [10] describe instrumental genesis as comprising two phase, instrumentalization, which involves the emergence and evolution of the instrument's artifact components, selection of pertinent parts, choice, grouping, elaboration of function, transformation of function, etc, and instrumentation, with its emergence and development of private schemes and the appropriation of social utilisation schemes. In other words, in the former the subject adapts the tool to himself while in the latter he adapts himself to the tool. While the teacher may provide a suitable environment, each student who uses CAS has to work out its role in their learning for themselves (see [7]). They have to learn to decide what CAS is useful for, what might be better done by hand, and how to integrate the two. As part of this process they will need to address: the differences between mathematical and CAS functioning; CAS use of symbolic notations and internal algorithm; the need to monitor the operation of the CAS (e.g., the syntax and semantics of the input/output, the algebraic expectation, etc); and to consider the difficulties of navigating between screens and between menu operations (see [15]).

A part of the process of instrumentation, the student adapting himself to the tool, is the nature of his/her interactions with the CAS representations. Thomas and Hong [13] present a qualitative analysis of how CAS can be perceived as an *observational tool* or an *action* or *activity tool*. In turn they propose that CAS representations may be either observed or acted upon, and distinguish between a range of procedural and conceptual interactions. For example, a student may observe from a CAS graph of $y = -x^2$ that the maximum value of the function is 0, what they call a *procedural property observation*. However, if the student can use the representation (and its associated functions) in a pointwise fashion to answer the question of whether the function's graph has a gradient of 2.35 and if so where, then Thomas and Hong (*ibid*) describe this is using it as a *conceptual process tool*. Acting on the representation to use it to solve the equation $-(x-5)^2 = 7$, for example by transforming the graph as a single entity, would constitute its use as a *conceptual object tool.* A teacher can only encourage a didactic contract leading to such conceptual uses of technology if they see it as more than mere computational or procedural tool; they also need to have the view that it is a teaching and learning instrument.

In this paper we discuss how an experienced secondary school teacher with no previous experience of using a CAS calculator began the implementation of CAS into her classroom. We examine the approach she took, her attitude to the CAS, some of its implications for her didactic contract, and the opportunities for mathematical thinking presented to the students.

## 2. Method

The research comprised a case study of Sharon (a pseudonym), who has been teaching mathematics for 20 years, mainly across years 9 to 13 (ages 14 to 18), and is currently a head of department in a large secondary school in New Zealand. Sharon initially volunteered to attend our professional development workshop over three weeks (3 sessions of 2 hours each) on using the TI-89, since she had no experience of it, and had only previously used a basic calculator before. This workshop covered both the operation of the CAS and provision of ideas on how it could be used to teach algebra and calculus. Following the workshop she agreed to take part in the research and was given a brief questionnaire on her perspective of the value of the CAS in teaching mathematics. We were then able to observe and video her teaching in the classroom, and she completed a diary of her teaching with CAS over a three weeks of teaching years 10-13. This included the mathematical content and her aims and objectives. The video was transcribed and all the data from the questionnaire, the lessons and the diary were analysed.

## 3. Results and Discussion

The results will focus in particular on Sharon's use of the CAS with her year 13 calculus class. There were 23 calculus students in this class, none of them had ever used the CAS calculator before, and each of them was supplied with their individual CAS calculator for the research period. Part of the expectations that a teacher has for the didactic contract is laid down in the layout of her/his classroom and the position adopted by the teacher. In this case the classroom was set up very traditionally with the students sitting individually in rows, as was usual for the class.

