Using LiveMath as an Interactive Computer Tool for Exploring Algebra and Calculus

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Abstract: Many mathematics educators in Singapore secondary schools are aware that *The Geometer's Sketchpad*, a dynamic geometry software, can be used to explore geometry. But most of them do not know of any computer algebra system (CAS) that can be used to explore algebra and calculus. Traditionally, most mathematicians, scientists and engineers have always used a CAS, such as *Maple*, to perform symbolic manipulations in order to solve algebraic and calculus problems. However most educators do not see any purpose in their pupils learning a CAS to perform symbolic manipulations, such as factorisation, differentiation and integration, when formal assessments still require them to perform such skills by hand. But with the advance of *LiveMath* (previously known as *Theorist* and *MathView*), an intriguing CAS that provides "a unique user interface that allows one to perform 'natural' algebraic maneuvers even more 'naturally' than one can achieve them on paper" (Kaput, 1992), there is now another way of using a CAS in the teaching and learning of mathematics, i.e., to explore algebraic and calculus concepts. Moreover the capability of *LiveMath* templates to be interactive even on Web pages opens up an exciting chapter in online mathematics learning. This paper looks at some examples of how educators can use *LiveMath* as an interactive tool for their pupils to explore algebra and calculus. It also provides some research evidence to suggest that the use of *LiveMath* for exploring mathematics may enhance pupil learning.

1. Introduction

The 1997 vision of the Ministry of Education (MOE) of Singapore, "Thinking Schools, Learning Nation", gave impetus for the infusion of the information technology (IT) initiative into the secondary school curriculum (Ministry of Education, 1998). Teachers were expected to follow the MOE directive of using 30% of curriculum time for IT lessons. How they did it was another matter. Some teachers clocked the required time by using PowerPoint presentations. One teacher even told me that she put up mathematics questions on PowerPoint for her students to do. In the B.C. comic strip on the next page (see Figure 1.1), Peter used the pencil, a new invention, in the same way that he used a chisel. Jensen and Williams (1993) observed that many educators used computers like the way Peter used the pencil. They exhibited a closed mindset about the use of information technology in the teaching and learning of mathematics when they used software that resembled electronic workbook pages. Is this the direction that mathematics teachers should go?

The most important question is whether pupils benefit from the infusion of IT in the school curriculum. It is the quality of the IT lessons that educators should be concerned about, not just the quantity. Dimmock (2000) pointed out that technology should not be seen as a final goal in itself but as a means of enhancing teaching and learning.

The Ministry of Education of Singapore launched the second IT Masterplan (or mp2) in July 2002 to make better use of technology to stimulate thinking and creativity among the pupils (Shanmugaratnam, 2002). A key feature of mp2 was to integrate IT into the design of a more flexible and dynamic curriculum by taking into account new teaching methods that were made possible by technology (Ministry of Education, 2003). This was a different approach from the current practice of using IT to support an existing curriculum. The mp2 would also seek to change the current predominant pedagogy of a teacher-centred use of IT to a more pupil-centred strategy for learning.

Figure 1.1 B.C. Comic Strip

This paper will examine various approaches of using IT in the teaching and learning of mathematics. Then it will look at some examples of how *LiveMath*, an interactive computer algebra system (CAS), can be used to explore algebra and calculus, even when it is used over the Internet. Lastly, this paper will provide some research evidence to suggest that the use of *LiveMath* for exploring mathematics may enhance pupil learning.

2. Various Approaches of Using IT

Taylor (1980) explored the use of computer technology in three different modes: as a tutor, a tool and a tutee. The tutor mode involved using computer assisted instruction (CAI) to tutor and drill pupils in basic procedural skills. Educators had been using this mode since the 1970s (Manoucherhri, 1999). The tool mode involved using computer software as a tool to explore and to investigate mathematics. It contained the widest range of possible applications and was the most active area for research and development (Jensen & Williams, 1993). The tutee mode involved pupils using some basic programming languages to teach the computer to perform certain tasks.

Streibel (1998) analysed three approaches to the use of computers in education: drill and practice, computer tutoring, and simulation and programming. His three approaches were rather similar to Taylor's (1980) three modes, suggesting that the Taylor's model is still relevant today, even after two decades of advances in technology.

