Using Hand-held Technology to Enhance Teaching and Learning of Mathematics

Kwee Tiow Choo
ktc@hjc.edu.sg
Department of Mathematics
Hwa Chong Junior College, Singapore

Abstract : Hand-Held Technology, such as Graphic Calculator (GC) TI83/TI83 plus, Computer Algebra System(CAS) TI92 plus/Voyage 200, Calculator-Based Laboratory^{TM} 2(CBL2^{TM}) and Calculator-Based Ranger^{TM} (CBR^{TM}), has allowed for more innovative ways in teaching Mathematics.

The introduction of Graphic Calculator TI83 / TI83 plus into A level Further Mathematics syllabus has certainly changed and opened up new avenues in the teaching of Mathematics. By incorporating this technology into the Mathematics classroom, students have a tool to facilitate their understanding of Mathematics and allow them to explore Mathematics concepts and theorems.

The TI92 plus and Voyage 200(V200) incorporate graphical and numerical features with a powerful computer algebra system. This CAS has added a new dimension in the use of technology for teaching Mathematics. By using CAS, students are introduced to a new technology which will definitely enhance and enrich their learning experience in Mathematics.

I will present some interesting questions which I encountered over the past few years when I used TI83 and V200 in teaching Mathematics.

Introduction

Hand-held technology has been has been around for a long time. What started out as the humble but trusty hand-held electronic calculator had gathered momentum, especially over the last decade. As in all technology changes, it has certainly affected many areas, including the teaching and learning of Mathematics. Current hand-held technology, such as Graphic Calculator TI83, TI83 plus, Computer Algebra System TI92 plus, Voyage 200, Calculator-Based Laboratory^{TM} 2(CBL2^{TM}) and Calculator-Based Ranger^{TM} (CBR^{TM}), has allowed for more innovative ways in the learning and teaching of Mathematics.

Currently only GCE A level Further Mathematics students are allowed to use Graphic Calculator (GC) T183. From 2006, T183 will be used in all GCE A level Mathematics papers. By incorporating this technology into teaching, students have an additional tool to facilitate their understanding of Mathematics and allow them to explore Mathematics concepts and theorems. There will also be implications for teachers. Berry (1997, p46) indicated that the future of teaching Mathematics with technology will be exciting for both teachers and students and it will change the learning style. He added that the use of technology as a tool to solve problems will allow more realistic problems to be solved and the use of technology in examinations will cause us to think what we are really trying to assess.

The use of hand-held technology in the teaching of Mathematics is not without controversy. Nevertheless there are no denying that hand-held technology has brought about positive changes in the teaching and learning of Mathematics.

Scope of Presentation

The focus of my presentation is to share some of my experiences and results when using hand-held technology in the teaching of Mathematics, with specific reference to Graphic Calculator T183, T183 plus, Computer Algebra System T192 plus, Voyage 200, Calculator-Based Laboratory ^{TM} 2(CBL2 ^{TM}) and Calculator-Based Ranger ^{TM} (CBR ^{TM}). In addition, I will also highlight some of the benefits and concerns in the use of hand-held technology in the teaching of Mathematics. I will also share the ways I overcame some of these concerns.

Benefits of Using Hand-held Technology

Many studies have been done that show evidences to support the use of technology in the teaching and learning of Mathematics. Among them, Burrill (pi) suggested that given supporting conditions, evidences indicate that handheld graphing technology can be an important factor in helping students develop a better understanding of Mathematics concepts, score higher on performance measures and raise the level of their problem solving skills. He added (pv) that there are indications that extensive use of the technology does not necessarily interfere with students' acquisition of skills. This is supported by Linda (1988 p76) who indicated that technology does help students to develop conceptual understanding and problem-solving abilities.

Some Concerns in Using Hand-held Technology in the Teaching of Mathematics

According to Waits and Demana (1999 p5), the major controversy is resistance to change from the pen-and-pencil method to the use of hand-held technology. They stated that one of the great problems faced in Mathematics education is communicating the real nature and value of Mathematics. The challenge for teachers is to communicate the true nature of Mathematics and build a case that appropriate use of technology will enhance the teaching and learning of Mathematics. This means teachers must realized that the pen-and pencil method is no longer the only way to teach Mathematics and they must examine on a case-to-case basis which topics or portions can be substituted by using hand-held technology to enhance the teaching process. They (1999 p7) added that teachers' fear of technology are natural and need to be addressed. However, with students using new tools, many things need to be changed. These include curriculum, tests and expectations. Teachers not willing to change will hence fear technology.