### 3.1 The Instrumentation Approach

One of the first things that a teacher new to using CAS, or indeed any other technology, has to decide is how they will structure its role in their classroom. This is a basic part of the addition to their didactic contract. Sharon chose to do much of her teaching to the whole class from the front of the room, using the TI overhead projection unit to project the screens of her calculator onto the screen. She primarily chose to maintain her control of the classroom situation by demonstrating at the front and getting students to follow her. This involved projecting her CAS calculator screen onto a white board using an overhead projection unit. In this way she was also able to overwrite the screen projection with data, such as the coordinates of certain points (see Figure 1). Many today would consider this too traditional an approach, and would argue that teachers should be seating the students in groups and facilitating group discussions and investigations rather than engaging in whole class teaching. However, in this research we were only interested in observing and describing the teacher's pedagogical approach and outcomes, with no intervention in mind, and hence we did not seek to change her style. The work covered over the period included the trapezium and Simpson's rules for approximating integrals, definite integrals, use of antiderivatives, and graphs of more difficult functions.

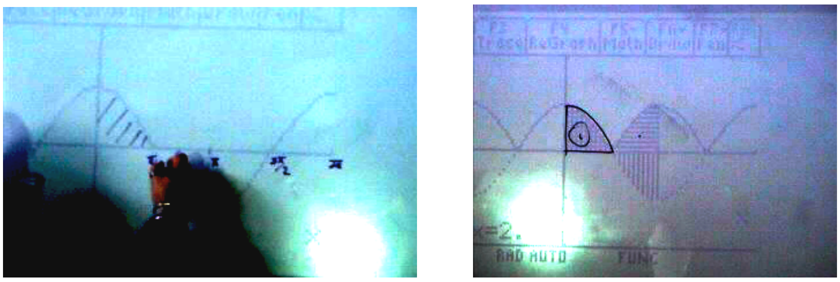

**Figure 1.** Using the whiteboard to overwrite the CAS screen projection.

She began the issue of assisting students' instrumental genesis by addressing their lack of familiarity with button pressing and menu operation. Initially she gave them directions on how to clear the memory and screen:

> Okay, so clear to your Home screen. Just turn it on. Clear your Home screen by going F1 8 and clear. Green F1, same deal. F1 8 and back in the Home screen. You might as well do F6 because you're orange in F1, so clear up from A to Z. So that means that anything that you may have put in before is now cleared up.

Other direct operational instructions on use of the CAS followed, as necessary, during the early part of the lesson:

> … you want to integrate the values, and to do that you go to F3 on your calculator. There you have your calculus-type things: differentiation, integration, limit. It goes all the way down to C, that is where you do all your differentiation equation-solving which we are *not* going to do today. Basically, that's all your calculus kind of stuff.

> Catalogue, and if you go to alpha A, you'll be in the As. If you go Alpha W you'll be in the Ws. Okay so that's how you go up and down. So get yourself back into Alpha A because we're going to be using absolutes.

> So the way to copy very handily, just use your 'up' arrow and highlight the integral in the first place, if you press enter it will repeat it down below. So you can pick up anything from above by highlighting and press enter. It will bring it down.

> If you want to go and change your scale, go F2 and something like *standard*. Get number 6, gets you ten, ten, ten, ten and you'll have to do your integral again.

Sometimes the instructions were very brief, such as making the crucial distinction on the TI-89 between the use of (–) for negative and – for subtraction.

> If you're having problems it's the negative at the bottom and not the minus sign. So, you've got to be careful don't use the minus sign for the negatives.

These instructions also extended to operational instructions on how to carry out some mathematical operations. For example, when considering $\int(x^2 - 2x)dx$, she said:

Put in what you're integrating *x* squared minus 2*x*. Now you have to tell it what to integrate with respect to. So then it's d*x*, means you're going to comma *x*.

and followed up with another example, performing the operations as the students follow (as shown by her words 'close *my* bracket'):

Alright, we'll try the next one. F3 integrate. You're gonna do *x* cubed minus 5*x* plus 1. What is your d at the end? You write comma *x*. Have you any more numbers? No, so we'll have C. And close my bracket, otherwise it's not going to work.

