Technology Used as a Tool for Mediating Knowledge in the Teaching of Mathematics: the Case of Cabri-geometry

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Abstract

The use of technology is nowadays often strongly recommended by institutions and curricula. But on the one hand it turns out that a large part of mathematics teachers remains reluctant of integrating technology into their teaching (Guin & Trouche 1999). On the other hand several investigations of students using technology that have been carried out over the past ten years show that students do not learn from simply interacting with technology. The design of adequate tasks, the role of the teacher plays a critical role in the success of integrating technology (Lagrange et al. 2001).

In this talk, we attempt to argue that it is possible

- to make use of technology for organizing good conditions fostering learning
- but that this organization must be based on several analyses, a mathematical analysis of the notions to taught, a cognitive analysis of the possible difficulties of students in learning mathematics but also in using technology.

1. The nature of mathematical objects and the crucial role of representations

As so often stated since the time of ancient Greece, the nature of mathematical objects is by essence abstract. Mathematical objects are only indirectly accessible through representations (D'Amore 2003 pp.39-43, Duval 2000) and this contributes to the paradoxical character of mathematical knowledge: "The only way of gaining access to them is using signs, words or symbols, expressions or drawings. But at the same time, mathematical objects must not be confused with the used semiotic representations" (Duval, *ibid.*, p.60). Other researchers have stressed the importance of these semiotic systems under various names. Duval calls them registers. Bosch and Chevallard (1999) introduce the distinction between ostensive objects and non ostensive and argue that mathematicians have always considered their work as dealing with non-ostensive objects and that the treatment of ostensive objects (expressions, diagrams, formulas, graphical representations) plays just an auxiliary role for them. Moreno Armella (1999) claims that every cognitive activity is an action mediated by material or symbolic tools. Kaput and Schorr (2002, pp.28-29) claim that the development of algebra in the history of mathematics was made possible by an entirely new mode of thought "characterized by the use of an operant symbolism, that is, a symbolism that not only abbreviates words but represents the workings of the combinatory operations, or, in other words, a symbolism with which one operates."

The activity of solving mathematical problems, which is the essence of mathematics, is based on both an interplay between various registers and treatments within each register. Each register has its own treatment possibilities and favors specific aspects of the mathematical activity. Besides registers, individuals may have recourse to tools for performing a mathematical activity and namely over the past years the recourse to technology has become very important in various domains of

mathematics. Tools allow operating on mathematical objects in specific registers, a tool making use of one or several registers. For example, Derive has mainly recourse to symbolic expressions but also to graphical representations in coordinate geometry. Dynamic geometry software as Geometer Sketchpad or Cabri-geometry are intended to draw variable geometrical diagrams on the screen of the computer but for example Cabri is also providing menus and feedback messages in natural language as well as dynamic markers such as blinking lines or points.
It is important to stress that the semiotic registers of these technological environments may deeply differ from what they are in a paper and pencil environment. It is especially the case with dynamic geometry software that offers diagrams of a very specific nature: variable diagrams that can be continuously modified while keeping their geometrical properties when dragged. The direct manipulation of diagrams has a visible spatial effect but has also a mathematical counterpart. The operations performed within the register of dynamic diagrams (that Duval calls treatment) have thus a specific nature and this leads to two assumptions that are currently shared by various research works and supported by empirical research. They will be presented in the next section.

2. Mathematical knowledge and instrumental knowledge

Two main hypotheses underlie our analysis of the role of technology in the learning and teaching processes.
First hypothesis: We assume that a tool is not transparent and that using a tool for doing mathematics not only changes the way to do mathematics but also requires a specific appropriation of the tool. In the last decade, some psychologists (Vérillon & Rabardel, 1995) have shown through empirical research, how the tool (also called artefact) itself gives rise to a mental construction by the learner using the tool to solve problems. The *instrument,* according to the terms of Vérillon and Rabardel, denotes this psychological construct of the user: "The instrument does not exist in itself, it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity." The subject develops procedures and rules of actions when using the artefact and so constructs *instrumentation schemes* and simultaneously a representation of the properties of the tool. A scheme must be understood as an invariant organisation of actions in a given class of situations. The notion of instrumentation scheme refers to an invariant organisation of actions involving the use of an artefact for solving a type of tasks.

Second hypothesis: tools like those offered by information technology embed mathematical knowledge (as for example already visible in Cabri from the denominations of menu items — perpendicular bisector, parallel line…—) and the use of such tools requires the integration of both mathematical knowledge and knowledge about the tool.

