Use of Geometer’s Sketchpad (GSP) to Teach Mechanics Concepts in A Level Mathematics

Toh Tin-Lam
Email: tltoh@nie.edu.sg
National Institute of Education,
Nanyang Technological University

Abstract: The use of Geometer’s Sketchpad to enhance students’ learning of Geometry concept in Secondary School mathematics has been well documented. In this note, the use of Geometer’s Sketchpad is extended to enhance the students’ learning of Advanced Level Mechanics concepts in Mathematics. Illustrations are selected from the topics Kinematics and Simple Harmonic Motion.

Introduction

The use of Geometer’s Sketchpad (GSP) to enhance students’ learning of Mathematics concepts has been well documented.

Leong & Lim (2003) found out, effective use of GSP was able to improve students’ spatial ability measured by the students’ performance in geometry achievement test. Hoehn (1997) gave sample activities of exploring worksheets that could be done with GSP. He suggested that activities that involve the proof of theorems on geometry could be done with the software. It is implied that more abstract results on geometry can be taught by focusing on improving students’ spatial ability. Hoehn further suggested that students could be asked to attempt to generalize existing theorems and state and verify their conjectures. All this is in line with our new Problem-Solving Approach in the Mathematics syllabus (Ministry of Education, 2000).

GSP can be used to enhance students’ understanding of some rudimentary kinematics concepts, especially in the abstract concepts related to Relative Velocity – a topic recently introduced into the Additional Mathematics syllabus (Toh, 2003; National Institute of Education, 2003). Teachers are urged to illustrate the concepts without getting students into the rigors of tedious mathematical computation, which is more suitable to be taken care at the advanced mathematics.

Mathematical Concept of Projection

The concept of projection is difficult for most students. The computation part may be easy. For example, a particle traveling with constant velocity v making an angle θ as shown in Figure 1 below.

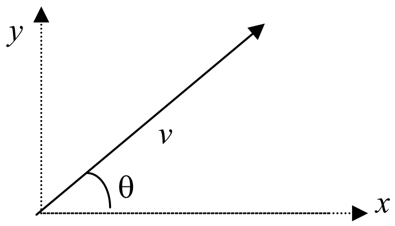

Figure 1

By direct computation, it is not difficult to see that the projection of the velocity v along the x- and y-directions are given as

$$v_x = v\cos\theta$$

and $$v_y = v\sin\theta.$$

However, the concept of the projection of velocities can be best applied in solving kinematics problems if the students have an intuitive feeling of the concept of projection. Here, GSP can play an important role in helping students to visualize the concept of projection.

Consider an elementary kinematics question for an illustration.

Example 1. Consider a boat traveling with a constant speed v making an angle θ downstream in a river (the river is enclosed by the two vertical parallel line) as shown in Figure 2 below. The width of the river is d.

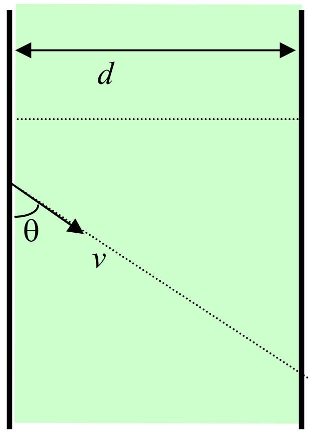

Figure 2

Students can be asked to derive an expression for the time taken for the boat to cross the river.

This problem can be solved by using projection of velocity (Toh, 2004). The use of GSP helps to illustrate the concept clearly.

Consider Figure 3 below. The point B represents the actual position of the boat that moves across the parallel river banks along the path (marked by the dotted line) making an angle θ downstream.

The point A is constructed such that it is the "projection" along the dotted line perpendicular to the two river bank. The movement of A is dependent on the movement of B, which is set to animate by moving across the river once. The movement always ensures that AB is parallel to the two parallel river banks.