We note here that since the adding of +C for the indefinite integral is not something the CAS does, the issue is addressed within the process of teaching the CAS commands. Essentially she says, there are no limits to enter in this question so we can close the bracket now, but don't forget that when you write this answer by hand you will need to add +C. Definite integrals were similarly dealt with, and linked to the by-hand +C. For example, for $\int_0^2 3x^{\frac{5}{2}}dx$, which is entered into the CAS as $\int(3\sqrt{\ }(x\text{^}5), x, 0, 2)$, she said:

So definite rules are exactly the same except instead of here, the C is going and because your calculator is reading, its usually from left to right. So if I say I want to go to the zero here, you need to feed in zero, comma and 2, close the bracket because it goes left to right. When you press equals it will come up the right way around okay.

This instructional style of mathematical presentation was also observed when the students were using the graphical representation of a function, not just on the algebraic Home Screen. In this case, a different technique for finding a numeric approximation to a definite integral using the graph.

Now magically, in F5 where there's lots of maths things.. Press F5 and down there at number 7 there is Integral. It integrates *f(x)* to the d*x*. On your graph press Integrate. What was the limit? We were going between 1 and 2. Press 1, enter, then upper limit, then 2, enter. And there is the integrated area that you are actually finding.

However, Sharon soon seemed confident that the students had quickly picked up many of these basic ideas, and by the end of the same lesson she was able to give quite different instructions:

Okay, who's got the answer to the first one? Have you integrated? Done your integral from pi and zero? What did you get? Integral between pi and zero? Go back to Home Screen.

You know that we write sine cubed *x* that is sine *x* to the power of 3. So that's what we have to tell the calculator. Ok, so you're going to have to key it in. This is five sine five times sine times cos *x*, so keep your scales like they were. Right, I'm going to leave you to graph that. Graph it. I would suggest that you should tidy up that screen. Ditch everything that you've done so far, clear your *y* screen so you haven't got a heap of other stuff and just do these two on your own.

This last set of instructions now has the students performing a number of operations alone. They have to enter the function $y=5\sin^3x\cos x$ in the form $y=5(\sin x)^3\cos x$, clear the screen, graph it, and then calculate two definite integrals, 'do these two on your own'.

### 3.2 The Mathematical Approach

While, as we have seen above, Sharon felt it necessary to approach the students' instrumental genesis quite procedurally, it would not be correct to assume that she was approaching the mathematical content this way too. Rather than concentrating on getting her students simply to reproduce procedural results, Sharon adopted into her didactic contract the requirement that students consider a conceptual perspective on some of the mathematics. She described one year 13 lesson in her diary as "revision of integration concepts of more complex graphs.", and for another lesson she wrote "…rather than just doing question with no concept of what is being calculated." Furthermore it seemed that she felt able to do this because of the power of the CAS, where "students were able to graph far more examples that they usually can without calculator."

One major example of this conceptual emphasis was a lesson in which she had the goal of helping students to see the difference between the value of a definite integral and the area between the graph of the function and the *x*-axis. Sharon used $\int_0^{\pi} \cos x dx$ in order to achieve this, and was able to ask the students to do this integral themselves:

> [Writes $\int_0^{\pi} \cos x dx$ on the board—See Figure 2a] Now the question is, I want you to graph it and find the integral, and find out everything you can about it. Now the question is, if I give you that question, what do you get? If I give you area bounded by cosine $x = 0$, and $x = \pi$ what's your answer then? [writes Area bounded by $\cos x$ $x = 0$ $x = \pi$—See Figure 2a] OK two questions. Off you go.

First, however, she did get them to change the scale on their graphing window:

> And if you want a nice cosine curve? Now before you do this one I'm going to make sure you change your screen. So shift, same function window, green F2. Right, because we are going to do trig, it would be nice to have it in radians, and not 1, 2, 3, 4, 5, 6. So we usually have our axis from minus 2 pi, and if you feed in negative 2, and press enter. If you do that you get a wonderful decimal negative 6 something, which is 2 pi.

The resulting screen is that shown in Figure 2b. She then asked them to calculate the value of the integral using the CAS on the Home Screen (see Figure 2c), where they obtained the answer 0.