An example of instrumentation schemes
Let us illustrate this claim with the example of the construction of a parallelogram in Cabri. Students are given two segments AB and AD and they are asked to construct the parallelogram ABCD. In a compass and ruler construction in paper and pencil environment, students would use a strategy based on the congruence of opposite sides. But in Cabri, almost all students use the strategy of constructing parallel lines to the given segments in order to obtain the fourth vertex C. It illustrates very clearly how much the preferred strategy is linked to the domain of efficiency of the tool. Constructing parallel lines in paper and pencil would be more tedious since the ruler and compass construction of a parallel line to a line is based on the construction of a parallelogram. In Cabri, the tool parallel line is available and students have a spontaneous recourse to it since the typical feature of a parallelogram for students is the parallelism of the sides. After parallel lines and

point C are constructed (Fig.1), then the two additional sides (or the polygon) have to be constructed and parallel lines must be hidden (Fig.2). In this sequence of actions, called by Verillon and Rabardel (1995) *scheme of instrumented action*, are intertwined both mathematical knowledge and knowledge of how to use the tool for fulfilling the task to produce the dynamic diagram of a parallelogram. The use of the tool affects not only the choice of the construction strategy but also the actions to be done. In Cabri, segments have to be constructed since a segment cannot be obtained as a part of a line (Fig.3).

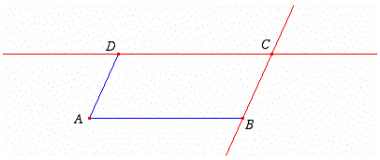

Fig.1

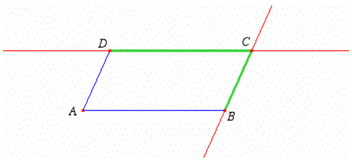

Fig.2

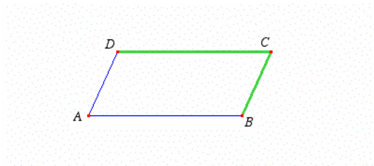

Fig.3

The mathematical knowledge of the user is thus another critical factor affecting the type of strategy that is used. Students are often successful in constructing a parallelogram in Cabri by obtaining the fourth vertex as the intersecting point of lines parallel to the given segments. But the teacher may expect another strategy, the use of a central symmetry around the midpoint of segment BD. This latter strategy is valid even when the parallelogram is “flat” whilst the former one would not provide a flat parallelogram. The central symmetry strategy is shorter than the parallel line strategy since the parallel lines have not to be hidden in order to make visible the only parallelogram. The scheme of instrumented action in Cabri attached to this strategy differs from the preceding one and clearly depends on mathematical knowledge. It involves the invariance of a parallelogram under central symmetry, a geometrical property which is not operational in a paper and pencil environment for constructing a figure. It is generally not proposed by students and must be introduced by the teacher. An instrumented task can thus be the source of reinforcing or introducing knowledge. This is an important issue related to the integration of technology into teaching.

As described above, a scheme of instrumented action involves actions directly linked to a specific use of the artefact. For example, in Cabri, in order to construct a parallel line to a segment, the user has to perform a sequence of elementary actions, selecting a menu, pulling down it, selecting the tool in the menu, showing a point and a line. Each of these actions requires the move of the cursor by using the mouse and clicking. The user has also to construct an invariant organisation attached to the sequence of these elementary actions. Such an organisation is called *scheme of usage* by Rabardel. A scheme of instrumented action involves several schemes of usage. The design of interface certainly affects the construction of schemes of usage. However mathematical knowledge is also involved in a scheme of usage. Below are presented two schemes of usage in Cabri requiring a functional view of a geometrical object, i.e. a conception of geometrical objects as a function of other objects.

At middle school or even high school, students do not have such a conception and therefore may encounter difficulties in using tools of DGE[1]. Constructing parallel lines in DGE requires for example designating with the mouse two elements of which a parallel line is function of, the direction (i.e. a line) and a point through which the parallel line is passing. Very often the students working with Cabri we could observe at middle school or even high school showed the direction

[1] In the paper DGE denotes Dynamic Geometry Environments

and were waiting for the parallel line to be drawn and did not understand why the computer did nothing. After a while they clicked anywhere in the screen and more than often they clicked on the line giving the direction. The obtained parallel line was thus coinciding with the line and could not be seen unless the cursor came close to the line and an ambiguity message was displayed "What object?". Understanding this message requires being familiar with the ambiguity notion in Cabri, i.e. a sophisticated knowledge of the tool. Very often students do not understand the situation they have created and a solution can be found only with the help of the teacher.

Another example can be given with the construction scheme of the midpoint of a segment. Very often the beginners show the position of the midpoint to ask the computer to construct the segment whereas they could show any place on the segment since the midpoint is defined as a function of the segment in a DGE and not as a spatial object.