It can thus be seen that both points A and B reach the other side of the river bank at the same time if AB is to remain parallel to the two river banks. In this example, the concept of "projection" can be perceived as imagining another particle crossing the river, but reaching the other side of the river at the same time. Whenever B has traveled a distance v per unit time, A would have traveled a distance of $v \sin \theta$ in the same time, while A and B reach the other shore at the same time. Thus, the time taken for A to cross the river = $\dfrac{d}{v \sin \theta}$, which is the same as the time for the boat B to cross the river.

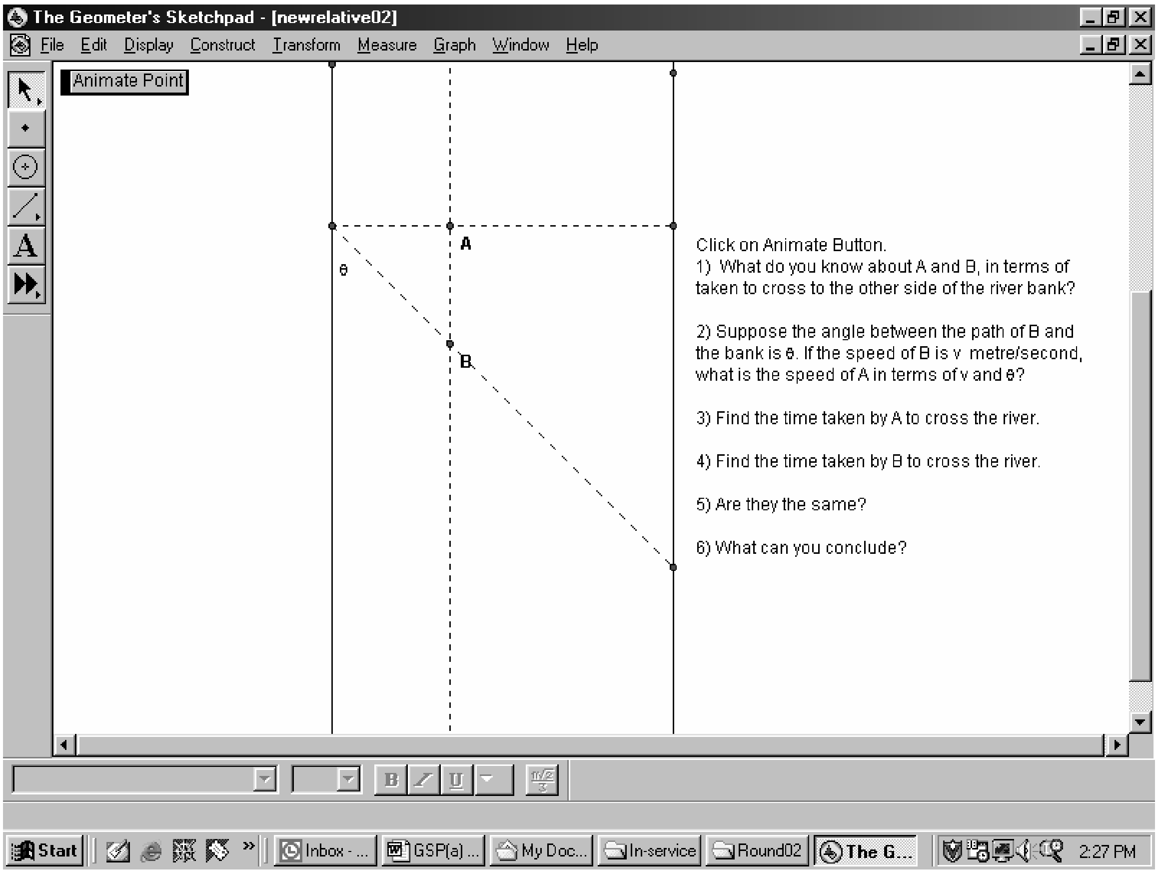

Figure 3 The crossing of river by the boat B and its projection A

The animation can be constructed such that it allows the angle θ to vary its value. In such situations, students can also be led to see that the closer the value of θ is to 90^0, the greater will be the speed of the projected particle A, hence the shorter time to cross the river. This is a visual interpretation of $\dfrac{d}{v \sin \theta} \geq \dfrac{d}{v}$ with equality if and only if $\theta = 90^0$.