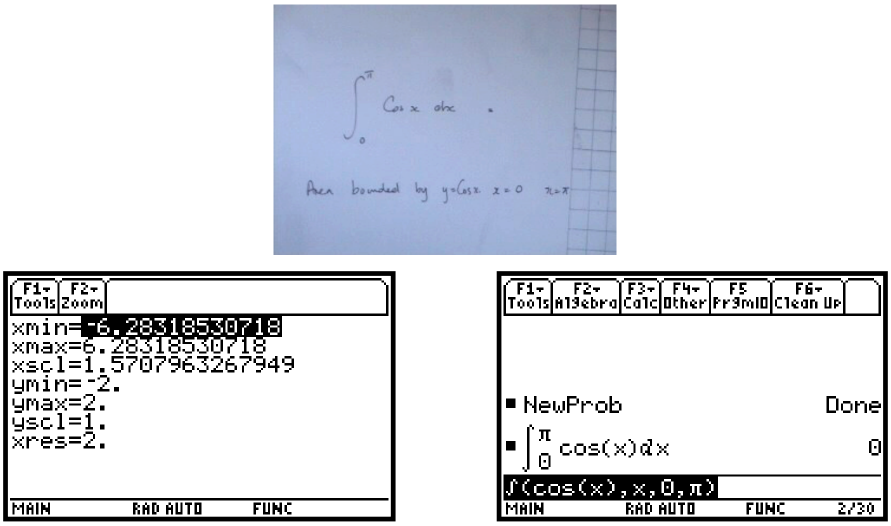


**Figure 2.** The comparison question; changing the graph scale and finding the value of the integral.

She then checked with them on their findings.

> Yes, it is zero. The answer to this is zero. Now did you do that on the Home screen? If you did it on the Home screen you would just get the answer zero. If you did it on the graph, you would have had a pretty shading come with it.

Clearly the students could see from the graphical representation used that the area for the function *f(x)*=cos *x* from $x$=0 to $x$=π is not zero, and so a discussion about why this is the case ensues.

> [Pointing at the screen—See Figure 1a] This is pi over 2, this is pi, this is 2 pi. So you've taught it to go from zero to pi. So it's just the difference between the 2 questions. If the question at the beginning of the [examination] paper says do this [ $\int_0^{\pi} \cos x dx$ ], you were just integrating, you don't care whether it's above the *x* axis or below. And you can see, here this area exactly cancels out that area [pointing to the shaded areas–See Figure 1b]. So plus anything, minus anything is going to be zero.

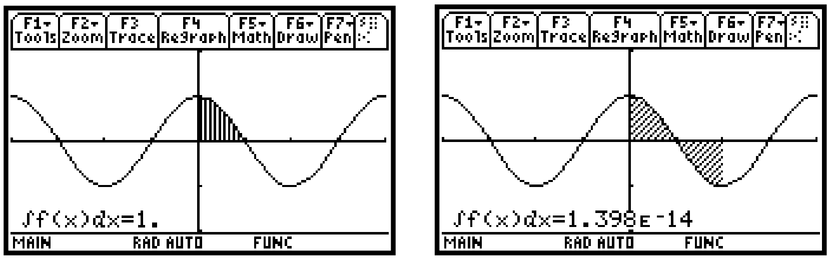


**Figure 3.** Calculating the area using the graph screen.

While it is true that the value is not (usually) exactly zero on the CAS even though the first area is approximated to 1 (see Figure 3), Sharon is here making a valuable link between the algebraic representation and its exact answer for the integral, and the graphical representation, with its approximate value for the area.

Having established the conceptual idea that a definite integral does not always gives us the area we require, due to intersections with the axis, it would be easy to leave the point there. However, from her workshop training, Sharon has realised that the CAS gives another opportunity, and she takes this up. To do so she leaves the current problem for an interlude on absolute value, something she is clearly excited about herself. This is an aspect of her didactic contract too. She has an expectation that her students will share the feelings of excitement in the mathematics that she feels.