The interface can make students aware of the necessity of showing the variable elements the object to be constructed is function of. Taking into account this difficulty of students, the designers of Cabri-junior (on the TI 83 Plus or Silver) decided to display under the form of a dotted object the temporary spatial position of the geometrical object to be constructed. This temporary position is determined by the first variable element already shown by the user and the current position of the cursor. When the user clicks the final variable element defining the object to be constructed, this latter object is displayed in the usual way (Fig.4). This new interface continuously informs the user and avoids the reaction of students saying that machine is not working.

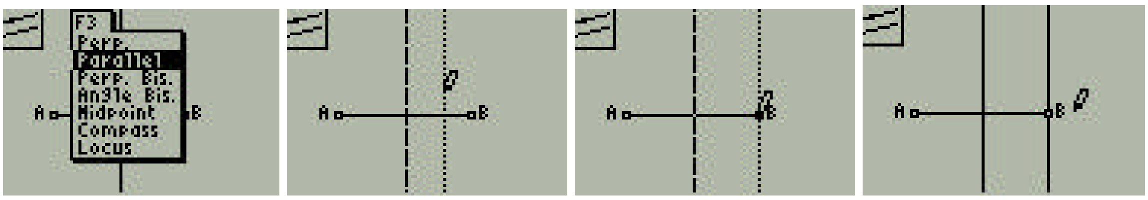

Fig.4 – The interface of the construction of a parallel line to a line in Cabri Junior

Briefly speaking, solving mathematical tasks in a technological environment requires two kinds of knowledge, mathematical and instrumental. Most of time, especially because ICT used in the teaching of mathematics embeds mathematics, both types of knowledge interact in the use of technology. It will be illustrated in the next section by some results coming from empirical investigations in the case of DGE. We decided to focus on the use of the drag mode that is certainly one of the most typical features of such environments.

3. Empirical investigations about the use of the drag mode

The way in which students drag as they solve problems was investigated by several researchers. Hölzl (1996) identified the "drag and link approach" in students' construction processes of Cabri diagrams. Students relax one condition to do the construction and then drag to satisfy the last condition. They obtain a diagram visually correct and want to secure the diagram by using the redefinition facility of Cabri. But often it does not work because of hidden dependencies that are not considered by students. As said above, they often are not aware of functional dependencies between objects. Although Hölzl does not refer to instrumentation, this "drag and link approach" would be called an instrumentation scheme in terms of Vérillon and Rabardel. The students constructed an instrumentation scheme not compatible with the functioning of Cabri.

Arzarello, Micheletti, Olivero, Robutti, Paola, and Gallino (1998 a & b) identified different kinds of dragging modalities that were not all referring to an organized experimentation: “wandering dragging,” “lieu muet” dragging, and dragging to test hypotheses. Wandering is just moving without a predefined aim for searching for regularities whilst Lieu muet dragging refers to dragging in such a way that some regularity in the drawing is preserved. The dragging to test hypotheses obviously presupposes that regularities have already been detected which are not systematically tested. Goldenberg (1995) notes that often students do not know how to conduct experiments and are unsure what to vary and what to keep fixed. Thus a student’s purposeful move from wandering dragging to lieu muet dragging represents a cognitive shift.

From these investigations, it appears that the power of the drag mode in exploration is not spontaneously mastered by students. It may also happen that the variability introduced by drag mode makes the task more complex. Below is reported an investigation on the construction of a proof by 9th graders (Abd El All 1996).

Complexity in the proving process introduced by variability
All students of a class (9th graders) were given the following tasks. They worked in pairs. The work of four pairs was observed and audio-recorded.
Task 1 (Fig.5)
Students were given a rectangle ABCD and the quadrilateral IJKL of the midpoints of the sides of ABCD in a paper and pencil environment. They had to determine the nature of IJKL and to justify their answer. All students found that it is a rhombus.
Task 2
Then they had to predict whether IJKL would remain a rhombus in any movement of B which does not preserve ABCD as a rectangle. All students predicted that IJKL will not be any longer a rhombus.
Task 3
They were given in Cabri a rectangle ABCD. Then they had to construct the circle with center D and radius AC and to redefine B as belonging to the circle. They were asked. “Is IJKL still a rhombus?” (Fig.6)