Through an elementary illustration with GSP, it is hoped that students are able to apply the concept of projection more confidently in solving elementary kinematics problems.

Example 2. The theory of simple harmonic motion (shm) is another difficult topic in Advanced Level mathematics for most students. It is well-known that the formula for a particle moving with shm is $\frac{d^2x}{dt^2} = -w^2 x$, where x is the displacement of the particle from an origin O, t is the time after the start of vibration, and w^2 is some positive constant. However, it is seldom illustrated the unique features of shm and how it differs from other types of motion which are non-shm. The use of GSP is able to assist the students to identify and appreciate the visual characteristics of shm.

By performing integration to the differential equation defining shm, we obtain the equation $x = A\cos wt + B\sin wt$ for some constants A and B. This means that the displacement x is a sinusoidal function of the time t. By using GSP, we can also illustrate how this is obtained.

Activity One: Visual Characteristics of shm and non-shm
By the end of this activity, students should be able to identify and describe shm (not merely deduction from the mathematical equations but from the actual motion of the particles) and identify that it is the projection of a particle moving with constant speed in circular motion.

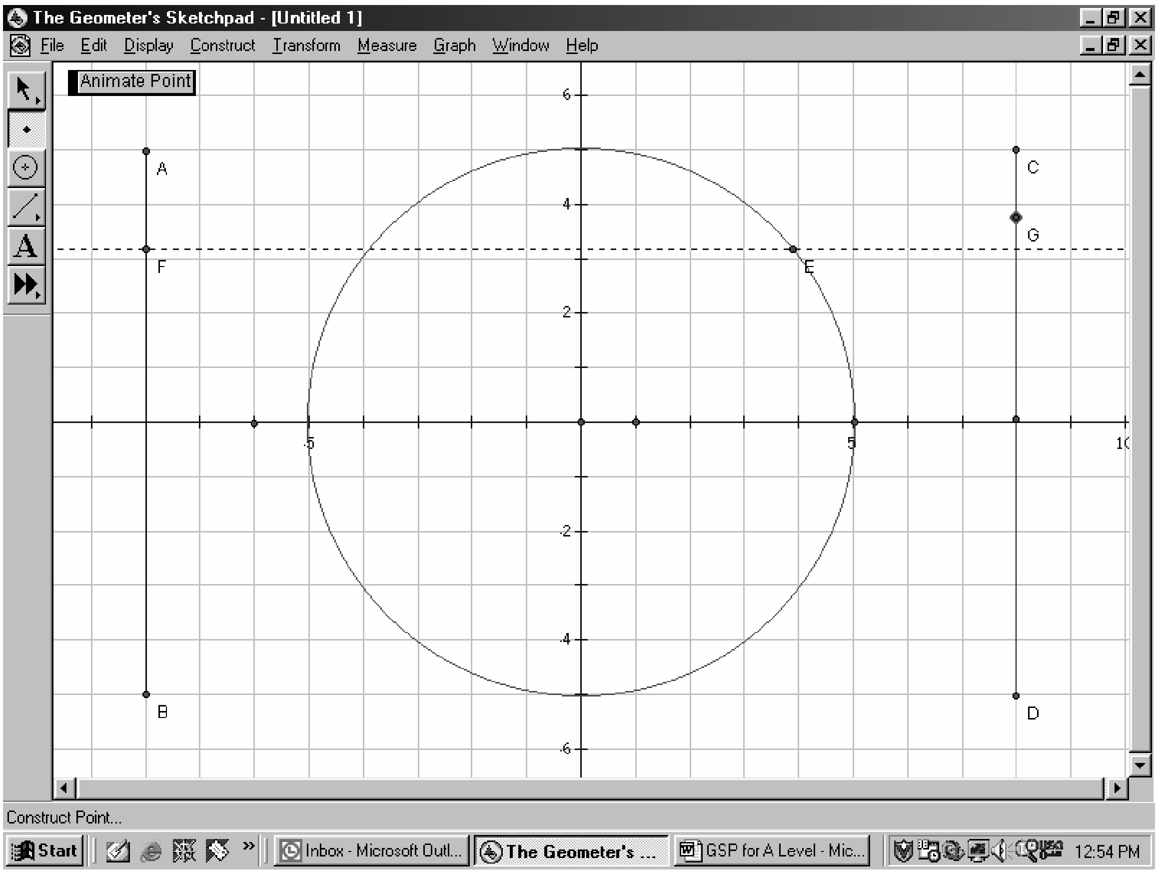

Figure 4 Particle F (being the projection of a particle in circular motion) moving with shm and particle G moving with constant speed

A circle and two parallel line segments AB and CD are constructed as in Figure 4 above, together with the point E on the circle while F is constructed the projection of E along the line segment AB. The line segments AB and CD are the projections of the circle along the vertical lines as shown in Figure 4 above. Another point G is another movable point on the segment CD. Then the circle, point E and the dotted line are hidden as in Figure 5 below. Points E and G are allowed to be animated (to move with constant speed along the circle and the line segment AB respectively) by the “ANIMATE” button. Both points E and G are animated with a constant speed on the screen, while F will move along AB as the projection of E.

Students can be asked to describe the differences of the two types of motion represented by the particles F and G, and asked to deduce which is more likely to be shm, explaining in the light of the equations involving shm. Here it illustrates clearly that for the case of shm, the particle comes to instantaneous rest at the two extreme points of the motion, and its speed increases as it moves towards the centre of shm.

Teachers can then show all the hidden objects and revert to Figure 4, and explain that in fact F describes shm and that it is the projection of a particle E moving with constant speed along a circular path.

