

ADDING A SPARKLE TO COORDINATE GEOMETRY IN 2D and 3D

DOUGLAS BUTLER

iCT Training Centre, Oundle School

Peterborough

United Kingdom

debutler@argonet.co.uk

Web: www.argonet.co.uk/oundlesch

Abstract

Of all the academic subjects, Mathematics has possibly the broadest range of computer-related applications which can help with visualisation and greater understanding. For many teachers this is the great escape from the chalk, but for many though it represents a serious challenge to their well established and perfectly effective teaching styles, throwing up significant training issues.

Some classrooms are now being fitted with data projectors, and new techniques have to be established to make the best use of them. The key is variety, mixing regular teaching strategies with occasional computer-generated images.

This presentation will explore the wealth of **Java applets** now available on the web, and the new capabilities of **Autograph v.3**, to bring new life into the teaching of vectors, transformations, calculus and trigonometry. It will also show how dynamic 3D objects (lines, vectors, planes, shapes and surfaces) can help students make sense of cross products and all the calculations involved in 3D intersections and shortest distances.

Finally, with students the world over tending to find the subject dry and uninteresting, and teachers thinking they have discovered better things to do with their lives, this paper will also try to make the point that technology can have a role to play here in rescuing this situation. Technology can help to make the subject appear more lively and relevant, and to add a sparkle to the lessons, thus making the process of teaching and learning the subject more fun, and more effective.

CAVEAT

Since this presentation largely consists of dynamic moving images, a static paper is not really the right medium to summarise the main points. Like a still photo of a theatrical performance in a newspaper, it can never expect to convey the spirit of the real experience.

New Classroom Teaching Techniques

Many teachers are ill-prepared to make the best use of computer-driven images as part of their teaching. They are anxious about things going wrong which will compromise the effectiveness of the lesson. Interactive White Boards are also being installed in schools and colleges in some developed countries, giving teachers yet another technological layer to master.

So far as content for helping with the teaching of coordinate geometry is concerned, there are various sources of material for the dynamic classroom, and this paper is concentrating on:

1. Java and Flash resources from the Web
2. Dynamic Software (Autograph v.3)

1. JAVA AND FLASH RESOURCES from the Web

A collection of good resources is maintained by the author on:

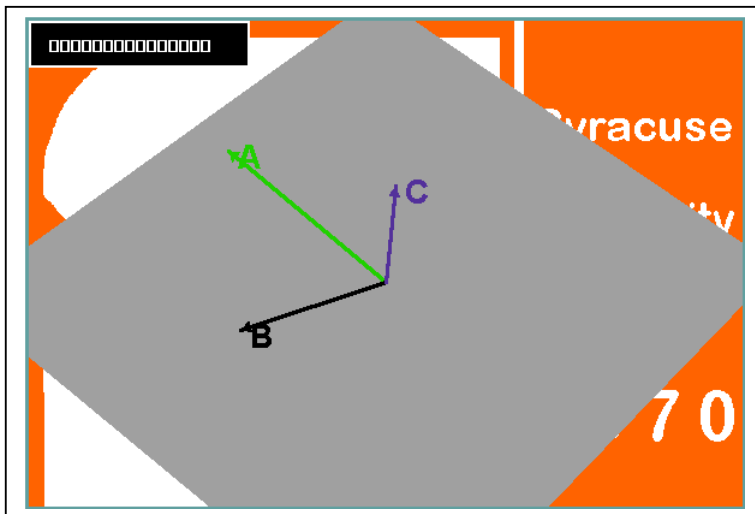
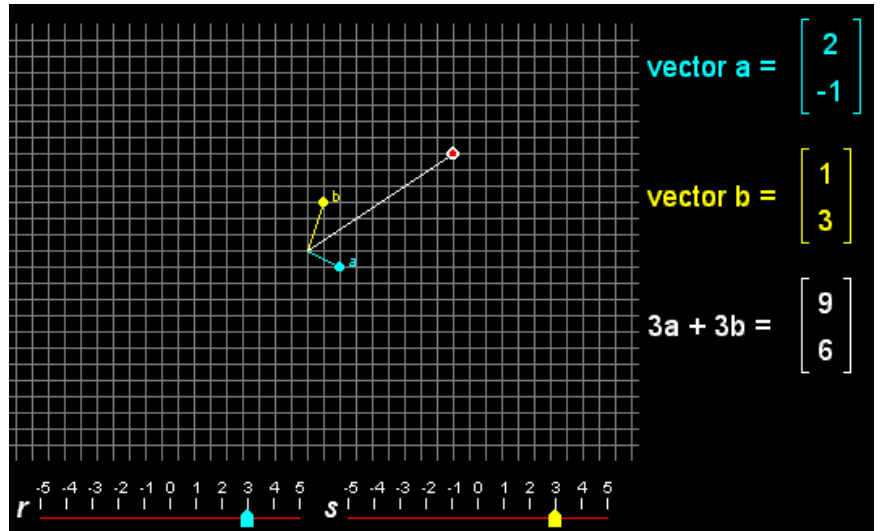
<http://www.argonet.co.uk/oundlesch/mlink.html>

