Developing Prospective Elementary School Teachers' Expertise with Dynamic Geometry

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Abstract: This study explored the use of prepared files and associated activities to introduce pre-service elementary school teachers to calculator-based dynamic geometry. In the activities the future teachers constructed altitudes, angle bisectors, medians, and perpendicular bisectors in triangles. They then animated the figures and wrote conclusions and conjectures based on the concurrency of the straight objects they had constructed and the vertex angle measures of the triangles. The pre-service teachers also constructed and measured triangles bounded by the constructed straight objects and the original triangles, and made conjectures based on these measures and re-animations of the figures. Results showed that the simpler layout of the calculator platform made it an appropriate technology for the elementary school grades and an excellent tool for geometric investigations. Application of a cognitive framework with four levels of tools and four access characteristics confirmed that the prospective teachers used the prepared files with the activities to develop their expertise with the tools and access features detailed in the framework.

1. Introduction

While it still may be an open question if one's comprehension of geometry is enhanced when dynamic geometry software is used, Cabri and similar packages offer a wide range of opportunities to explore and solve problems. The use of dynamic geometry also widens the range of accessible geometric constructions and solutions by offering tools that are inaccessible or unavailable in paper and pencil geometry. As a result, dynamic geometry environments expand the scope of possible activities, and provide a route to deeper reflection and more refined exploration and heuristics than is often the case with traditional geometry.

Santos-Trigo and Espinosa-Perez [10] documented the usefulness of Cabri on tasks that led students to detect relationships and to verify properties that connected distinct geometric configurations. Within the process of constructing and elaborating Cabri figures, the researchers found that students had opportunities to examine parts of configurations and eventually propose conjectures and ways to support them. In this context the software played an important role in actually verifying properties of figures. The researchers concluded that Cabri could be a powerful tool in identifying and examining geometric properties in auxiliary constructions, and in observing and exploring mathematical relationships.

A key feature of Cabri is its flexible and insightful use of the drag mode [12]. Although one of Cabri's major contributions is the dynamic visualization of geometrical relations available using this drag mode, it is important to emphasize that students need ample time and structure to discern a geometric property from its various spatial-graphical representations. Laborde [4] found that teachers (even those who were novices in using technology) immediately exploited this contribution by asking students to form conjectures about properties. But, such learning tasks required deliberation and attention to issues of instructional design prior to the actual introduction of dynamic geometry technology in the classroom.

Another recent study by Leung and Lopez-Real [5] found that Cabri helped students in defining and identifying geometrical properties and the dependencies between them. Particularly, these researchers noted that the drag mode in Cabri seemed to open up a region of proximal solutions between students and the dynamic geometry environment in which students' insight and understanding developed via open investigation and experimentation. Leung and Lopez-Real also found that an important feature in the cognitive scheme that resulted from dynamic geometry was the interaction between the person engaged in the mathematical task and the dynamic geometry environment. This interaction, according to the researchers, created a web of meaning that students could draw upon to construct their own meaning for certain aspects of mathematics.

These same researchers (Leung and Lopez-Real [5]) also constructed a dynamic visualization model for dynamic geometry environments to stimulate students' cognitive processes. This model carried the idea of a holistic figural concept with an inherited duality, and the duality related the non-linear reasoning of the dynamic geometry environment with the traditional linear, deductive reasoning of symbolic formal systems. The researchers suggested that students activated these dual logical modes through their engagement in the dragging schemes within a dynamic geometry environment.

Mariotti [7] reported on a teaching experiment in which constructions with Cabri were also aimed at developing theoretical thinking in geometry. In part, the study was motivated by a review of the literature that showed there was no general consensus about the contribution of dynamic geometry in the development of theoretical thinking. Mariotti found that Cabri's dragging mode introduced specific criteria of validity in construction problems since the constructed figure was valid if and only if the figure was stable under dragging. As a result, Mariotti's teaching experiment showed that using construction tasks in a Cabri environment served as an important means for accessing theoretical geometry.

In a similar vein, Scher [11] explored dynamic geometry constructions and reported on how they fostered insights into geometric theorems. Scher's findings particularly emphasized that students were motivated to generalize theorems by setting the figures in the theorem into motion. This animation of the hypotheses of theorems permitted students to uncover relationships that static counterparts in text form did not reveal.

2. Purpose

The primary purpose of this study was to build on the preceding research base in order to introduce prospective elementary school teachers to a calculator-based dynamic geometry environment through activities on concurrent points and triangles. Secondary purposes of the study were to: (1)

reduce the time required for the prospective teachers to perform dynamic geometry on a calculator so as to allow the introductory activities to be completed during one class period, and (2) use the introductory activities as a means of synthesizing results about the concurrency of straight objects related to triangles. During the activities the prospective teachers would have the opportunity to explore animated dynamic-geometry constructions that were designed to foster their insights into traditional theorems about altitudes, angle bisectors, medians, and perpendicular bisectors of triangles. Moreover, by setting these constructions in motion, the future teachers would also have the opportunity to generalize the theorems and uncover relationships that were not accessible from static counterparts in texts.