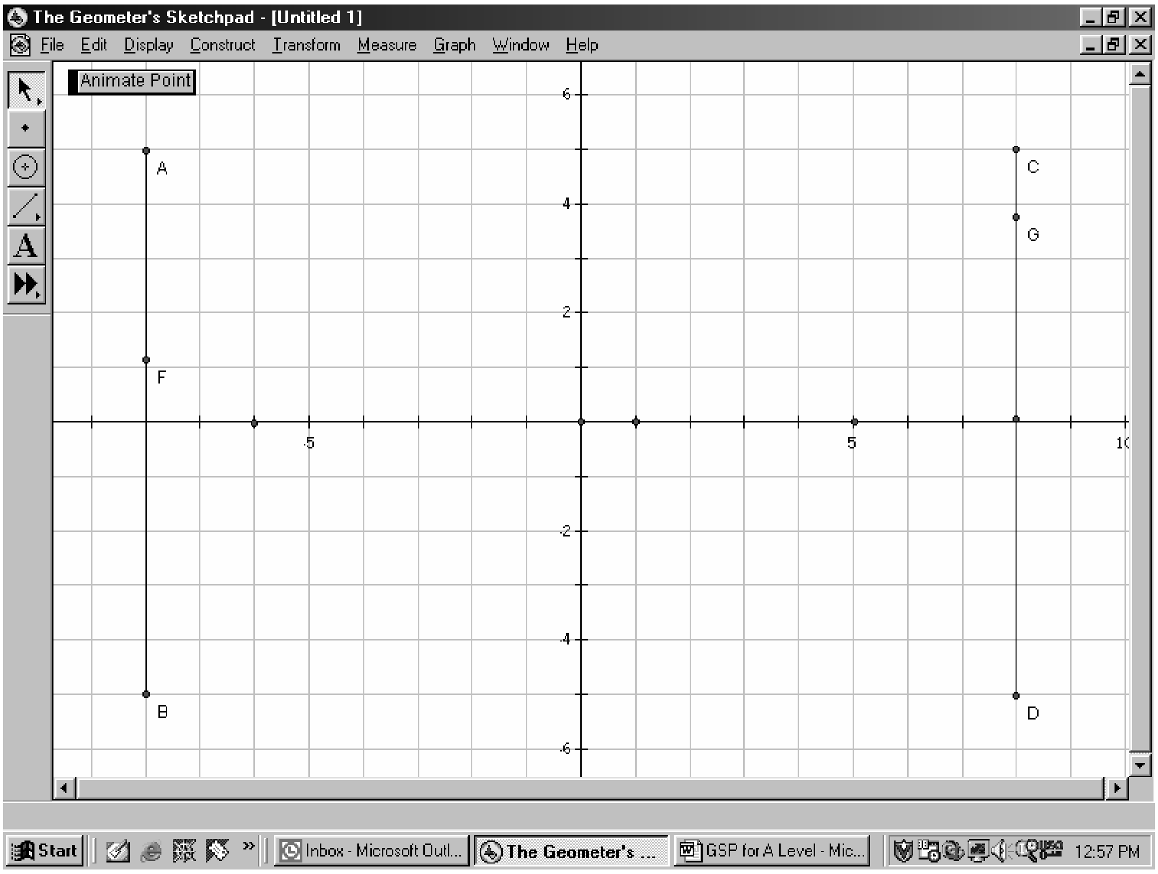

Figure 5 The motion of particles F in shm and G with constant speed with the circular motion of E removed from Figure 4

Activity Two: Displacement-time Graph

The next process of tracing the displacement-time graph for the particle F moving with shm. It can be traced out as illustrated in Figure 6 below. Let a point J runs with a constant speed from left to right (hence it is proportional to the time from the start of the motion) along the *x*-axis, while F starts with its shm along the path AB. Trace the point H, which is the intersection point of the vertical line through J and the horizontal line through F. The path of H is traced and it could be seen that the path taken by H is a sinusoidal graph.

This gives a visual illustration that the function of x versus t is in fact a trigonometric function, which is obtained as $x = A\cos wt + B\sin wt$ by solving the differential equation for shm. The choice of A and B is dependent on the position and the speed of the particle F at the start of the motion.

The next question to ask will be when the point G, which moves with constant speed, is traced its displacement-time graph, how it will look like. The same could be done, and it is illustrated in Figure 7 in the next page.

