Enabling Teachers to Perceive the Affordances of a Technologically Rich Learning Environment for Linear Functions in order to Design Units of Work Incorporating Best Practice

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Abstract: A technology rich teaching and learning environment affords new ways of engaging students in learning mathematics. Teachers and students equally have to learn to become attuned to the affordances of technologically rich learning environments. This paper reports preliminary findings from an Australian Research Council funded linkage project. This three year project aims to enhance mathematics achievement and engagement by using technology to support real problem solving and lessons of high cognitive demand. A design research methodology is being used to develop lesson sequences incorporating best practice. Findings from the first cycle of design research for the development and implementation of units for teaching linear functions in Year 9 will be reported. These are from classes in two schools where quite different approaches were taken to the teaching. A selection of electronic technologies was available to the teachers including graphing calculators and laptops with the image digitiser, *GridPic*. Student responses are presented to a hidden function task. These responses are discussed together with the results of document analysis of teacher work programs and student workbooks, and analyses of student and teacher interviews, lesson observational data, video tapes of student task solving, and verbal reports by teachers of their practice. The purpose of the research is to establish what it is that enables teachers to perceive, attend to, and exploit affordances of the technology salient to their teaching practice and likewise for students in their learning about function.

1. Introduction

The purpose of the present research is to contribute to an empirically grounded theory of the teaching and learning of functions in secondary school classrooms where there is a technologically rich learning environment. This study is part of the RITEMATHS project, funded from 2004-2006 by the Australian Research Council Linkage Scheme. The project involves The University of Melbourne and the University of Ballarat in partnership with Texas Instruments (TI) and six schools. The aim is to enhance mathematics achievement and engagement by using technology to support real world problem solving and lessons of high cognitive demand in secondary mathematics classrooms. Discovering how best to use technology is currently a priority research theme in mathematics education around the world (English, 2002) and in the RITEMATHS project. In particular, it is assumed technology can be beneficial to student learning and most effectively used to promote higher-order thinking (Wenglinsky, 1998).

The methodology used in this research, and by the project overall, comes under the umbrella of design research where design is used as a strategy for developing and refining both theories and practice (Burkhardt & Schoenfeld, 2003). Design research methodology is being used to develop lesson sequences incorporating best practice. Findings from the first cycle of design research for the development and implementation of units for teaching linear functions in Year 9 will be reported here. These findings are from classes where quite different approaches were taken.

Within the RITEMATHS project two forms of technology, *real-world interfaces* and *mathematics analysis tools*, are being utilised. The first of these, for example, software enabling analysis of video and data logging devices, can bring a virtual world into the classroom for analysis. The second form of technology, the mathematics analysis tools such as graphing calculators and spreadsheets, do computational work whether it be algebraic, graphical, or numerical nature.

Various technologies are available to the classes in this study. Many affordances[1] that would be useful in the teaching and learning of function are offered by these technologies. However, the realisation of any affordances depends not only on "the technological tool, but [also] on the exploitation of these affordances embedded in the educational context and managed by the teacher" (Drijvers, 2003, p. 78). To empower teachers and enable students to take up these affordances, both must "learn to perceive a perceivable affordance, that is, learn to become attuned to" (Scarantino, 2003, p. 954) what specifies it. From a research perspective it is necessary to establish what cues exist in technology rich environments enabling the realisation of offered affordances.

2. Functions and Graphing Technology

Without doubt, functions have an important place in the secondary mathematics curriculum (Yerushlamy & Shternberg, 2001). Marjanovic (1999), whilst discussing reform in the high school mathematics curriculum during the first half of the twentieth century, proposes "the most important innovation at that period was the introduction of the concept of function into the secondary school, emphasising the dominant place of that concept in contemporary mathematics" (p. 43). Graphing technologies provide more than alternative ways of doing the same old things; they allow us to do new things, including accessing and linking multiple representations almost simultaneously (Kaput, 1992; Pea, 1993). The production of graphs has changed from a time consuming and often difficult activity to a simple and quick occurrence, whether one begins with an algebraic or numerical representation. However, the role of teachers is vital as it is they who "must make prudent decisions about when and how to use technology and … ensure that the technology is enhancing students' mathematical thinking" (National Council of Teachers of Mathematics, 2000, p. 24).