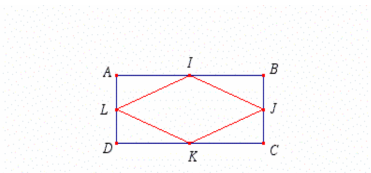

Fig.5

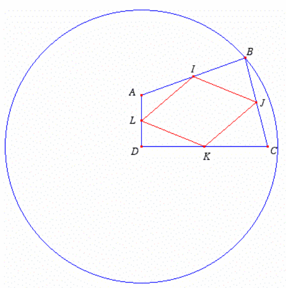

Fig.6

The sequence of questions was designed with the intention to favor the need of having recourse to proof. In a computer environment, the need for proof cannot any longer be favored by the uncertainty of the result. It may arise for intellectual motives because the student wants to know why a phenomenon takes place. As pointed out by the Piagetian perspective, a means of provoking this intellectual curiosity may be caused by conflict between what the learner believes or predicts and what actually happens. Such a conflict may be achieved by asking the students to predict properties of the diagram before allowing them to check on the computer, as in this problem. In task 1, we expected that students would prove that IJLK is a rhombus by using the specific properties of a rectangle (theorem of Pythagoras, properties of reflection, congruence of right angle triangles) rather than using the more general property of the midpoint segment that is valid even if ABCD is no longer a rectangle. In task 2 we expected them to predict that IJKL is no longer a rhombus as they probably would have justified in task 1 that IJKL is a rhombus by using properties of a rectangle. In task 3 they should be very surprised by observing that IJKL remains a rhombus and would be eager to understand why. This is why they were not asked to justify what they observed. We expected that from the strength of the contradiction would arise the need of justifying.

It is exactly what happened. Students were so surprised to discover that IJKL was a rhombus that they became eager to prove why without being asked in an explicit way to do so. However it took time for them to construct a justification. We could observe that the variability of the diagram created several difficulties for students. We comment here the effect of variability on the use of a theorem. Some students did recognize that IJ was the segment joining the midpoints and evoked the property of this segment but they were not sure about the validity of using the theorem when the diagram moved. V. and L. for example evoked the theorem of midpoint segment but did not dare using it. Pushed by the observer, they selected a triangle and V. looked carefully at the triangle and the midpoint segment when point B was dragged. She expressed her satisfaction:
"The theorem of midpoints moves, yes it moves. It works even if we move"
L. confirmed: " the midpoint theorem it works"
V.: "it works the same way"
V. even tried to justify the invariance of the property in the drag mode:
"they are all the same because there is always the same length. AC it is two times that. It is always two times that. It is always two times that and it works there all the time even if we move anyway."
A student of another pair wrote at the end of their proof: "As DB is always the radius, this proof is always right" and then the partner added: "for any position of B"

For these students a proof seems to be carried out only for a particular instance of the diagram. From the work in Cabri arose for them the problem of the shift from proving on one instance to proving on all instances. According to Netz (1999) Greek proof was rather done on a generic example than in a general case. The validity of the general statement was claimed at the end of the proof in the final part called Sumperasma. The expression of the validity claimed by students for all instances obtained by the drag mode can be compared with the expression of the sumperasma in the Greek proof.

In this example, Cabri provided a window (Noss & Hoyles 1996) on the conceptions of students about proof but the complexity introduced by the variability of the diagram acted as a catalyst for change in this conception for students such as V and L who became aware of the fact that a theorem may be valid for a moving diagram since the relations between elements remain unchanged. They learned from the complexity brought by the computer environment that offered to the students another window on mathematics (Noss & Hoyles *ibid*.). This point of view was supported by several researchers (CAS used as a lever to promote work on the syntax of algebraic expressions Artigue, 2002, p.265, Lagrange 2002, p.171, or DGE as assisting pupils to explain the properties of a rhombus and to distinguish them from those of a square, Hoyles & Jones 1998).

4. The medium role of technology for teaching mathematics

The idea of computer environments as reifying abstract objects and structures originates from the notion of microworld (Minsky-Papert 1970, Thompson 1985) in which it is possible to explore and experiment on representations of abstract objects as if they were material objects. The same potential is often considered in computer environments such as CAS, spreadsheets or Dynamic Geometry environments. They offer working models on which the users can carry out actual experiments corresponding to the thought experiments they can perform on abstract objects. But if the thought experiments on abstract objects are not available (as it is often the case for learners), a crucial question about learning is whether such environments could favor an internalization process of the external actions in the environment.

The idea of internalization process is not new and was present in the Vygostkian theory of semiotic mediation and of tool. Vygostky considered that signs and tools belong to the same category of mediators of human activity and as such are fundamental elements in the process of constructing concepts. He coined the difference between technical tools and psychological tools (that he also called signs) by considering their respective functions. The function of a technical tool is externally oriented and helps acting on the outside environment to change it whereas the function of a sign is internally oriented and contributes to change the mental constructions of the individual. Vygostky described the internalization process as a process transforming a technical tool into a psychological tool.

