

# The Use of Graphics Calculator in Teaching and Learning Transformations, Polar Curves and Matrices

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This is a sharing on how graphics calculators can be used in the teaching of 'A'-level topics both involving and not involving graphs. To explore graphing functions, specific examples are given using transformation of graphs and polar coordinates. With some scaffolding and guidance on calculator keystrokes, students are encouraged to explore how graphs can be translated, scaled and reflected. Students also learn to appreciate the beauty and diversity of polar curves. In non-graphing topics like matrices, an example on how graphics calculators can be used in application problems to stimulate students' interest in Mathematics will be discussed. Teachers' feedback and students' responses are also noted.

## Introduction

A revised 'A'-level syllabus and curriculum will be introduced in Singapore Junior Colleges in 2006. A significant change in the Mathematics syllabus is the incorporation of graphics calculators in assessment. Presently, the use of graphics calculators is allowed in the GCE 'A'-level Further Mathematics examinations but the questions set are to be graphics calculator neutral, meaning students will still be able to solve them without the use of graphics calculators. With the revised syllabus in 2006, examination questions may not be graphics calculator neutral. Hence, it is even more important that teachers and students prepare themselves to teach and learn effectively with graphics calculators.

Besides the fact that examinations are starting to require the use of graphics calculators, another motivating factor for the use of graphics calculators in schools should be the enhanced learning experience for students. Burrill (2002) examined eleven studies that compared students (ranging from ninth grade to university) who used handheld graphing technology to those who did not and concluded that the use of handheld technology had a positive impact on student performance. In particular, there is improvement in students' conceptual understanding and ability to apply mathematical concepts especially in the topics of functions, graphs and in solving word problems.

Another advantage of graphics calculators compared to other mathematical software like Graphmatica and Geometer Sketchpad is the accessibility of the tool to students. Lessons conducted using graphics calculators need not be confined to the computer laboratories. In fact, students can also be given guided worksheets to learn independently or in small groups at their own pace and time. With these in mind, the following sections will serve to provide specific examples on how graphics calculators can be used in the topics of transformations, polar coordinates and matrices. The commands and keystrokes described in the following sections are applicable to the TI-83 Plus graphics calculators from Texas Instruments.

## Transformations of graphs

This simple worksheet on transformations (Appendix 1) only require basic graphics techniques and thus can be used both in a big group lecture or in a classroom setting since there will not be much need for troubleshooting. If not all the students have graphics calculators, the teacher has the flexibility of conducting the lesson as a demonstration while students make use of the worksheet to note down observations and draw conclusions.

In their secondary schools (13-16 years old), students are exposed to single linear transformations such as  $y = f(x+c)$  or  $y = f(cx)$  but not a combination of both such as  $y = f(2x-1)$ . From the graph of  $y = f(x)$ , students need to perform a scaling of factor  $\frac{1}{2}$  parallel to the x-axis, followed by a translation in the x-direction by  $\frac{1}{2}$  unit. Many students find this idea hard to grasp and they tend to commit the error of translating by 1 unit in the x-direction after scaling. For the past 2 years, the different graphs were illustrated using a graphics calculator to lecture groups of about 120 students so students were able to observe what exactly happens during the transformations. In addition, graphics calculators make it easier to present many graphs and examples to students to help in their understanding.

From the teachers' feedback, students seem to make less mistake of the kind described above than before in assignments and tests. Here, the graphics calculator is used as a tool for students to learn as they tend to forget soon after hearing but seeing helps them to remember better.

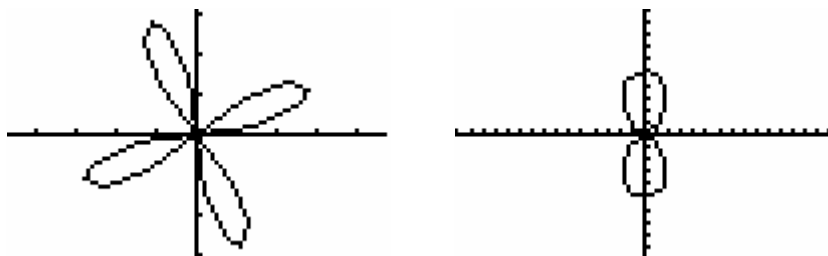
## Graphs of polar curves

Having been exposed to only the rectangular coordinate system in secondary schools (13-16 years old), students tend to have some initial difficulties in sketching polar curves. The worksheet in Appendix 2 is designed to provide some scaffolding for the students, first guiding them through the calculator keystrokes required to graph a polar curve and then letting them practice graphics other polar curves on their own.