Noss and Hoyles describe tools as "active" (1996, p. 58) and suggest that "for a tool to enter into a relationship with a user, it must afford the user expressive power: the user must be capable of expressing thoughts and feelings with it. It is not enough for the tool to merely 'be there' … it must enter into the user's thoughts, actions and language … . Expressive power opens windows for the learner, it affords a way to construct meanings" (p. 59). Use of a different tool does not, however, guarantee improved student learning. Mathematical technologies afford new ways of teaching and learning. In addition, the "potential to reshape the learner-teacher relationship is only part of the picture" (p. 6). The use of technology allows the researcher a new view. The technology "offers a window … . It affords a view of the meaning-making process, a glimpse of learning which is often difficult, if not impossible to catch. [It] provides a screen on which learners can express their thinking, and simultaneously offers us the chance to glimpse traces of their thought" (p. 6).

3. Methodology

The research methodology of the RITEMATHS project is that of design research experiments (Feuer, Philips, Shavelson, & Towne, 2003). Design experiment approaches "include significant efforts to change educational practices, generally with some innovative materials as well as a reorganisation of the activities of teaching and learning. They also include significant efforts to understand processes of learning and teaching in the situations where the new materials and practices are being used" (Greeno, 1998, p. 21). One goal of design experiments in general, and this research in particular, is of "obtaining information and understanding that can support changes in resources and activities that would strengthen [teaching] practice" (Greeno, 1998, p. 22). Design experiments operate through iterative cycles of preparation and planning, development of a design,

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[1] For a discussion of affordances see Brown, J., Stillman, G., & Herbert, H. (2004). Can the notion of affordances be of use in the design of a technology enriched mathematics curriculum? In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010, Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1* (pp. 119-126). Sydney, Australia: MERGA.

implementation, and evaluation and refinement involving the researchers and teachers as partners (Greeno, 1998). Intended research outcomes from this project include creating a set of design principles for effective learning environments which bridge abstract mathematics and the real world, exploiting the affordances of technologies; and an understanding of real-world software interface features that promote students' engagement and support mathematical thinking.

The purpose of this methodology is to "develop theories" (Cobb, Confrey, diSessa, Lehrer, & Schuable, 2003, p. 9). The situation reported here involves classroom experiments where a research team works collaboratively with teacher partners in the search for "supporting new forms of learning in those specific settings" (p. 10) to develop increased understandings of "the process by which students develop a deep understanding of particular mathematical ideas, together with the types of tasks and teacher practices that can support learning" (p. 10). Teacher participation as a co-designer, implementer, and evaluator in a design cycle based on theory and practice is the key to "confronting everyday classroom, school, and community problems that influence teaching and learning and adapting instruction to these conditions; in recognising the limits of theory; and in capturing the specifics of practice and the potential advantages from interactively adapting and sharpening theory in its context" (Feuer, et al., 2003, p. 25). Through this ongoing partnership and iterative cycles of design and implementation, "insights into the complexity involved in developing skills and knowledge, and … the role that teachers play in capitalising on the affordances of learning materials" (The Design-Based Research Collective, 2003, p. 6) are gained.

Phase 1: Preparation and Planning

At the time of writing two teaching experiments are in progress. These began after an initial phase of preparation and planning involving meetings of the research team, the partner teachers, and personnel from the technology supplier where theoretical frameworks (e.g., Stillman & Galbraith, 2003) were introduced. Collectively, the RITEMATHS team met for four days prior to the data collection described in this paper. These meetings were for one, two, and one day. They involved demonstration and use of various electronic technologies, including graphing calculators, CAS, data loggers, applets and dynamic geometry applications. There were opportunities for discussion, sharing, and reflection of teaching and learning ideas. One session involved participants considering: *Why teach linear functions?* and *How can we use the graphing calculator environment to do this?* All schools decided to focus on Year 9 initially. Two schools were to teach linear functions in the first half of the year with the remaining schools addressing this curriculum area later in the year. Within these two schools, data were collected from one class (referred to as School A and School B). From the initial meetings the following criteria for the selection of learning activities for a unit of work introducing linear functions became apparent: (a) the use of context, (b) technology use, and (c) emphasis on the importance of language. In keeping with the project focus, the unit must include activities involving electronic technologies either as real word interfaces or as analysis tools or both.