This Vygostkian approach has been adopted by Bartolini Bussi and Mariotti (1999) for artefacts or computer environments (as described in Mariotti 2002). In the case of CAS and DGE incorporating mathematical knowledge, it seems possible that teaching contributes to the internalization process from the external tools offered by the environment to the construction of the meaning of the mathematical concept. This has been done by Mariotti (*ibid*.,p.713) about the notion of geometric construction by using the external tools, drag mode and history command of Cabri. As told by Mariotti, “the temporal sequence of the constructions’ steps represents the counterpart of the logic hierarchy between the geometric properties of a figure.” “The availability of an external tool referring to the procedure of the construction in its temporal sequence very often contributed to the production of a description and a correct justification of a construction.”. But as argued by Lagrange (2002, chapter 3) about the use of CAS, the design of the tasks proposed to students is critical in order to foster a conceptualization process. The task must introduce a problem that can be appropriated by the students and be experienced by them as a real problem that they attempt to solve by involving knowledge and not for meeting the expectations of the teacher —corresponding to what Brousseau (1997) calls adidactical situation—.

An example : a pointwise conception of figure and geometrical transformation

An example related to the learning of a pointwise approach of geometrical transformation will be given below as an illustration. Two points of view are useful in the use of geometrical transformations: a global point of view and a pointwise point of view. The former allows the use of conservation properties about figures: a reflection transforms a straight line into a line, a circle into a circle, two parallel lines into two parallel lines. The latter allows to determine or to construct the image of figures that cannot be reduced to simple figures the image of which is known. It is also useful when no global property is available about the transformation. In both cases, a fundamental property is used: a figure is a set of points. This conception of figures is far from natural for students beginning high school. The problem is that it is not operationally practical in paper and pencil environment. But it is possible in Cabri to define a transformation with a pointwise approach: the transformation is a macroconstruction which provides a point when the initial object is a point. Obtaining the image of a figure is done by using the tool Locus on the image of a variable point of the initial figure. The creation of a variable point on the initial figure and the use of Locus are the external counterpart in Cabri of the following definition based on a pointwise approach of the figure and of the transformation: the image of a figure is characterized as the set of points images of the points of the initial figure. Jahn (2000) made use of this possible mediation in Cabri of a pointwise approach in a teaching sequence in a grade 10 class. An unknown transformation X was given as a black box (a macro construction providing the image of any point). It was actually an oblique symmetry. Students had to determine how to construct the image of a point by exploring the effect of the macro-construction. All succeeded. Then they had to find the image of a circle under transformation X. As this latter did not preserve length, the only way to construct the image was to obtain it as a locus of a variable point of the circle (Fig.7). Cabri could offer visual feedback showing that a circle could not be the image of the initial circle. A point on the initial circle did not have its image on the supposed image circle. The task offered good conditions for the construction of a solution by the students.

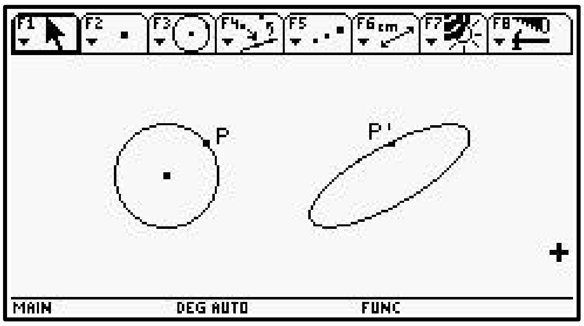

Fig.7 – Locus of the image P’ of a variable point P moving on a circle

In this sequence, it was intended to mediate the pointwise approach to geometrical transformation through the tools macro-construction and Locus. However, although the tool Locus was already introduced in the teaching sequence, its scheme of usage was not yet constructed by the students. “They had great difficulties in understanding the order in which the inputs should be selected to successfully apply the Locus tool” (Jahn, *ibid.*, p.I-101). But on their own they had recourse to the tool Trace and produced the trajectory of the image of a variable point of the circle. Then the teacher intervened to favor the move to a mathematical interpretation of their actions in Cabri under the form of the pointwise characterization of the image mentioned above. He reinforced the use of tool Locus with the arguments that a trace cannot be saved whilst a locus can or that a trace cannot move when the initial circle moves. The scheme of usage of Locus required the construction of a mathematical meaning of the pointwise view of the notion of image. *A contrario* this meaning was constructed by means of the mediation of image by the combination of the drag mode and the tool Trace (another scheme of usage).Then when other transformations were presented to students of this class, even outside the Cabri environment, from themselves they claimed that it was necessary to check whether the transformation preserves collinearity or not. This was a sign of the internalization process of another point of view than the global one, and very likely of a pointwise approach (as stated in further activities). This example highlights the embedding of mathematical and instrumental knowledge. It also shows the importance of the role of the teacher.