> Now we're going to do something very exciting and we're going to draw that absolute value graph. So go to the Y= screen. Go to your Y screen. Now leave cos there, use your tick F4 to turn anything else off. Now go to catalogue: absolute cos *x*… If you're graphing absolutes, it'll get you to graph $y = |x|$. If you've got *y*=*x*, you've got 45 degrees, what does $y = |x|$ look like? It is your happy 'V' that gets reflected up here. So can we in our brains have a picture before we press the magic button?

Here she uses a well-known function $y = |x|$ to get the students to think ahead about the result of using the absolute value of cos *x*. Doing this they obtain two graphs on the CAS screen (see Figure 4).

> So first it will graph the cosine for you, then it will graph on top of it. Now if we want to look at that one harder, I will make my original one, dotted. So if we go to F6, we can make the original one dotted, graph it again so the original cosine curve you get will be dotted, and there, solid over the top is your absolute…Integrate lower limit zero, upper limit pi, and it's doing those two, right. So if you know that, what's this area here? Just 1, because 1 plus 1 equals 2. 1 minus 1 equals zero.

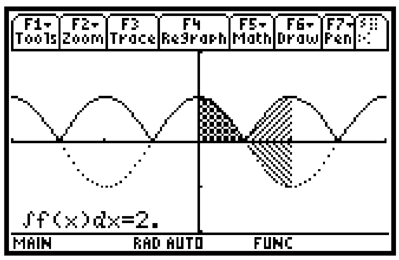


**Figure 4.** The use of the absolute value function for finding areas with CAS.

This method (the resulting screen is shown in Figure 4) of obtaining the exact value of the area between the graph and the *x*-axis is directly attributable to the CAS, and is not available as a by-hand method. Hence it forms a crucial part of the techniques needed for proper instrumentation of the CAS (see [7]).

Later Sharon used a question on a definite integral to bring in the concept of odd and even functions. After discussing a question to calculate the area of the region bounded by the graph of $y=5\sin^3 x \cos x$, the $x$-axis and the lines $x = 0$ and $x = \pi/2$, she introduced the concept:

> Right, now this brings us on to something about sine cubed. You know that we write sine cubed $x$ to the power of 3. So that's what you've got to tell the calculator…So F5, down to number seven, your lower limit zero and your upper limit pi divided by 2. Takes a while. It says it's busy down at the bottom, don't get impatient. It's doing a lot of work for you, just be nice to it.

Here we see an aspect of the instrumentation and the didactic contract. Sharon is encouraging a positive relationship with the CAS, talking about it as if we have to converse with it, 'So that's what you've got to tell the calculator.", and also that we have to form a friendly, positive working relationship with it, "don't get impatient", and "It's doing a lot of work for you, just be nice to it." Next the integral was calculated as 1.25 (see Figure 5), again using the graphical (approximate) mode.

> So you've got the area 1.25. Now we can change this into lots of sorts of questions. They could tell you that that area is that. They could ask the integral between here and here. The straight integral between there and there is going to be. Now try it with pi. Try to integrate it between zero and pi.

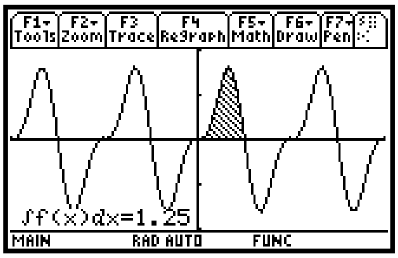


**Figure 5.** A graphical representation approximation of the integral of $y=5\sin^3 x \cos x$, $x=0$ to $x=\pi/2$.

Sharon then uses the fact that the integral from zero to pi is zero as a starting point for the discussion on rotational symmetry (or what she calls point symmetry—actually an integral from $-\pi/2$ to $+\pi/2$ would probably have been a better option here too).

> Have you got zero? How's it cancelled out? … Or change this so that they both have point symmetry. Now if they said, that this is similar and taken to be an odd function—an odd function means that it does have that point symmetry. Odd functions will exactly match that. So if they were to say to you is this an odd function, you'll say no. Because you have not got this area the same as that area. It may look like it, but an actual fact no it is not of because those two are not exactly the same.