Phases 2 & 3: Development and Implementation of a Design: The Teaching Experiments

School A: One teaching experiment was conducted in a coeducational Years 7-12 regional secondary college in a large country town. One teacher, TA1, decided to implement a linear functions unit of work with one class in semester 1, 2004, and then evaluate and refine this for semester 2 with a second class. The unit was chosen by other project researchers but the teacher had freedom to implement it in a way that best suited the class. The experiment comprised approximately 20 single lessons of 50 minutes duration delivered over a 5 week period with eighteen Year 8 students (13 year olds) in a Select Entry and Accelerated Learning Program. The students are undertaking Year 9 studies in mathematics and other subjects. The unit consisted of pages 1-20 of the student edition of *Graphic Algebra: Explorations with a function grapher* (Asp, Dowsey, Stacey, & Tynan, 1995). The publication aims to "improve the teaching of algebra, … and encourage the use of computers and calculators in the teaching of mathematics" (p. vii). The students were to be introduced to graphing calculators as a mathematics analysis tool for the first time during the experiment. The focus of the material was to "develop student understanding of function; use graphs to teach properties of functions; use graphs to solve problems from realistic contexts; link different representations so that students learn to move easily between tables of values, algebraic expressions, and graphs; and to teach concepts and skills needed for computer or calculator graphing" (p. ix).

Prior to utilising the text, the class undertook one week of line graph interpretation from their usual text. This included plotting coordinates including those for functions such as $y = x + 1$ as well as interpretive tasks involving reading linear graphs to answer questions. The unit proper began with the situation where a girl is making lemon squash to sell at a school fete. The cup volume, selling price, and price of ingredients are provided. Students are asked questions such as 'how much profit will she make if she sells 23 cups?' Students then complete a table of values and plot the resulting points on graph paper. They are told the points are linear and to draw a line through them. Questions are then asked about the information provided by the graph including via extrapolation and the consideration of negative profit. The equation of the function is then given and students graph this with their graphing calculators and use its various features to answer questions, some identical to those already answered by hand. Using graphing calculators students produce graphs of functions to answer further questions and compare profits. Similar problems are presented involving mobile phone charges, international shoe sizes (attempted by only some students) and arm span versus height (where class data collected is used as the starting point rather than worded/algebraic information). Finally, students are provided with graphs of linear functions in a particular viewing window that create a pattern. Students use their graphing calculator to reproduce the given creation using linear functions. This is extended to include functions with restricted domains. Teacher TA1 omitted the section, *Linear function graphs and constants* where the general form of the linear function is addressed. Students had access to the graphing calculators only during class time. Their homework was unrelated to the topic which is the usual practice in this school.

School B: The other teaching experiment took place in a coeducational Years 7-12 private college in an outer suburb of a large Australian city. This school has 4 Year 9 classes and 2 teachers currently involved in the project. Both teachers, TB1 and TB2, worked collaboratively to develop the linear functions unit for 2004. This unit built on their previous work and collaboration with researchers in the RITEMATHS project. Data were collected from one class consisting of 25 Year 9 and 4 Year 8 students taught by teacher TB1. School B students have used laptops from Year 10 for some time and this has been extended to Year 9 this year. Ownership of TI-83 graphing calculators is expected in Year 9 and students have used them since the beginning of the year.

TB1 and TB2 in their focus on utilising technology in high cognitive demand tasks developed learning experiences around the following cycle: question, data collection, data interpretation (numerical, graphical, algebraic), response, new question. This cycle was the organising principle of their initial activities as the understanding of task complexity became an important issue for them. Prior to unit implementation, both teachers worked together to increase their skills and understanding of various educational technologies. This included extension of a dynamic geometry application to create dynamic applets which have been implemented as part of the Year 9 mathematics program prior to the beginning of the linear function unit.