The interventions of the teacher are essential for making possible the construction of a correspondence between mathematical knowledge and knowledge constructed from the interactions with the computer environment. Because as pointed out by the instrumentation theory, the meaning constructed by the student when using the artefact, may differ from what is intended by the teacher, the interventions of the teacher are critical to let the meanings evolve towards culturally shared meanings of mathematical knowledge. This may be done by using collective discussions in classrooms as proposed by our Italian colleagues (Bartolini Bussi & Mariotti, *ibid*.).

Below are reported two teaching experiments based on a semiotic mediation process.
The first one deals with the notion of operative status of a statement in a deductive step (hypothesis or conclusion). The second one deals with the notions of independent and dependent variables as well as the notion of graph of a function.

In both experiments, the design of tasks in the technology based environment fulfills a double role,

- contributing to the construction of a priori expected instrumentation schemes being the counterpart of mathematical objects and operations
- proposing problem situations for which a specific aspect of the notion to be taught which is object of teaching is a solution means (adidactical situation in terms of the theory of didactical situations, Brousseau 1997).

Mediation of the operative status of a statement in a proof in Cabri Junior
Duval (1991) analyzes a deductive step in a proof as made of three parts

- the premises or hypotheses
- the rule of inference (theorem or definition taken from a corpus accepted by the community in -and for- which the proof is done)
- the conclusion.

He also pointed out on the function (called by Duval *operative status*) of a statement in a proof. A statement in a step may have the function of hypothesis or of conclusion but in both cases the epistemic value of the statement is true. This is why it is so difficult for students to assume for a while that only the premises are true whilst the conclusion cannot be considered as true. It is difficult for students to accept that what is important in a proof is not the truth of a statement but its operative status (given in the data, proved and thus being part of potential premises, not yet proved). As a consequence students often do not see a difference between a theorem and its converse, both referring to the same objects and the same relations. They do not understand the function of hypothesis and of conclusion in the statement of the theorem and therefore are not able to distinguish between hypothesis and conclusion as often noticed by teachers.

The drag mode of dynamic geometry can be used to introduce two critical aspects for understanding the distinction between hypothesis and conclusion:

- a dissymmetry in action that allows to make the distinction between hypothesis and conclusion
- the variation that transforms the static truth into a dynamic phenomenon in which the consequence of the action becomes true when conditions are satisfied.

Example in Cabri-Junior (Fig.8)
Construct any quadrilateral ABCD (more general hypothesis), its diagonals and the midpoint of each diagonal. Drag any vertex A, B, C or D so that the midpoints are coinciding (variation of one element in order to obtain an additional condition). The focus of the control is on the superimposition of the midpoints.
The visible effect is the change of shape. The obtained shape is a parallelogram.

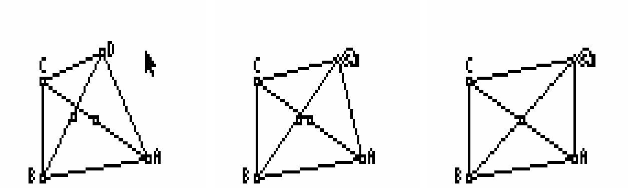

Fig.8 – Dragging vertex D in a quadrilateral until making midpoints of the diagonals coinciding

Changing the condition on an object by dragging it (here the diagonals), implies a visible change on other objects (here the parallelogram). The condition is what the student is directly changing. The visible effect is the result of the implication. The condition plays the role of the hypothesis. The effect plays the role of the conclusion. The link between condition and effect introduces a causal effect oriented from the hypothesis to the conclusion.
A teaching experiment was recently conducted with 8^{th} graders (13-14 years) in France (Coutat 2003). An initial test was given to all students of the class in order to select a sample of 10 students who do not distinguish a theorem and its converse. The ten students were then faced with tasks to be done in Cabri Junior. They worked in pairs. Then final tests were given to the whole class about the same topic.

In the initial written test, students had to group the statements referring to the same theorem in proposed statements in two cases: the midpoint segment theorem and the right angle inscribed in a circle. As an example, let us present the first question about the midpoint segment theorem.

> " Draw a triangle ABC and M the midpoint of side [AB]. Draw a line passing through M and parallel to side [BC]. What do you notice?
> Among the following properties, indicate those which seem to correspond to the displayed property. Justify your choice.
> - In a triangle if one draws the parallel line to a side passing through the midpoint of another side, then this line is passing through the midpoint of the third side.
> - In a triangle a parallel line to the third side is passing through the midpoints of the two other sides.
> - In a triangle, the line passing through the midpoint of a side and parallel to a second side intersects the third side into its midpoint
> - In a triangle, if a line is passing through the midpoints of two sides, it is parallel to the third one."