Burrill (pi) added that simply providing teachers with information about how the technology functions is not likely to result in effective integration in the classroom. To overcome this, substantial professional development and support is necessary for teachers to make informed decisions about how to best use handheld technology in their classroom. He (pii) also stated that researchers indicated concern about students' over reliance on the technology, accepting results at face value with little critical thinking.

The last point was one of my main concerns in introducing technology such as CAS to students as they may exploit the technology and simply use the calculator to get answers, instead of understanding the concepts and using the traditional analytical method of getting answers. This concern has been expressed by Macintyre and Forbes (2002, p30) when they make reference to Taylor (1995) that obtaining solutions through machine calculations rather than by human mathematical analysis will ultimately reduce creativity and problem solving skills rather than facilitate them.

Overcoming Some of the Concerns

The strategy I have adopted to overcome some of the concerns highlight above, namely teachers' fear of technology and students misusing the technology is to conduct sharing sessions for teachers and to give worksheets for students. Conducting sharing sessions is probably the fastest way to introduce and equip teachers to make use of CAS. As for worksheets for students, it allows me to ascertain that students are familiar with the CAS. In addition, the worksheets can be designed such that it minimizes the opportunity for students to use the calculator merely as a tool to get answers.

Personal Experience in Using Hand-held Technology

Much time and effort were spent exploring the capabilities of the CAS and how relevant aspects can be incorporated into the O and A Level Mathematics syllabus. Once they have been identified, sharing sessions and worksheets were worked out for teachers and students.

In working out the materials for the sharing sessions for teachers, besides familiarizing them with the features of the CAS, the emphasis was on how CAS can be introduced into the current Mathematics syllabus. I shared a numbers of different scenarios where CAS could easily be used to enhance the teaching of Mathematics. These are:

a. Solving algebraic equations and inequalities.
b. Drawing of Inverse Functions.
c. Drawing of vertical line.
d. Some interesting questions in integrations.
e. Simple Transformations.
f. Simulation of simultaneous motions described by the curves cycloid and trochoid.
g. Illustration of existence of composite function.
h. Illustration of limit of functions.
i. Illustration of Maclaurin's series.
j. Illustration of family curves in differential equations.
k. Illustration of co-web or staircase diagrams in fixed point iteration.
l. Exploration of some graphical designs.
m. Using TI92 plus and CBR™ to teach transformation of graphs.

(Some examples are in the appendix).

Worksheets were also worked out as reference materials to aid the teachers.

As for the students, 32 mathematically inclined students were selected and invited to participate in a case study project on the use of CAS in curriculum. The objective is to see the effect of CAS infused curriculum on students' achievements and their attitude towards technology. Lessons were conducted to familiarize them with the features of the CAS and subsequently worksheets were given to them.

Results

The time and effort spent were worthwhile. Feedback from the sharing sessions with teachers shows that some found it useful to incorporate CAS when preparing their lessons. However there were others who felt inadequate when using the equipment. With time and further exposure to CAS it is likely that such feelings of inadequacy can be overcome, given their strong mathematical grounding.

As for the students, the preliminary feedback was that they found it refreshing that technology is use to enhance the teaching of Mathematics. They were very impressed with the algebraic, numerical and graphical capabilities of the CAS. Many certainly welcome such a move and were eager to explore new capabilities of the CAS, such as writing their own program, playing games and how to download programs from the computer. It must be noted that the 32 students involved were mathematically inclined and this minimizes the concerns expressed in other research e.g. Lagrange (1999, p52) that students may lacked the familiarity with the technology needed to really use it to support their mathematical activities and learning.

I have also observed that some students, when they realized that they cannot use V200 in their tests and examinations, were no longer very keen on the CAS. This was similar to observations by Macintyre and Forbes (2002, p50) who remarked that once student ascertained that they couldn't use them for exam purposes, they stopped using them - preferring to work without the CAS calculator.

On a personal basis, I find it a very enriching experience. What was most satisfying was the enthusiasm shown by students with the new technology in enhancing the learning of Mathematics, and how they make use of this new tool and knowledge to further explore their area of interest. This has encouraged me to look into new technological tools, such as the Calculator-Based Laboratory™ 2(CBL2™) and Calculator-Based Ranger™ (CBR™), and explore how they can be used as new frontier in the teaching of Mathematics.