3. Challenges and Opportunities in Teaching Cabri on Different Platforms

Although working with dynamic geometry on a calculator is somewhat similar to working with dynamic geometry on a computer, there are substantial differences. Calculator-based dynamic geometry presents some unique challenges and opportunities for the novice user, even if this user has had substantial experience in computer-based dynamic geometry.

A general challenge in using dynamic geometry in classrooms concerns the time required for teachers to develop expertise with the software. Mackrell [6] found that an impediment to using dynamic geometry in the classroom was the steep learning curve for both students and teachers in using this software to its full potential. To alleviate this concern, Mackell reported that prepared files worked well to dampen the learning curve for those without previous exposure to Cabri. In these classes, the files provided students assistance in getting started, and once the students did start, they were able to perform successfully on Cabri activities. Thus, prepared files made the extensive construction, measurement, and animation capabilities of Cabri readily available to new users of the software.

Another general challenge when teachers use software-generated knowledge to solve geometric problems concerns cognition and dynamic geometry environments. These environments, despite the power they afford, are not blank checks for cognition. Harris [2] found that appropriate problems for dynamic geometry are those that are designed using a cognitive framework that synthesize cognitive tools and a corresponding set of access characteristics. Using four functional levels for cognitive tools (support, sharing, engagement, and hypothesis testing) along with four cognitive access characteristics (types of knowledge, representation, retrieval, and construction), Harris demonstrated that a framework configured from these components exhibited substantial compatibility with publishers' descriptions of dynamic geometry packages. Hence, by using prepared files in their initial work with Cabri, new users had the potential to employ tools that augmented their cognition at each of the four functional levels and process knowledge using the four access characteristics identified in the Harris' framework.

As previously noted, one of Cabri's major contributions is the dynamic visualization of geometrical relations available using the drag mode. The absence of a mouse to move the tool icons and to select and drag constructed objects is one aspect of dynamic geometry on calculators that impacts the environment available for manipulating or exploring geometry on calculator-based systems. Moving tools into position and dragging with the four-directional keys on a calculator keypad is often more cumbersome than using a computer's mouse. The smaller screen size and the limited number of tools, constructions, and manipulations available in the pull-down menus on calculator-based dynamic geometry are also limitations that are readily evident to users with experience on computer-based programs. Despite these drawbacks, calculator-based dynamic geometry environments have a number of advantages.

Portability, compactness, and affordability are conspicuous advantages of the calculator over the computer, and for an elementary school teacher these benefits alone often mean the difference between whether students have access to mathematical technology or not. As a result calculator-based systems enable prospective elementary school teachers to have experiences with affordable classroom technology that is appropriate to the needs of their future students. Calculator-based dynamic geometry systems also enable students to explore geometric questions both independently and in-groups, thereby allowing for considerable instructional flexibility.

Both computer and calculator versions of dynamic geometry software allow for the use of prepared files, and these files allow the user to benefit from using each of the four levels of cognitive tools and each of the four cognitive access features identified by Harris [2]. Due to this benefit even novice users can readily participate in activities that synthesize significant and meaningful geometric results.

Consequently, the present study explored the feasibility of introducing prospective elementary school teachers to a calculator-based version of the Cabri dynamic geometry package called Cabri Junior. The simpler layout of the Cabri Junior environment (as compared to Cabri or other computer-based packages) makes it an appropriate technology for the elementary school grades, as well as an excellent instrument for prospective school teachers to utilize in their own investigations in geometry.

4. Methodology

To investigate the feasibility of using Cabri Junior activities about special points of triangles with prospective elementary school teachers, two pilot tests were conducted. The first pilot test was carried out in a formal geometry course for prospective secondary school teachers. The 32 students enrolled in the class worked on the Cabri Junior activities as part of one class period in the course. Although these students had previous experience with dynamic geometry, the Cabri Junior activities were their first experiences with calculator-based dynamic geometry. Time for students to complete the activities was limited to 40 minutes due to technical problems caused by an inoperable overhead projection panel. Attempting classroom repairs on the inoperable panel and locating a replacement panel resulted in a loss of approximately 20 minutes of the time that had been planned for students to work on the activities. Partly due to the reduced time available to complete the activity, students' responses to the questions on the activity sheets showed considerable variation. Percent of correct responses ranged from 65 to 88 percent and none of the students were able to complete the activities. Participants in this pilot study also contributed significantly to the study by completing open-ended evaluation forms that facilitated further refinement of the activities.