However many educators, including Taylor (1980) himself, were not keen on the tutor mode. The main concern was pedagogical (Noss & Hoyles, 1996). Many educators subscribed to the constructivist view that knowledge could not be transmitted from teacher to learner but constructed by the learner himself (Jonassen, Peck, & Wilson, 1999). So they, including Taylor (1980) himself, preferred to use the tutee mode where learners could construct their own knowledge by building their own models using some programming languages or applications (Jonassen, 2000). Nevertheless, recognising that the tutee mode could be very time consuming, especially when educators had a syllabus to cover, Jonassen (2002) had no objection if the learners used pre-built models or templates to explore mathematics, instead of designing the models themselves. This leads us back to the tool mode which is a good compromise between the preferred tutee mode and the time constraint faced by many educators to cover the syllabus in time for their pupils to sit for some formal assessments.

3. LiveMath

Mathematics educators in Singapore have been exposed to software which were designed to deliver the existing curriculum, for example, the *Dynamic Math Series*. They have also been exposed to open tools such as *The Geometer's Sketchpad*, a dynamic geometry software, which can be used to explore geometry; and *Graphmatica*, a graphing software, which can be used to explore graphs. Some have also used *Excel* to teach statistics. However many of them are not aware of any open tool which can be used to explore algebra and calculus.

Software vendors have tried to propagate the use of *Maple*, a computer algebra system (CAS), to teach algebra and calculus, but teachers are not receptive of the idea because they do not see the purpose of pupils using the software to perform algebraic manipulations such as factorisation, differentiation and integration, when formal assessments still require them to perform such skills by hand (Yeo, 2003). There are some educators who advocate the use of a CAS to compute symbolic manipulations and therefore argue for a change in formal assessments where pupils are allowed to use handheld CAS-enabled calculators (Connors & Snook, 2001; Forster & Mueller, 2001; Monaghan, 2000). However it is not the purpose of this paper to debate the issue. This paper is more concerned with how educators can use a CAS *now*, instead of waiting for any change in formal assessments. So we need to look at what else we can use a CAS for. In line with what was said about the tool mode in Section 2, can we use a CAS to explore algebra and calculus?

Traditionally, most mathematicians, scientists and engineers have always used a CAS, such as *Maple*, as a computing tool to perform symbolic manipulations in order to solve algebraic and calculus problems (Asp & McCrae, 2000). But *Maple* is non-interactive. For example, the user has to key in a new equation and re-plot its graph every time. However, with the advance of *LiveMath* (previously known as *Theorist* and *MathView*), an interactive CAS, the user just needs to key in an equation and plot its graph once. Then the user can just change the values of the variables in the equation and everything that is linked to the equation, including its graph, will be changed automatically and instantaneously. Even Kaput (1992) was intrigued by an earlier version of the software because it provided "a unique user interface that allows one to perform 'natural' algebraic maneuvers even more 'naturally' than one can achieve them on paper" (p. 534). This interactive feature allows the learner to explore algebra and calculus without having to re-key every single equation and re-plot every single graph.

An example is to use *LiveMath* to investigate the nature of the roots of a quadratic equation. The teacher can set up a *LiveMath* document or template (see Figure 3.1) to calculate the discriminant and to plot the graph of the quadratic curve (Yeo, 2001a). The pupils will just need to change the values of the coefficient of x^2, the coefficient of x and the constant, according to the equations given in the worksheet designed by the teacher. The discriminant and the graph will then change instantaneously, enabling the pupils to explore the nature of the roots and to infer the conditions for the different types of roots to exist. The pupils do not even need to know the technical aspects of the software, thus saving a lot of curriculum time that will otherwise be spent teaching them how to programme the algebra. The pupils can then focus on the exploration rather than spending time trying to set up the template (Yeo, 2002). The teachers can also use existing resources, such as *Maths Online* (Yeo, 2001a; 2001b), which contain pre-designed *LiveMath* templates and worksheets.