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Appendix

Some investigative works using TI92 plus or V200

I) Ask students to key in $x^2 - 1 = (x-1)(x+1)$, they get answer "True"

Ask students to key in $x^2 - 1 = (2x-1)(x+1)$, before they enter to get the answer, ask them to make a guess on the answer.(Most will say "False") Enter to get the answer, ask for comment. Figure 1.
Reason:
The first equation is an identity. So it is true all the time.
The second equation is not totally false because it is true for some values of x.
This is a good example to illustrate 2 types of commonly seen questions in Mathematics: ie. ‘proof of identity’ and ‘solve equation’

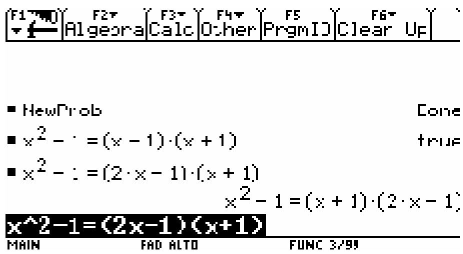

Figure 1

II) Ask students to key in $\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$, they get answer "True".

Ask students to key in $\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$, before they enter to get the answer, ask them to make a guess on the answer. (Most will say "True") Enter to get the answer, ask for comment. Figure 2.
Reason: Attempt to apply to imaginary numbers the ordinary rule for division of radicals is not valid.

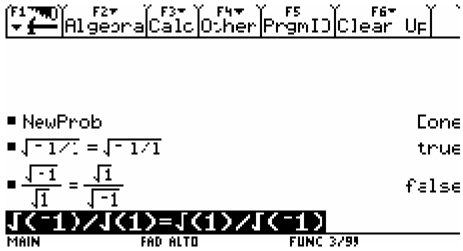

Figure 2

III) Ask students to key in $\sqrt{x}\sqrt{\frac{1}{x}}$, they get the same expression as answer.

Ask students to key in $\sqrt{x}\sqrt{\frac{1}{x}}\Big| x > 0$, they get '1' as answer.

Ask students to key in $\sqrt{x}\sqrt{\frac{1}{x}}\Big| x < 0$, before they enter to get the answer, ask them to make a guess on the answer. (Most will say '1') Enter to get the answer, ask for comment. Figure 3.

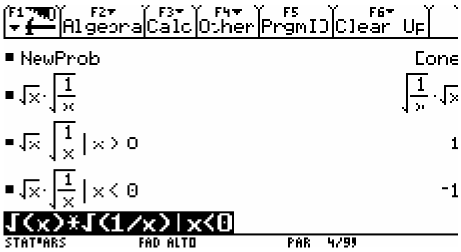

Figure 3

IV) Ask students to key in π*2, they get answer"2 π"
Ask them to key in π*2. (Note that there is a decimal point), before they enter to get the answer, ask them to make a guess on the answer (Most will say 2 π). Enter to get the answer, ask for comment. (This is a good example to show students that once they have decimal values in their intermediate steps, final answer must be in decimal). Figure 4.

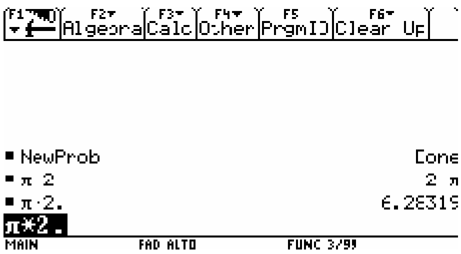

Figure 4

Using TI83, TI92 plus or V200 to draw vertical line and inverse function

Introduction: After teaching Cartesian curves and parametric curves, this is a good exercise to see if students can apply what they learn.

Task: To draw vertical line $x = 4$

- Give students a few minute to try out using either the GC or CAS
- Show them the methods: (a) for GC , (b) for CAS, (c) and (d) for both GC and CAS
  a) Use GRAPH, then 2nd PRGM and choose:4.Vertical
  b) Go to CATALOG, choose LineVert, and key in 4
  c) Use parametric equations, key in *x=4, y=T*
  d) Use Cartesian equation, key in y=9999(x-4)

Task: To draw inverse function $y = \sqrt{x}$

- Give students a few minute to try out using either the GC or CAS
- Show them the methods:

Using GC: Figures 5 and 6.

a) Set Cartesian mode, key in y1= $\sqrt{x}$, enter $\boxed{\text{GRAPH}}$, then $\boxed{\text{2nd}}$ $\boxed{\text{PRGM}}$ and choose 8.DrawInv, key in y1

b) Set parametric mode, key in x1t=t, y1t= $\sqrt{t}$, x2t=y1t, y2t=x1t

Using CAS:

a) Set Cartesian mode, key in y1= $\sqrt{x}$, enter $\boxed{\text{GRAPH}}$, then F6, and choose :3.DrawInv, key in y1(x)

b) Set parametric mode, key in xt1=t, yt1= $\sqrt{t}$, xt2=yt1(t), yt2=xt1(t)