In addition, teachers TB1 and TB2 commissioned the development of an image digitisation program, *GridPic* (Visser, 2004), which they have incorporated into their linear functions unit. This program incorporates the ideas of digitising images and data collection in a way that could be best integrated into the teaching practice at this school. The application goes "beyond the blackbox approaches of finding coordinates and the regression equations" (TB1). After selecting a picture (e.g., a stairway with rails, see Figure 1), *GridPic* allows the user to click on any point on the image. The coordinates, with reference to an overlaid Cartesian axes system, are automatically listed in a table of values. To find a model of the set of points the user selects from ten options including straight line, quadratic, and vertical line. A general algebraic equation for the selected model appears. The selection of straight line gives $y = ax + b$ and allows the user to enter values for either or both of a and b and then select "show line". Using guess, check, and refine the user can determine a function that best fits the points. In addition, the picture can be hidden by the user to produce a view similar to a graphing calculator screen. The use of *GridPic* "had a profound effect on the way in which the topic was presented" and "raised the importance of the visual and tabular information as aids to understanding the concepts of the algebraic representation" (TB1).

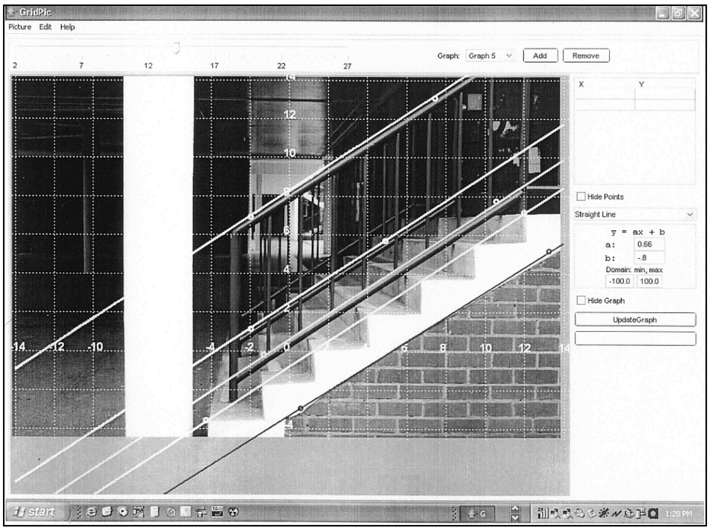

Figure 1. A stairway with rails imported into *GridPic* with several linear functions displayed.

The teaching experiment comprised six single lessons of 50 minutes and three double lessons over a four week period towards the end of Semester 1, 2004. Activities using *GridPic* (initially focusing on a range of photographs of stairs) included identifying picture features located at given coordinates, determining equations to represent other features (e.g., stair rails), considering the number of points required to determine a line uniquely, specifying if a set of lines were parallel or intersecting and in both cases the relationship between their equations. Students were asked to suggest a method, other than refinement of guessing, that would suggest the slope of a line, measured by a in the equation of the line. Particular emphasis was given to recognition of any given line as having a negative, positive, or zero gradient, knowing that both the gradient and the y intercept needed to be identified in order to find the equation of a given line. Various methods for determining the gradient were used with no method being given higher status than another. Students were expected to be able to understand any method used by other students. Although *GridPic* was used initially as a real world interface, the use of graphing calculators as mathematics analysis tools, and by hand methods were integrated seamlessly throughout the unit.

Phase 4: Evaluation and Refinement

Once the teaching experiments were completed in the two classes, students working in pairs were asked to complete a task called *Hidden Function* using one graphing calculator per pair. The task was designed by this researcher. Data were collected in the form of written scripts in response to the implementation of the task, video recordings of one student pair attempting the task from each school, classroom observations and field notes made during and after the task, and audio recordings of the subsequent post-task student and teacher interviews. In addition, classroom observations by this researcher in School B, teacher reports to the RITEMATHS team meetings, retrospective reports by other research team members who observed lessons in School A, and one student workbook from each school supplement these data in the retrospective analysis that follows. Bardini, Pierce, and Stacey (2004) report other findings from School A.