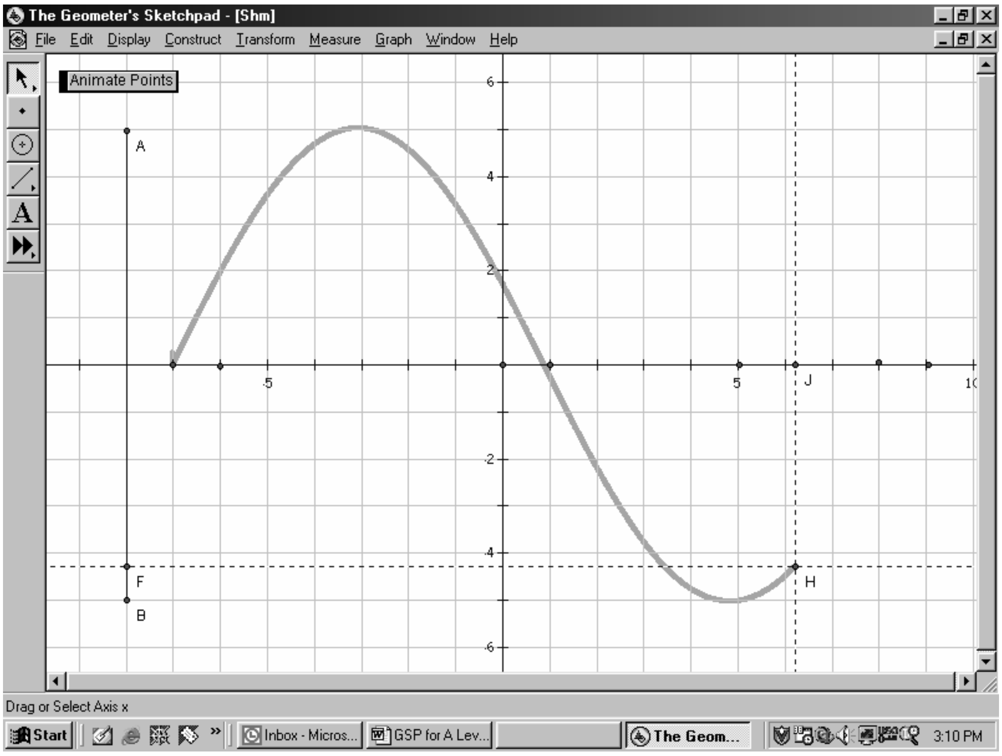

Figure 6 The displacement-time graph of a particle moving with shm along AB

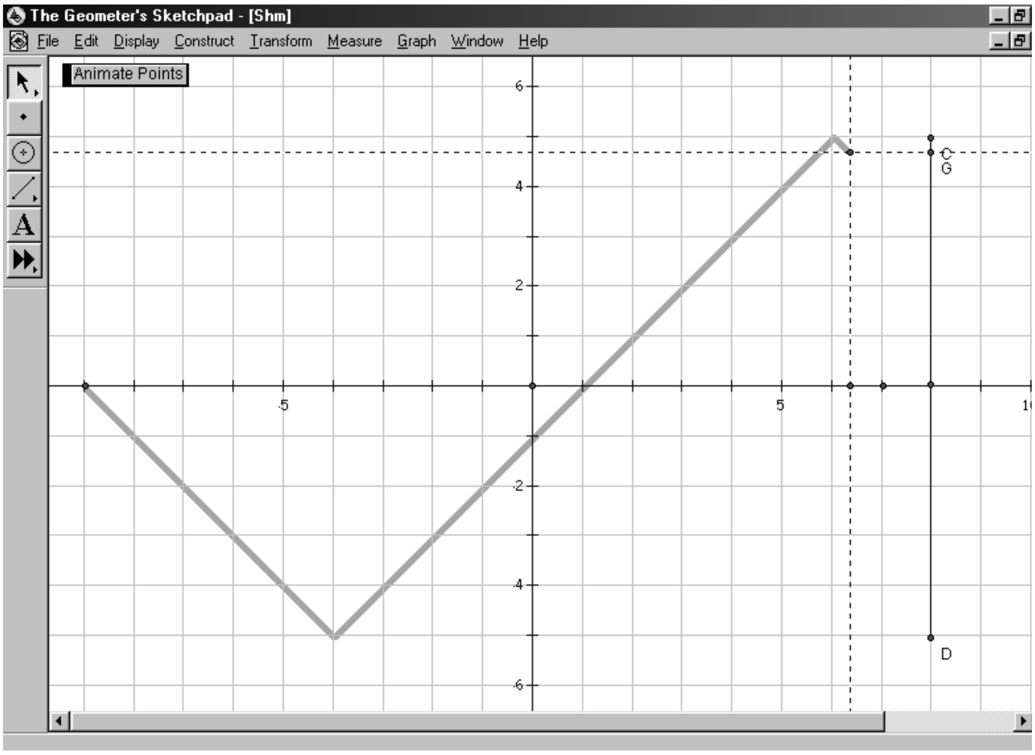

Figure 7 The displacement-time graph of a particle moving with constant speed along CD

Thus, it could be seen from Figure 7 that for the particle G moving with constant speed along CD, its displace-time graph that is traced out consists of straight line segments and not a sinusoidal graph as in Figure 6.

Conclusion

This is the initial effort to incorporate IT into the teaching of Advanced level mechanics. We have thus illustrated the use of GSP to make concepts in geometry and some mechanics concepts more visual so as to enhance students' learning of some advanced concepts in applied mathematics.

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