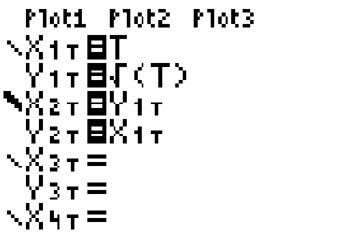

Figure 5

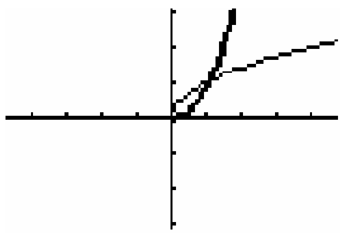
Figure 6

Using TI83, V200 to teach composite functions

The followings are good examples to enhance students understanding of existence of composite functions.

Example 1:

1. Key in the followings: Figure 7.

   $y_1 = x^2$

   $y_2 = \sin(x)(x \geq 0)$ for TI83

   $y_2 = \sin(x)|x \geq 0$ for V200

   $y_3 = y_1(y_2(x))$

   $y_4 = y_2(y_1(x))$

2. Change the style of y_4 to be thick.
3. Enter $\boxed{\text{GRAPH}}$ to see y_3 and y_4. Figure 8.
4. Explain why there is graph for y_3 in the domain $x \geq 0$ only.
   (Reason: domain of composite function fg is domain of g)
5. Enter $\boxed{\text{2nd}}$ $\boxed{\text{TABLE}}$ to see the values of the functions to explain step 4. Figure 9.

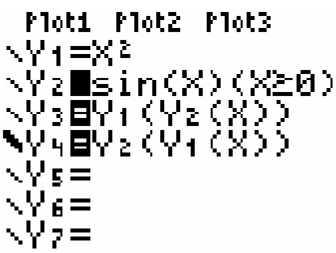

Figure 7

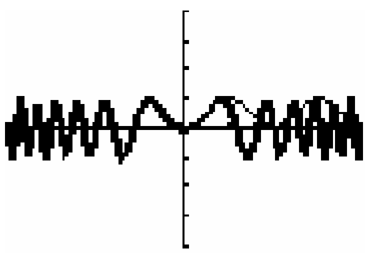
Figure 8

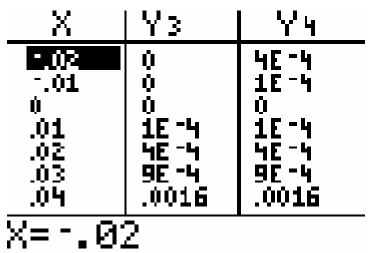

Figure 9

Example 2:

1. Key in the followings: Figure 10.

   $y_1 = x^2$

   $y_2 = \sin(x)(x < 0)$ for TI83

   $y_2 = \sin(x)|x < 0$ for V200

   $y_3 = y_1(y_2(x))$

   $y_4 = y_2(y_1(x))$
2. Change the style of y_4 to be thick.
3. Enter $\boxed{\text{GRAPH}}$ to see y_3 and y_4. Figure 11.
4. Explain why there is graph for y_3 in the domain $x < 0$ only.

   (Reason: domain of composite function fg is domain of g)
5. Explain why there is a thick line on the x-axis for y_4.

   (Reason: composite function does not exist)
6. Enter $\boxed{\text{2nd}}$ $\boxed{\text{TABLE}}$ to see the values of y_4 to explain step 5. Figure 12.

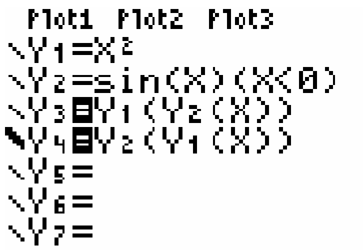

Figure 10

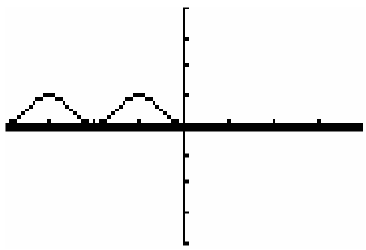
Figure 11

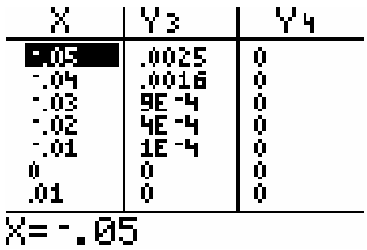

Figure 12

Example 3:

1. Key in the followings: Figure 13.

   $y_1 = x^2$

   $y_2 = \sin(x)(x \ge 0)(x \le \pi)$ for TI83

   $y_2 = \sin(x)|x \ge 0$ and $x \le \pi$ for V200

   $y_3 = y_1(y_2(x))$

   $y_4 = y_2(y_1(x))$
2. Change the style of y_4 to be thick.
3. Enter $\boxed{\text{GRAPH}}$ to see y_3 and y_4. Figure 14.
4. Explain why there is graph for y_3 in the domain $0 \le x \le \pi$ only.