The *Hidden Function* Task: The broad aim of this task is to determine student understanding of the links between the parameters and the linear function $y = ax + b$. The specific purpose is to ascertain student understanding of the relationship between two variables when information is available numerically but students are restricted to determining which sets of ordered pairs are known by selecting specific values for the independent variable using a graphing calculator. To support implementation of the task this researcher designed the graphing calculator program, *Hidden*, in order to generate random linear functions and ensure that the algebraic representation of these functions is not directly accessible (i.e., visible).

The task was undertaken as part of a regular mathematics lesson in both classes. At the beginning of the lesson, this researcher provided a broad description of the purpose of the task to be attempted and demonstrated the use of the graphing calculator using a ViewScreen. This demonstration included how to access and run the program, how to select the TABLE feature, and how to enter and display table values. Student pairs were provided with a Task Sheet and a Record Sheet.

Using the graphing calculator, each pair of students ran the program, *Hidden*. In addition to generating a random linear function, this program sets the TABLE SETUP so that both the independent and dependent variables of the random linear function are not automatically displayed, as shown in Figure 2. Each of these values needs to be ‘asked’ for. The user can enter any value for the independent variable, (in TABLE view, the user enters any value and presses enter). The value of the dependent variable is not automatically displayed. To observe the corresponding values of the dependent variable, the user moves the cursor to the desired location and selects ENTER.

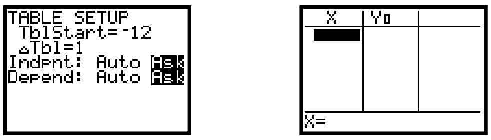

Figure 2. Settings for TABLE SETUP and resulting view of the TABLE.

When the program, *Hidden*, is run the user is presented with four options: Initial, Easy, Medium, and Hard. Selection of Initial/Easy results in a random integer from 2 to 8 (inclusive) being generated for the parameter a in the general linear function $y = ax + b$. A second random integer from 2 to 10 inclusive is generated for the parameter b. If Medium is selected integers in the range of -10 to 9 and -20 to 20 are generated for a and b respectively. The Hard option generates rational values for a in increments of 0.2 for the range -10 to 20 and for b in increments of 0.5 for the range -10 to 10. Reselection of the program and a new menu item generates subsequent functions. After the student pairs have correctly identified at least four hidden functions and are confident that they have a successful method for identifying the algebraic representation of any hidden function generated by the program, they are asked to explain this method in writing.

4. Retrospective Analysis

Student understanding of the general linear function

A number of approaches were used by students in determining the algebraic representation of the function in the *Hidden Function* Task. These methods related to the identification of a and b in the general linear function $y = ax + b$. A description of each category is shown in Table 1. In School A five of the nine pairs found a by recognising a 'going up by' pattern in the y values and using this as the 'multiplying' number in the equation. Three of the pairs at School A explicitly found the difference between the y values corresponding to consecutive x values. Another pair at School A systematically found the differences between two consecutive pairs of y values, again where $\Delta x = 1$. Twelve of the 14 School B pairs used a difference method to determine the value of a. Of these, three pairs found the difference $y(1) - y(0)$; four pairs found differences where $\Delta x = 1$, however, it can be inferred that $\Delta x = 1$ was not a requirement for their solution; and a further five pairs explicitly demonstrated their understanding that they could determine the value of a using any pairs of coordinates. One pair of this last category also used linear regression to identify one function and to check their result for another. Two School B pairs used substitution to determine the value of a using one set of values after first identifying the value of b. At both schools the most common method of identifying b was through recognition that b corresponded to $y(0)$. This method was used by four School A pairs and twelve School B pairs. At School A, three pairs used substitution (after identification of a) and two pairs used trial and error. At School B, two pairs used substitution with one of these pairs using formal written methods to undertake this process.