About one third of the students thought that all statements were the same or that statements 1, 3 and 4 were the same. Very often no justification was given or students wrote that all statements were true.

Then the selected students worked in pairs under the guidance of an experimenter in an one and half hour session. They were given two kinds of tasks: formulating a theorem from manipulations in Cabri Junior and associating manipulations in Cabri Junior to the statement of a theorem.

Tasks 1

> "Create a triangle ABC then construct the perpendicular bisector of [BC]. Then construct the altitude of triangle ABC passing through A.
> What manipulation do you have to do to make the perpendicular bisector coinciding with the altitude? What is the consequence of this manipulation on triangle ABC?
> Formulate in terms of if…then the corresponding property."

> "Create a triangle ABC, draw two perpendicular bisectors of this triangle then the circumscribed circle of the triangle. What manipulation do you do in order to make the centre of the circle coming onto a side of the triangle? What becomes the triangle?

Formulate a theorem under the form *If… then* that you can associate to the manipulation that you have done."

Tasks 2

"If a parallelogram has a right angle, then it is a rectangle"
What manipulations can be associated in Cabri Junior to this statement in order to make apparent the distinction between hypothesis and conclusion ?

"A quadrilateral with two opposite sides parallel and equal is a parallelogram."
Same question

After each task the experimenter intervened to point out the link between hypothesis and actions and the link between conclusion and the consequence of the actions and proposed the converse theorem in which students were asked to find hypotheses and conclusions by evoking the action on Cabri.

The drag mode is the key element in the mediation process offered by Cabri Junior. It allows to start with a diagram not satisfying the conditions (this is quite impossible in paper and pencil environment) and to act on the diagram to make it satisfying one (or several) condition(s). The dynamic change of the diagram makes visible the effect of the condition. In tasks 1, students must produce formulations at two levels:

- at the level of actions and manipulations
- at the theoretical level in mathematical terms.

These tasks are intended to facilitate the correspondence between the actions in Cabri Junior and mathematics. In tasks 2 students must analyze the statement of a theorem in terms of hypothesis and conclusion and Cabri offers a visualization of their analysis.

All five pairs did not encounter problems in the manipulations with Cabri-Junior in finding what they had to move in order to obtain the conclusion. Formulating a theorem was not so easy for some of them who needed the help of the experimenter. Generally the formulations of theorems kept some elements coming from the manipulation such as "If one moves … then…" or "In a triangle if the altitude and the perpendicular bisector are superimposed, then one obtains an isosceles triangle" or "If… then the triangle becomes isosceles" "If… then the triangle will be isosceles". The dissymmetry introduced by the manipulations in Cabri-Junior is reflected in those formulations. The working session on Cabri Junior was short (1,5h) and it is clear that the internalization process takes much more time and a longer sequence of tasks. The students seemed to be able to produce a hybrid formulation of a theorem combining mathematical terms and some elements attached to the manipulations. It is important also to know whether they were able to distinguish hypothesis and conclusion outside the Cabri environment. After the session each pair was given by the experimenter a theorem and its converse and asked to say whether they were identical or not. Almost all students found them identical but as soon as the experimenter evoked Cabri, students were able to simulate in thought what they could do in Cabri and gave a correct answer.
In the final tests (given 3 weeks after the session), that turned out to be difficult for all students of the class, the group of the Cabri students differed from their classmates in that seven of them were able to find the hypotheses and the conclusion in a theorem. In a test about finding theorems identical to a given theorem in a list of statements the students of the Cabri group had definitely better results than the others (as presented below Fig.9), although they were the students who initially encountered the greatest difficulties.

Written Test

For each of the proposed formulations, select the formulations which seem to be equivalent to the given theorem, i.e. which have the same hypotheses and conclusion.

1 – If a quadrilateral has its diagonals intersecting into their midpoint then it is a parallelogram.
- A parallelogram has its diagonals intersecting into their midpoint
- A quadrilateral that has its diagonals intersecting into their midpoint is a parallelogram.

2 - In a triangle, if the altitude is coinciding with the perpendicular bisector, then the triangle is isosceles.
- A triangle that has a perpendicular bisector coinciding with its altitude is an isosceles triangle.
- An isosceles triangle has its altitude and its perpendicular bisector are coinciding
- A triangle is isosceles if an altitude and the corresponding perpendicular bisector are coinciding

3 – In a triangle, if a line is parallel to a side and intersects a second side into its midpoint, then it intersects the third side into its midpoint
- In a triangle, a line intersecting two sides into their midpoints is parallel to the third one.
- In a triangle, a line intersecting a side into its midpoint and parallel to another side intersects the last side into its midpoint.
- In a triangle, a line that is parallel to a side et and that intersects a second side into its midpoint is passing through the midpoint of the third side

Tasks	1	2	3
Group without Cabri J	100%	46%	31%
Experimental Group	100% (9/9)	66% (6/9)	77% (7/9)

Fig. 9 - Table of the proportion of correct answers

Mediation of the notions of dependent and independent variables and of graph of a function

Let us present an example of each kind of task in a DGE taken from our work on functions and graphs of functions (Mariotti et al. 2003*)* that took place in French and Italian schools (grade 10) under the form of a long term teaching sequence (Falcade 2003).