Following the first pilot test, the activities were revised to include more explicit instructions for constructing the straight objects, finding the areas of triangular regions, and opening prepared files to transition between activities. The activity sheets were also revised to make the written instructions more self-explanatory and easier for students to use on their own in the event of future projection equipment failures.

The second pilot study was conducted with a class of eight pre-service elementary school teachers who were enrolled in a geometry content course for teaching middle grades (US grades 5-8) mathematics. During this pilot study there were no equipment problems and once again none of the students had prior experience with calculator-based dynamic geometry. Students were encouraged to work together in completing the activities. An instructor and a teaching assistant were in the classroom throughout the class period and students were also encouraged to ask questions about any aspects of the activities. The students appeared to freely ask questions during the class period, and evaluation of the completed activity sheets showed that each student scored 100% correct. However, students continued to have difficulty with part of the activity involving the centroid that asked students to construct triangles on existing segments and to measure the area of these triangles. Also, students sometimes asked about how to continue when the calculator's Automatic Power Down (APD) feature turned off the calculator.

If APD turned off the calculator while a Cabri Junior animation was in either the AUTO or STOP mode, then to continue, the student had to turn on the calculator, press the APPS key and select **Cabri Junior** and then press the F1 or Y= and the ENTER keys to return to the animation displayed as it was before the APD turned off the calculator. Similarly, if the APD shut off the calculator while Cabri Junior was in a construction mode for **Perp.**, **Perp. Bis.**, **Angle Bis.**, or **Midpoint** (entered with the F3 key), or while Cabri Junior was in the **Segment** construction mode (entered using the F2 key), the student needed to restart the calculator and again reinstate the mode in which the calculator had previously been operating. These machinations complicated the process of completing the activities for some of the students who participated in the second pilot study. As a result of these problematic APD power downs, during later uses of these activities, the instructors illustrated the procedures to follow to regain the previous screen after the calculator was turned off while Cabri Junior was set in the animation or the construction modes.

To address concerns reported during the second pilot test, the activity sheets were again revised. This revision focused on making the sheets easier to use by: (1) elaborating the key strokes and other procedures to construct triangles and measure the areas, (2) enlarging the font and adding check-off boxes before sets of procedures, and (3) adding preliminary instructions about how to proceed after the APD feature shuts off the calculator. This revised version of the activity sheets was used for the study reported in this paper.

Students who participated in this study worked on four activities. An instructor used an overhead projection panel to project the image of a calculator's display onto an overhead screen to introduce the activities to the students. The introduction illustrated the use of the **Open** and **Animate** features in the F1 menu and the construct perpendicular (**Perp.**) feature in the F3 menu. Particularly, the instructor demonstrated how to open the figure used in the first activity (**ACT01**), how to construct the altitudes of a triangle, and how to animate and stop the animation of the triangle and its altitudes. The instructor also demonstrated how to restart the calculator and regain an existing display after an automatic power down (APD) by the calculator.

Following the introduction, the prospective teachers used Cabri Junior's animation feature to complete the first set of tasks and answer the corresponding questions about the altitudes of a triangle. Once they had recorded their responses to the questions, the students were asked to proceed to the next activity on the sheets, an activity involving the construction and animation of the angle bisectors of a triangle. Following the angle bisectors activity, the future teachers were asked to complete activities and answer questions concerning the perpendicular bisectors and the medians of triangles. In each of the activities, prepared files enabled the students to animate the triangle and the constructed straight objects. These animations permitted the future teachers to observe not only changes in the positions of the points at which the constructed straight objects were concurrent, but also the measures of each of the vertex angles in the triangle. Animations consisted of moving the top vertex of the given triangle up and down the length of a hidden segment that extended from the top vertex to the side opposite the top vertex. By running the animation students could observe the top vertex angle change in measure from about 75º to 180º. Students could stop the animation by pressing the 2^{nd} key followed by ENTER. Animation could be started again by pressing the 2^{nd} key, moving the cursor to the top vertex until a double-headed arrow appeared and pressing ENTER.

The final activity on the sheets provided the pre-service teachers with an opportunity to extend their experiences with Cabri Junior's features by constructing triangles formed by the centroid and two vertices of the original triangle. After these constructions were complete, the students were asked to use Cabri's **Area** feature to measure the areas of each of these smaller triangles. Students were then asked to record these measures and to comment on why the centroid could be regarded as a center of gravity.

5. Results and Discussion

A total of 13 prospective elementary school teachers participated in this latest administration of the activities. During this administration, students were encouraged to work independently on the activities. Students were also encouraged to ask questions of either the instructor or a teaching assistant who were in the classroom at all times while students worked on the activities. Students' scores on the activities and their overall scores in the course are shown in Table 1.