Table 1
Approaches to Hidden Function Identification: $y = ax + b$

Method	Selected Student Pair Responses to Hidden Function Task
Identification of b	
Trial and error	Then plus or minus until it is the right number.
Substitution	"Next we worked out the y intercept or (b) value by: substituting it in for the two points." They substitute (2, 12) into $y = 4a + b$ (sic) to find $b = 4$.
$y(0) = b$	"When you enter 0 in the x column, the y value that comes up is what b is in $y = ax + \underline{b}$.
Identification of a	
"Goes up by"	"Every time x goes up, y goes down by 4."
Substitution of any point, b known	Once the b value is found remove it from equation and y answer. Then work out the remaining number by dividing y by x. [i.e., $a = \left(y(x_n) - b\right) / x_n$]
Difference methods	
$\Delta x = 1$	"The jump is the gradient"
$\Delta x = 1$ but some evidence this is not necessary.	We then progressed up the ladder by ones to see the pattern in the y intercepts (sic). This then made it easy to find the equation because whatever y values increase by is the number you multiple x by."
Data collected shows explicit evidence that pair knows any two points can be used	"Firstly we worked out the gradient: To do this we worked out the y and x changes by the formula: $\frac{y_2 - y_1}{x_2 - x_1}$." [They find the gradient between (2, 12) and (4, 20), they then use the points (6, 28) and (8, 36) to confirm the gradient is 4.]
Linear regression	"I know it is kind of cheating but LINREG never lies!"

With respect to multiple representations in student approaches to finding the hidden functions, the predominant representations used were the numerical and algebraic. It was noticeable that the graphical representation was used in only two instances both at School B. This is somewhat surprising given in their previous work with graphing calculators this representation was commonly used. Perhaps task format contributed to this, especially for School A pairs where the major focus was pattern searching. Although Teacher TB1 was adamant that a multiple representation approach had been taken, the School B pairs even though looking for an algebraic rule tended to stay within the numerical representation. However, only three of the pairs made use of the numerical representation of their own volition beyond what was demanded by the task.

Use of Mathematical Language during the Hidden Function Task

With regard to the language of linear functions and in particular use of the terms *gradient* or *slope* and *y intercept*, clear differences are evident between the two classes. No student pairs at School A referred to either term in their solution script or post-task interview. This is not surprising as teacher TA1 reported not specifically teaching *y* intercept and slope but said some students were told about slope in one-on-one interactions. However, this teacher did introduce the term *domain* when using the graphing calculator in the extension to the *Creations* activity. For School B, nine student pairs referred to at least one of the terms and five referred to both. This suggests that students at School B were more likely to consider the generated data as the numerical representation of a linear function, whereas, students at School A tended to see the data as a numerical pattern that had a rule, as was confirmed in post-task interviews.

Teacher Reflections on the Hidden Function Task

Both teachers thought the task was appropriate for their classes to undertake. Teacher TA1 noted, "it was a good one to do. It was a bit like, here is the line, create the equation. Here's a set of numbers, look for a pattern and having them try to work out the equation." Teacher TB1 commented: "it fitted into the program beautifully, … the session continued the idea of think and think and apply". With regard to specific comments about the task, Teacher A1 notes, "it was from a different angle. Certainly not in context or anything like that." TA1 stated, "I think it fitted in very well with what they actually had been doing". For this class, "the table of values had always been horizontal across the page (however, the vertical tables) didn't seem to give them any trouble". TA1 continued, "it was interesting that … some of them still had not picked up that zero and one were fairly critical values to put in for *x*. Some of them, most of them, just picked up that you needed a sequence to actually get the actual multiplicative value, but they hadn't really picked up the zero for *x* was the important one to put in". The selection of values by the students in TB1's class bought the response, "we've discussed the importance of the zero value, not that they necessarily go there directly, they don't like zero as a number. They like to start at 1, 2, 3, 4. They think that zero is not a number". With regard to the task in general, TB1 stated that, "they have looked at tables of values as a means to identify whether or not points are on a line but they haven't gone the other way—here is the table of values, now let's generate from that". Both TA1 and TB1 made clear the intention to integrate use of the task into future teaching of functions.