The Mathematics lessons are conducted using a lecture-tutorial system where students are first taught a new concept in lectures (120-180 students) and problems are discussed in tutorial classes of 20 to 25 students. This worksheet serves as a 40-minute follow-up tutorial to 3 hours of lecture where students learn about polar coordinates, the techniques for sketching polar curve  $r = f(\theta)$  by making use of the graph of  $r$  against  $\theta$  and computation of area bounded by polar curves.

In the lecture, students were not exposed to cases such that  $r < 0$  for polar curves as this is not required in the A-level syllabus. During the graphics calculator lesson, students realized that the graph shown on their calculator for  $r = 2\cos 4\theta$  is different from the one they have done in the lecture. In groups of 3 or 4, the students tried to sketch different polar curves and brainstorm possible reasons for the difference in the curves. When they sketched  $r = \cos \theta + 1$ , they observed that the curve sketching method learnt in lectures produced a graph consistent with the one they obtained using their graphics calculators. Some groups were able to come up with the constraint that was considered in the lectures, that is,  $r \geq 0$ . At the same time, students also figured out how the graph is drawn in the cases of  $r < 0$ .

In the 40 minutes lesson, students were also encouraged to observe some of the distinct features of polar curves like number of petals, lines of symmetry and spiral curves and relate these features to the equation of the curve. As an example, figure 1 shows the solutions to Exercise 1(a) and (b) of the worksheet. Students can first observe how the number of petals in the graphs is linked to the equations of the curves. They can then observe when the tangents at the pole and lines of symmetry occur.



**Figure 1:** Graphs of  $r=3 \sin 4\theta$  and  $r=2-3 \cos 2\theta$  for  $0 \leq \theta \leq 2\pi$

Although the graphics calculator does not provide equations of the tangents at the pole or lines of symmetry but through observation, students were able to guess what these equations are. As the lecture was conducted beforehand, this serves to reinforce what was taught as this topic is totally new to the students, resulting in students finding this topic difficult and tedious to handle. This graphics calculator worksheet, together with knowledge on key features of polar curves, gives students the confidence they need to venture out and sketch the curves on their own.

Also included in this worksheet is the method of computing area bounded by polar curves using **fnInt**( feature of the graphics calculator. This allows students to check their computed answer against that of the graphics calculator. In fact, the **fnInt**( feature is applicable to definite integration problems in general and not only restricted to this topic.

One of the problems encountered in conducting the hands-on session was that some of the students were unfamiliar with the keystrokes and they ended up spending more time fiddling with the keystrokes than learning. This can be overcome by giving students the detailed worksheet (appendix 2) that guides students through the necessary steps required to sketch a graph before the lesson so students can familiarize themselves with the keystrokes.

In addition, due to the fact that the resolution of the graphics calculator is not high, some of the graphs seem disjoint. Students also realized some limitations of the graphics calculator such as the absence of asymptotes on the screen and distortion of graphs due to poor window setting or screen resolution. Thus, in this case, the graphics calculator serves as a tool that provokes thinking and encourages questioning from the students.

## Fun with matrices

The worksheet in Appendix 3 is adapted for the graphics calculators from an activity Evans & Johnson (1994) designed using Derive. This worksheet can be used as an introduction to matrices as it features ideas like matrix multiplication and inverse. However, it requires knowledge on modular arithmetic, which is outside the current syllabus. Thus, it will be good to use this as an enrichment worksheet to enhance students' interest in the topic. An application in the area of

coding is chosen, as many young minds are excited and intrigued by spies and breaking of coded secret messages.

The worksheet provides some background information on coding and guides students through encoding a message. In the process, students will learn how to define, recall and carry out operations on matrices. Their critical thinking skills are put into use in trying to figure out how to reverse the encoding process to decode a message. They will also have a chance to showcase their creativity in challenging their peers with messages they have encoded.

The role of the graphics calculators in this case is to speed up the tedious computation process of matrix inverse and multiplication so that students are free to understand and appreciate an application of Mathematics in everyday life.