   (Reason: domain of composite function fg is domain of g)
5. Explain why there is graph for y_4 in a certain domain for x.

   (Reason: a restriction of domain is given by the calculator to make composite function exist.
   Question: What is the restricted domain?)
6. Enter $\boxed{\text{2nd}}$ $\boxed{\text{TABLE}}$ to see the values of y_4 to explain step 5. Figure 15.

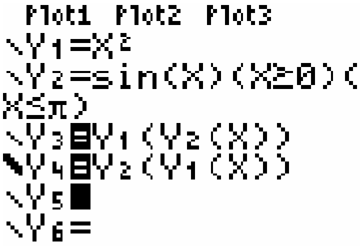

Figure 13

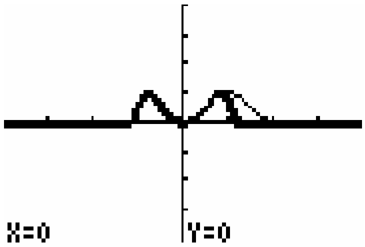

Figure 14

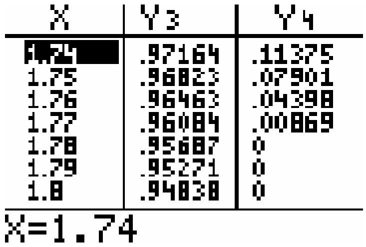

Figure 15

Using TI92 plus or V200 to illustrate Maclaurin's Series

1. in **F3,Calc**, select **9:Taylor(**
2. key in **sin(x),x,1)**
3. key in **sin(x),x,3)**
4. key in **sin(x),x,5)**
5. key in **sin(x),x,7)**
6. we get 4 expansions of sin$x$, up to various degrees. Figure 16.
7. cut and paste them into **y=** (highlight the function using the cursor control, then use ♦ C and ♦ V to paste it into **y=**). Figure 17.
8. select sin$x$, and one of the expansion,(remember to change the style in **F6,Style)** and enter ♦ GRAPH
9. we can see how close the 2 curves when the degree of the expansion is getting higher and higher. Figures 18 and 19.

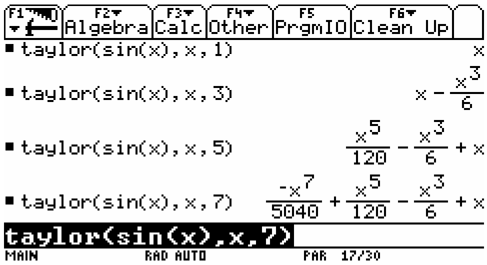

Figure 16

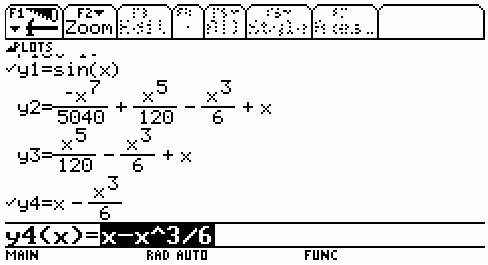

Figure 17

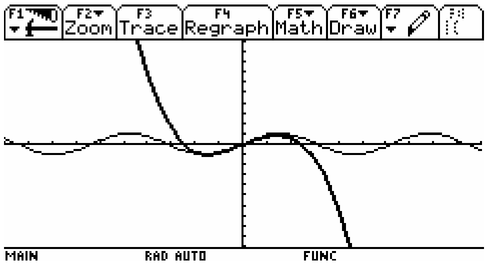

Figure 18: up to x^3

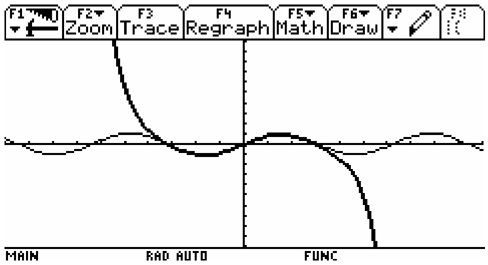

Figure 19: up to x^7