Table 1: Students' Scores on the Activities and in the Course

STUDENT NUMBER	ACTIVITIES SCORE	COURSE SCORE
1	95	99
2	80	78
3	90	91
4	85	90
5	96	89
6	76	77
7	60	93
8	95	81
9	91	77
10	53	79
11	60	68
12	48	82
13	85	59

The students' mean score for the activities was 78 and the median score was 85. These scores were similar to the students' overall mean and median scores in the course, both of which were 81. This result provided an indication that the activities in this study were comparable in difficulty for these students to the other material covered in their course.

The future elementary school teachers who participated in the study were also asked to describe (1) the advantages of using the calculator instead of the computer to do geometry in the classroom, and (2) how they might use calculator geometry in a primary school classroom. Students' responses to the first question revealed that most of the prospective teachers found the calculator was easier to use or more user-friendly than a computer for dynamic geometry applications. Moreover, six of the thirteen future teachers reported that they believed the calculator was a more appropriate technology than a computer for primary school students to use with dynamic geometry.

Students' responses to the question about how they might use calculator geometry in primary school classrooms showed that most of the future teachers felt that calculator geometry was most useful as a tool for developing geometric results similar to those on the activity sheets that they had just completed. Several respondents further indicated that geometry on the calculator enhanced elementary school children's potential for visualizing changes or relationships between points, angles, and/or triangles.

The quantitative findings provided evidence that Cabri Junior and the prepared files lent cognitive support and shared the cognitive load required for the future teachers to complete the activities. Responses to the qualitative questionnaire items also indicated that the calculator-based Cabri provided opportunities for both cognitive engagement and conjecture or hypothesis testing during the activities.

The pre-service teachers' performance on the activities also provided evidence that their Cabri-assisted cognitive access spanned knowledge that ranged from understanding and comprehension of geometric terminology to application and synthesis of exploratory results. Along with these types of knowledge, the Cabri-augmented cognitive access also showed evidence of satisfying the representational, retrieval, and construction requirements of the activities in this study. Consequently, the study's results confirmed that the prospective teachers used the Cabri activities to develop their expertise with the tools (support, sharing, engagement, and hypothesis testing) and access characteristics (types of knowledge, representation, retrieval, and construction) identified in the Harris (2000) framework.

In the USA, many State mathematics standards and frameworks require both knowledge of geometry and knowledge of dynamic geometry software. Accordingly, State accreditation programs typically require this knowledge base for teachers. For example, the standards for the New Hampshire Council of Teacher Education specifically state that teachers starting in grade 5 should be able to "use a variety of tools, physical models, and dynamic geometric software to explore geometric relationships"[9]. The California Department of Education (CDE) states that teachers must possess knowledge of geometric concepts and skills in its required content standards for elementary and middle school mathematics teachers. The particular concepts and skills are made more explicit in CDE's grade 5 Mathematics Content Standards, which state that students should be able to identify, describe, and classify properties of, and relations between, geometric figures, and to do so using drawing software along with other tools [1].

Further support for the notion that future elementary school teachers need to develop expertise with dynamic geometry comes from national groups as well. The National Council for Accreditation of Teacher Education (NCATE) requires knowledge of geometry in both its program for Elementary School Mathematics Specialists and its program for Middle School Mathematics Teachers. It specifically states that teachers must be able to use "geometric modeling to explore and analyze geometric shapes, structures, and their properties"[8].

The nature and extent of NCATE's requirements concerning prospective elementary school teachers' knowledge of geometry and dynamic geometry software are also reflected in the findings of other study groups. Improving Measurement and Geometry in Elementary Schools (IMAGES), an initiative of the Eisenhower Consortium for Mathematics and Science Education is one such group. IMAGES noted that the theoretical Van Hiele levels of geometric development predicted that most students in grades K-3 were at a visualization (level 1) stage in geometric development and most students in grades 4-5 were at an analysis (level 2) stage of development. To insure that new secondary school students are adequately prepared for further study, IMAGES reported that "it is important that elementary school teachers provide their students with experiences that will help them move from level 1 to level 3 (an informal deduction stage) by the end of eighth grade [3]." Level 3 in the Van Hiele's theory is one in which students can recognize properties of figures and follow logical arguments using these properties. Consequently, IMAGES suggested that an instructional strategy for developing informal deduction was to use models and drawings from dynamic geometry software as means to discover generalizations and counterexamples. This approach to informal deduction served as a basis in the activities reported in this paper. As a result, these activities are examples of the types of learning and preparation that future elementary school teachers need in their pre-service programs of study.

In addition to examining the types of learning and preparation that teachers must complete prior to using dynamic geometry, the results of this study point to the need to investigate the benefits of using dynamic geometry systems. If, as demonstrated in this study, prospective elementary school teachers can recognize opportunities for using dynamic geometry within their classrooms, then an explicit detailing of benefits of these systems is apt to help teachers sustain this new form of instruction. Such research would add to our current understandings of the benefits of dynamic geometry systems and elucidate their roles as effective and powerful learning environments.

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