Task 1

Points A,B and P are free points in the screen of Cabri. An unknown macro-construction provides a fourth point when showing A, B and P as input. The task of the students is to say whether it is possible to move directly each of these points and what points move when each of those points is dragged (Fig.10).

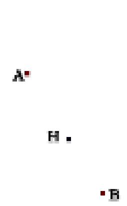

Fig.10

This task obviously introduces in the artefact the counterpart of dependent and independent variables and offers an external means for distinguishing between both. This task turned out to be a reference situation for the students. This external means was evoked by a pair of students later during the teaching sequence who hesitated between two expressions when they had to express symbolically that P is function of Q in a geometric figure: P= f(Q) or Q= f(P) ? They evoked without performing it what should happen when dragging these points. The instrumentation scheme for recognizing a variable depending on another one is the learning aim of the task.

Task 2

Students were asked to propose a geometrical construction of H starting from the three points A, B and P. Two solving strategies leading to a construction were possible:

- either by using Trace observing that H is moving on line (AB) when P is dragged and that line (PH) was always perpendicular to line (AB) (Fig.11)
- or by considering H as intersection point of two traces obtained when moving two of the free points and determining the obtained traces as curves depending only on the two free points that are not dragged (Fig.12)

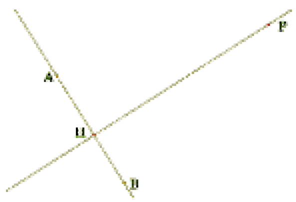

Fig.11

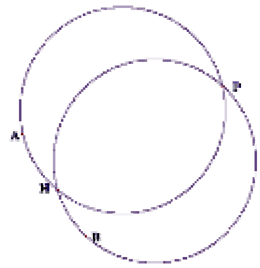

Fig.12

The notion of H as a result of a construction process starting from P is an implicit tool of the first strategy (process aspect of function).
The notion of H as intersection of curves is the implicit tool of the second strategy as well as the recognition of the geometrical curves (circles and line) from their appearance. The notion of image set as obtained when varying the independent variable is underlying this strategy.
Of course solving task 2 also involves an instrumented activity (using adequately the combination of drag and trace). This shows that very often a task may fulfill a double role creating the external tool and requiring a mathematical construction.
All students used the second strategy in Italian and French schools that allowed the teacher to introduce the notion of image set.

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5. Conclusion

Two issues must be stressed at the end of this paper: the one dealing with the role of the teacher in the interplay between mathematics and the environment, the other one with the computer environment.
Ruthven (2002) mentioned the difficulties of designing tasks in computer environments between two extremes: on the one hand proposing tasks with great technical and conceptual demands but with the danger of being didactically uncontrolled, on the other hand developing a carefully controlled student experience with the danger to give ‘keystroke by keystroke’ instructions to students. The interplay between the mathematical dimension and the instrumental dimension is one way of solving this dilemma but only partly. The collective discussion and interventions of teacher play a critical role under various aspects such as:

- transforming what has been done by students in the task in something mathematically legitimate by introducing mathematical terminology, helping students to formulate in mathematical terms
- evoking the environment to help the students when they have difficulties when back to paper and pencil.

The instrumental dimension has always been present in the history of mathematics, be it in the treatments operated within registers as claimed at the beginning of this paper (cf. quotation of Kaput & Schorr, §1) or in the use of the available technology. Paper technology and printing technology played certainly an important role in the development of mathematics by facilitating the representation of mathematical ideas and the expression of relations by spatial configurations in the sheet of paper, not only in geometry but also in arithmetic and algebra. Dynamic geometry environments introduce a spatial representation of another key feature of mathematics, the variability. We are only at the beginning of taking advantage of the semiotic mediation potential of this new dimension in the teaching of mathematics. But even in this era of great possibilities offered by dynamic geometry environments, there is a need of pursuing the reflections on the design of interface. The deep intertwining of the mathematical and instrumental knowledge in the use of computer environments illustrated at several places in this paper implies that interface features certainly affect the construction of instrumentation schemes by the user and thus the construction of mathematical knowledge.

We would like to conclude by stressing the importance of introducing prospective teachers to the instrumental dimension of the use of technology and of making them aware of the critical role of interface features.

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