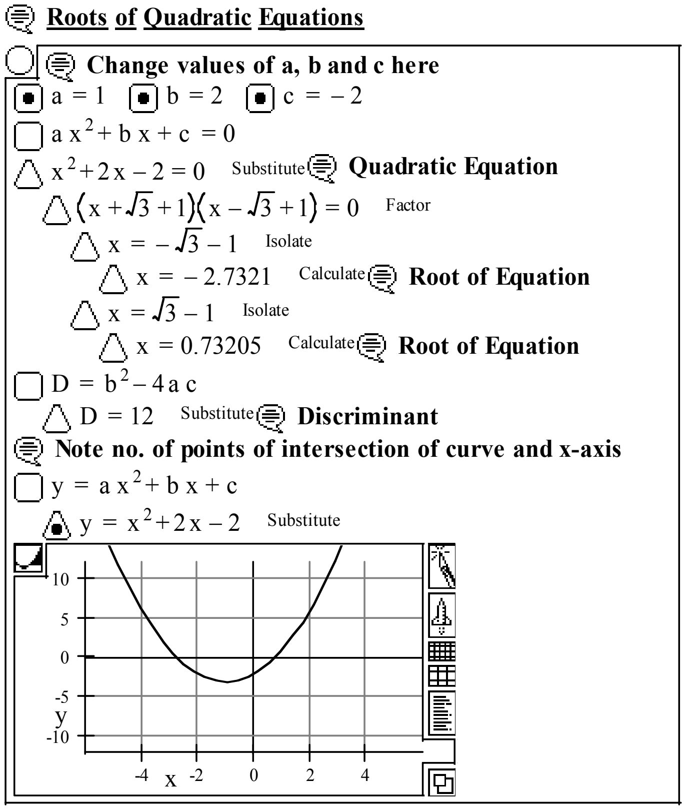

Figure 3.1 A *LiveMath* Template

4. LiveMath Internet

The wonder about *LiveMath* templates is that they are interactive even on the Internet. The software allows the teacher to create a Webpage that contains the template and the pupils can still change the values of the variables of an equation and everything that is linked to them will also change instantaneously. This opens up an exciting chapter in online learning.

Many existing mathematics lessons on the World Wide Web are still non-interactive. They consist mainly of texts and diagrams with no opportunities for interaction. This is what Roberts and Jones (2000) called the naïve model of online teaching: putting lecture notes on the World Wide Web. There are also Internet resources that contain some animations but the user cannot control them. The best are still Java applets that allow the user to manipulate them in order to explore mathematics. However these are mainly designed for geometry lessons. But with the advance of *LiveMath Internet*, pupils can now explore algebra and calculus over the World Wide Web.

The following shows an example of a *LiveMath* Webpage involving calculus (Yeo, 2001a). The template is on the left and the worksheet is on the right of the Webpage (see Figure 4.1). The purpose of this lesson is for pupils to explore the nature of a stationary point and to infer by induction the first derivative test.