## **Conclusion**

In addition to being a sophisticated calculator that helps students with their work, the graphics calculators can be further used as a teaching tool both in big group lectures and in a classroom setting. It can be used as a demonstration tool in lectures and a platform for individual or group work in classrooms. In the above examples, the graphics calculators were used as a visualization tool, a query-provoking tool and a tool to minimize tedious manual computation. In order for the usefulness of the graphics calculators to be fully exploited, teachers need to take a bold step forward to design, experiment and share lesson plans and worksheets using graphics calculators.

## **References**

- [1] Burrill, G. (2002). *Handheld Graphing Technology in Secondary Mathematics: Research Findings and Implications for Classroom Practice*. USA: Texas Instruments.
- [2] Evans, B & Johnson, J. (1994). *Linear Algebra with Derive*. USA: Wiley & Sons, Inc.
- [3] Soon, S (2001). *A Companion to Further Mathematics (Pure Math): TI-83 Plus*. UK: Oxford University Press.

## Transformations- Translation, Scaling, Reflection

### Define and Display Graphs

1. Press  $\boxed{Y=}$  to display the Graph Editor.
2. Enter  $x^2$  for  $Y_1$ . Use  $\boxed{X,T,\theta,n}$  to key  $x$ .
3. Press  $\boxed{\text{GRAPH}}$  to display graph.
4. Press  $\boxed{\text{ZOOM}}$  to zoom in or out of the graph or press  $\boxed{\text{WINDOW}}$  to change the range.
5. Now display the graphs of  $y = (x-1)^2$ ,  $y = x^2$ ,  $y = (x+1)^2$  on the same screen by using  $\{\}$ .
6. Use  $\boxed{\text{TRACE}}$  to observe the coordinates of points on the graphs.
7. Complete the table below with your choice of  $f(x)$ .

$y (a>0)$	Replacement	Description of transformation
1(a) $y = f(x+a)$	$x$ is replaced with $x+a$	
(b) $y = f(x-a)$		
2(a) $y = f(x)+a$		
(b) $y = f(x)-a$		
3(a) $y = f(ax)$		
(b) $y = af(x)$		
4(a) $y = f(-x)$		
(b) $y = -f(x)$		

## Polar Coordinates- Graphic Calculator Worksheet

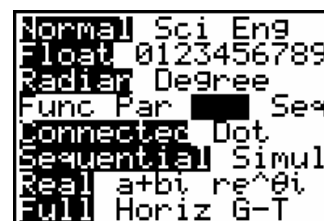
### Set calculator to Polar Graphing Mode

Switch on the calculator. This blank screen is the Home Screen.

1. Press  to display the mode screen.

2. Select **Radian** in line 3 and **Pol** in line 4.  
Now the calculator is in Polar Graphing Mode.

Press  to go to the Home Screen.



### Define and Display Polar Graphs

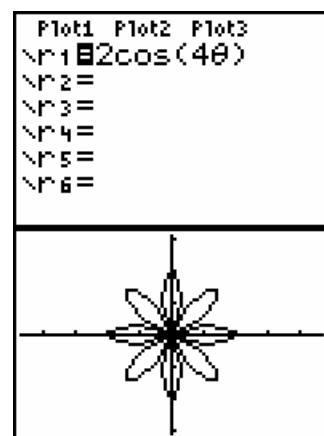
**To key in the equation:** e.g.  $r = 2\cos 4\theta$ , for  $r \geq 0$  &  $0 \leq \theta \leq 2\pi$ .

1. Press  to display the Polar Graph Editor.

2. Enter  $2\cos(4\theta)$  for  $r_1$ . Use  to key  $\theta$ .

3. Press  display graph.

4. Press   to select **ZDecimal**  
A screen like the one on the right will appear.



Note that the calculator will also include the portion of the graph where  $r < 0$ .

**For  $r \geq 0$ ,** we set  $2\cos 4\theta \geq 0$  or simply  $\cos 4\theta \geq 0$  as our condition.

5. Go back to Polar Graph Editor by pressing .

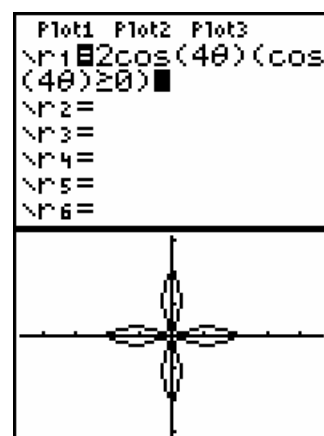
Press   to go to the end of the equation  $r_1$ .