Here the fact that a graph with symmetry about the origin has equal areas, but opposite in sign, is used to motivate the concept of an odd function. In order to complete the picture an example of an even function was considered, namely, $y=\sin^2 x \cos x$.

> Find the area between one arc of the curve $y=\sin^2 x\cos x$ and the $x$–axis. Pretty boring question, but a more complicated question might ask "Find the area and prove whether this is odd or even or.. what can you say?"

The graph in Figure 6 was considered with comments related to the concept of its period, without actually answering the question of whether it was even or odd for the students. This was probably because the lesson was near its end.

> Let's just have quickly a look at what's the period while we've got this up here. Is it a completely a different thing? What's the period? How long does it take for one complete cycle?…so you are going to have to have a period of two pi, that's how long it is long it is going to take to be sure.

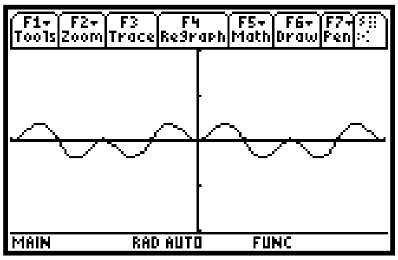


**Figure 6.** The graph of an even function $y=\sin^2 x\cos x$

With a different, year 11 class, Sharon considered lots of examples and she commented "I would either show graphs & ask for equations or show equations and ask for graph". In this way she was able to use the CAS to allow students to generalise transformations of the form $y = A(x + B)^2 + C$ applied to the graph of $y = x^2$. She wrote in her diary that "Students were able to graph far more examples than they usually can without calculator—so they became more sure about transformations of all graphs." It was clear that she saw the ability to generalise, whether it was for odd/even functions or graphing quadratic functions, as a very important part of understanding mathematics that she wanted to impart to her students. This also formed part of her didactic contract with her students.

## 4. Conclusion

What can we deduce from this study of Sharon's teaching with CAS? Firstly, we can see that it is possible for an experienced mathematics teacher, with no classroom experience of CAS teaching, to accommodate the CAS into her didactic contract. In this case one embracing quite a traditional method of teaching. She was able to include within the bounds of her contract a method of helping students cope with facets of their instrumental genesis of the CAS, especially with the use of buttons and menus. She was able to have as an aim to get students to consider the CAS as an instrument that could assist them with conceptual understanding of mathematical ideas, and not just as a black box procedural tool. As part of this she employed, and welcomed, novel CAS methods, such as using the absolute value of a function, as part of the techniques her students required for CAS instrumentation. Another method she was keen to use in this process was to get her students to generalise by abstraction of properties from multiple examples. When we asked her what advantages she saw to teaching with CAS she wrote "Clear/well organised menus. Great hopping from screen to screen. Table/graph/sequence/functions." Thus another value she saw in the CAS was the ability to move between multiple representations of concepts. Such representational fluency has been described as a very important part of mathematical understanding (see [8]), and she used the CAS to enhance it.

How did Sharon summarise her progress in teaching with CAS? Following the initial professional development workshop she wrote on her questionnaire that she "Went from zero to hero!!! Feel very comfortable using it—even with students." Her level of comfort was also shown in the summary she wrote in her diary after the three weeks teaching with CAS in the classroom, it said, in part:

> Most days at least once or many more I was able to use [the] calculator to explain different problems for students. Each day I was able to see more & more use I would have for the calculator and OHP… Having the calculator with OHP allowed me to see how much use I could get from this technology. But now I need more lessons too!

Sharon was not just comfortable with the CAS, but perhaps even more important, she had gained an enthusiasm for its use, and was able to see further applications of it for herself. We believe that what had started her on this personal voyage of discovery was our professional development workshop she attended, that gave her not only the CAS skills she needed but also a range of practical ideas that she could try in her own classroom, along with the support to do so. Such professional development is a continuing need for most teachers, and we consider it to be a vital area that should be focussed on if technology use is ever to fulfil its true potential for learning in the mathematics classroom.

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