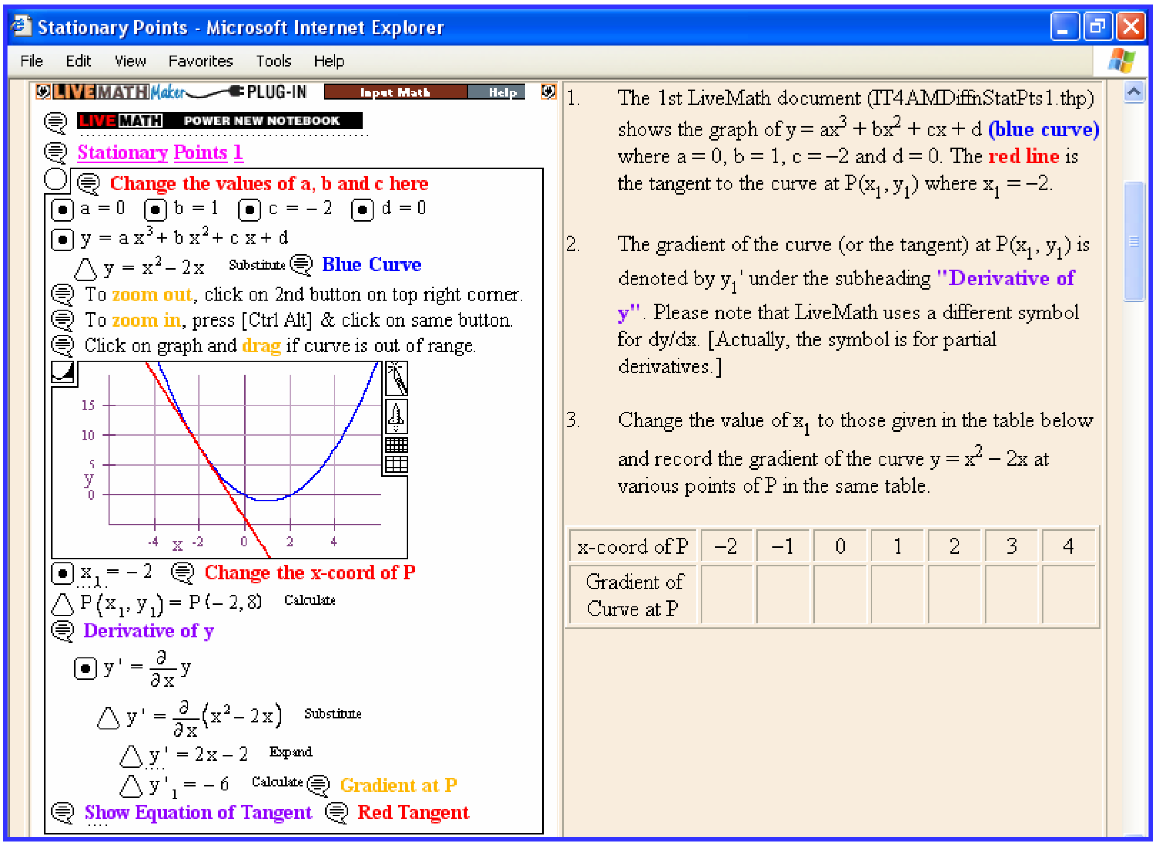

Figure 4.1 A *LiveMath* Webpage

5. Research Studies

Most local research studies on the use of IT in mathematics education centred on using a piece of graphing or geometry software to explore graphs and geometry respectively (Yeo, 2003). In fact, there was only one local study that actually involved the use of a CAS, namely *LiveMath*, to explore algebra (Yeo, 2003). Its findings showed a significant positive difference in both the conceptual and procedural knowledge of pupils using *LiveMath* to explore exponential and logarithmic functions as compared to pupils in the control group.

Most overseas studies on algebra and calculus usually involved the use of a graphing or a CAS-enabled calculator to perform symbolic calculations (Barton, 2000). There were very few, if any, that involved the use of *LiveMath* to explore algebra and calculus. Nevertheless, critics, such as Oppenheimer (1997), had dismissed many of these research studies as “more anecdotal than conclusive” (p. 48) because these studies did not control for other influences such as teaching methods and the latter was critical because computerised learning inevitably forced teachers to adjust their style and the better performance of the experimental group might well be due to a different pedagogy rather than the mere use of IT.

As a result of this important objection, the control group in Yeo's (2003) study used a non-IT guided-discovery approach to explore the topic, so as to keep the pedagogy constant. Therefore the findings of the study seemed to suggest that the better performance of the experimental group might be due to the use of IT and not due to any difference in pedagogy. However this was a small study and more research on this area needed to be done before we could come to any viable conclusion. Nevertheless, for the pupils involved in the study, the use of *LiveMath* did enhance their learning. These students were also more engaged in their IT lessons because the interactive *LiveMath* templates had captured their attention.

6. Conclusion

The advance of an interactive CAS, namely *LiveMath*, has changed the way CAS were used in the past. Other than using a CAS as a traditional computing tool to perform algebraic manipulations, educators can now use the interactive CAS, *LiveMath*, as a cognitive tool for their pupils to explore algebra and calculus in order to construct their own knowledge. Educators can even put their interactive *LiveMath* templates on the Internet so that Web-based algebra and calculus lessons need no longer be just "dead" lecture notes. *LiveMath* has indeed made mathematics come "alive"!

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