Enter  $(\cos 4\theta \geq 0)$ .

Use    for  $\geq$ .

6. Display the graph by pressing .

In this graph, only the portions where  $r \geq 0$  is displayed.



### To Change Domain:

If the domain is  $-\pi \leq \theta \leq \pi$  instead,

7. Press . Change  $\theta_{\min}$  to  $-\pi$  by keying    .

Similarly, change  $\theta_{\max}$  to  $\pi$ .

**To make graph appear bigger or smaller:**

8. For bigger graph, press

9. For smaller graph, press

Do Exercise 1.

**Area bounded by Polar Graphs**

**To check accuracy of your answer for the area bounded by a polar curve found by integration:**

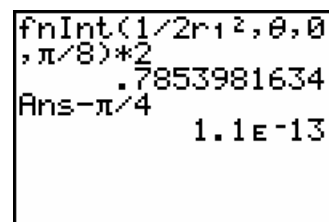
E.g. to find the area bounded by 1 loop of the graph  $r = 2 \cos 4\theta$

The area is given by  $\frac{1}{2} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} (2 \cos 4\theta)^2 d\theta$ . This is equal to  $4 \int_0^{\frac{\pi}{8}} \cos^2 4\theta d\theta$  which will work out to be  $\frac{\pi}{4}$ . To check the correctness of your integration,

1. Go to the Home Screen, by pressing  [QUIT].

2. Press   to select **fnInt(**

3. Key in the content of the bracket as shown on the right:  
To get  $r_1$ , press   to highlight Y-VARS.



Press  to select **3: Polar**, followed by  to select  $r_1$ .

4. To check, take the decimal answer by pressing [ANS] to subtract  $\frac{\pi}{4}$ , you should get close to 0.

**Exercise 1**

Graph the following equations for  $0 \leq \theta \leq 2\pi$  &  $r \geq 0$

(a)  $r = 3 \sin 4\theta$                       (b)  $r = 2 - 3 \cos 2\theta$                       (c)  $r = \frac{2\theta}{\pi} - 1$ .

(d)\*  $r^2 = 3 + \sin 2\theta$                       (e)  $r = 2 \sin^2 \theta$                       (f)#  $r = |1 + 2 \sin 2\theta|$

\* enter  $\sqrt{3 + \sin 2\theta}$  for  $r_1$ . Note that no condition is required as  $\sqrt{\quad}$  means that only the positive square root is considered.

# for the modulus (absolute) function, press   to select **NUM**, then  to select **abs(**. No condition is required.

## Fun with Matrices

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One common method of coding is the Hill Code. We add the three symbols , . ? to the alphabet to make the total number prime, and each alphabet is assigned a number as follows:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z	,	.	?	
15	16	17	18	19	20	21	22	23	24	25	26	27	28	

### To encode a message

To illustrate how it works, let us encode MAKE ME DISAPPEAR using a Hill 3-code with

encryption matrix  $A = \begin{bmatrix} 3 & 7 & 5 \\ 9 & 2 & 11 \\ 4 & 1 & 3 \end{bmatrix}$ .

**Step 1:** Remove the spaces and separate the letters into groups of 3. (Add arbitrary letters to the end if necessary.)

MAK EME DIS APP EAR

**Step 2:** Assign each letter its appropriate number from the table above and arrange these numbers in the columns of matrix  $B$  as shown:

$$B = \begin{bmatrix} 12 & 4 & 3 & 0 & 4 \\ 0 & 12 & 8 & 15 & 0 \\ 10 & 4 & 18 & 15 & 17 \end{bmatrix}$$

**Step 3:** Calculate the product  $AB$  modulo 29 to obtain the code matrix,  $C$ .

#### (a) To define matrix A in the graphics calculator

Press  [MATRIX] to select MATRIX EDIT and choose [A].

Change matrix dimensions and matrix entries by pressing  after each entry.

Repeat the procedure to define matrix  $B$ .