Affordances of the Graphing Calculator taken up by the Students

Also of interest in the analysis were the various affordances of the graphing calculator in the solution of this task and the rate at which these affordances were taken up. Some affordances of the graphing calculator for this particular task and the apparent uptake (number of pairs) were to: (a) generate the hidden function using the program and table (all), (b) enter the predicted algebraic equation of the hidden function as y_1 and compare table values (3), (c) use the homescreen for calculations (several), (d) use the homescreen for storing or recording values (at least 1), (e) enter generated data in lists and determine the linear regression equation (1), (f) graph the hidden function (0, 2 by hand). As all pairs identified the hidden functions, the affordances not utilised were clearly not essential for success. However, the question remains as to whether the use of some of these may have enabled implementation of a more efficient method or way of justifying results.

Teacher Reflections on their Unit of Work

Teacher A1 was satisfied with the unit of work and will be implementing it with a Year 9 class with some changes. This teacher reported that the students "really liked the calculators" and will continue to use them in the unit. A lesson that specifically looks at the effect of the parameters of the function on the graphical representation will be added. When undertaking the *Creations* activity much greater emphasis will be placed on drawing together student observations after undertaking their creations and generalising and formalising the effect of the parameters on the graphical representation of the linear function. Teacher A1 wants the students to develop better understanding of links between the various representations of the linear functions.

Teacher B1 has the clear aim that students be able to "algebracise situations" and "to use solving techniques of graphical, numerical, and algebraic methods". In addition, they should be able to recognise situations (e.g., "where a linear intersects a quadratic") "which they cannot solve algebraically" and then use both the graphical and numerical representations to solve. Further, students are expected to consider the difference between approximate and exact solutions. This teacher is adamant that *GridPic* has affected the teaching approach. "*GridPic* has changed inherently the way in which I taught it … . The table is used now to generate data rather than to generate points and plot the lines". Furthermore, "we started with the visual, which *GridPic* allows, [then] with the numerical which they then tried to algebracise to that pattern". The positive response to this unit extends to the team of Year 9 teachers, "certainly the anecdotal evidence back from the other teachers is saying they are happy with the way it is pulling together". "They [the students] are just starting to algebracise away with what happens with turning off pictures and moving [to more abstract situations] but they still have that image in their heads. They still know what a positive gradient looks like, they don't have that issue. They firmly know the importance, I think, of the intercept". For TB1 the implementation of the unit of work meant, "I prioritise content differently. Starting from the visual means I have to change my teaching. The numeric table becomes more important … . The table becomes the transition … . The table links back to Year 8 work—using blocks [to find patterns] … . The table allows students to have a functional focus (y against x) and shows why [for example] the graph is negative". The School B teachers have exploited the affordances of the electronic technologies in their classroom so that the table has assumed a new role. Changes to this unit for its next implementation are of only a technical rather than pedagogical nature, for example, ensuring all student laptops have the required programs correctly installed. The next major focus for the teachers at School B is on the intersection of two lines and inclusion of contextualised examples where both left and right hand sides of the resulting mathematical equation are algebraic expressions.

5. Conclusion

This preliminary study is, as Labree (2003) describes, the beginning of the discourse as to what is. These preliminary findings, together with those of Bardini et al. (2004), are a small step towards attempting to gain greater insight into the thinking of the students in two classrooms and what the teachers of these classes want their students to learn when introducing linear functions. All students in this study were able to generate pairs of numerical values that allowed them to identify the hidden functions. However, for students to progress from pattern recognition to a full understanding of the general linear function explicit emphasis on "generalising what it means" (TA1) is needed. The use of a real-world interface to bring a virtual world into the classroom for analysis impacted on the teachers' conceptualisation of the task of teaching linear functions at one school. It was a catalyst for a reconceptualisation of the links between the numerical, graphical, and algebraic representations and how a teacher might use the affordances offered by this and other technology to alter the past emphases on particular representations. In particular, the numerical representation has taken on a more important role as "the lynchpin for the algebra" (TB1). Information gained from these two implementations of quite different linear function units of work will be used to generate the beginning of a hypothetical learning trajectory (Drijvers, 2003) for function in secondary schools emphasising multiple representations in a technologically rich learning environment.

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