covering many aspects of the mathematics curriculum. Here are some free resources to help with the teaching of VECTORS in 2D and 3D:

An illustration of the linear combination of 2 vectors in a 2D plane.

This comes from "Waldo's Maths Pages" (by Ron Barrow, UK), part of a suite of fully interactive Java pages for the secondary and college curriculum.

<http://www.waldomaths.com>



The Cross Product illustrated dynamically from a page by Syracuse University.

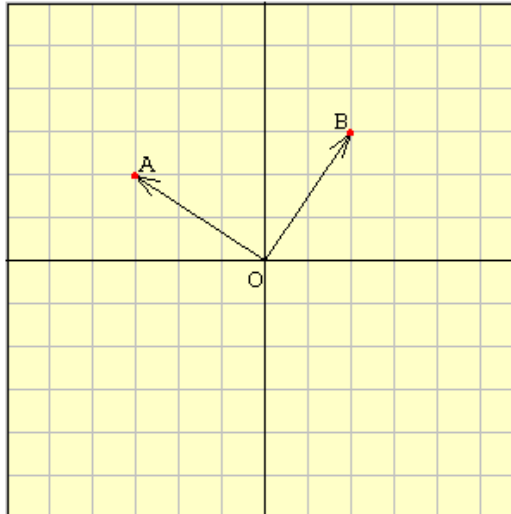
Here, either vector 'A' or 'B' in the plane can be dragged about and the position and sign of the cross product 'C' noted. What positions of 'A' and 'B' will make the cross product go out of sight?

<http://www.phy.syr.edu/courses/java-suite/crosspro.html>

$$\begin{pmatrix} -3.0 \\ 2.0 \end{pmatrix} \cdot \begin{pmatrix} 2.0 \\ 3.0 \end{pmatrix} = 0.0$$

Init Next

(C) 1996-2001 IES



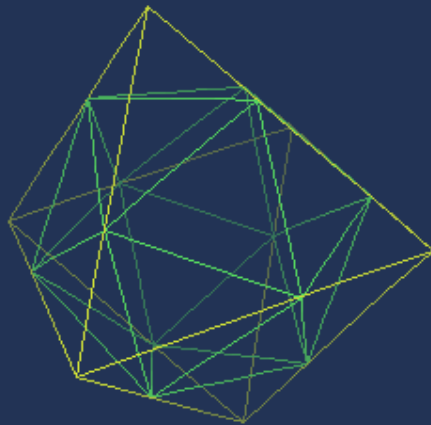
The Dot Product can be explored on this site: drag 'A' or 'B' about, and observe under what circumstances the dot product = 0, or 1 or -1.

This is one of nearly 300 remarkable applets offered to the mathematical community by Manipula Maths in Japan:

<http://www.ies.co.jp/math/java/>

Icosahedron

included into [octahedron](#)



(The image is an applet: you can click and drag it).

The 12 edges of the octahedron are split with 12 points according to the golden ratio (1.6180339887498948482...). Those 12 points are the vertices of the icosahedron.

Venturing into 3D has a particular fascination. The programming language Java has allowed mathematicians with a creative imagination to explore a wide variety of topics in 3D coordinate geometry for display on the web.

Here the mind of an Italian enthusiast Gian Marco Todesco is let loose on the five regular polyhedra and their inclusions.

Models can be spun round, and in this case there is another opportunity to discover that elusive constant, the Golden Ratio.

<http://www.divideo.it/personal/todesco/java/polyhedra>

2. THE USE OF AUTOGRAPH in the teaching of coordinate geometry

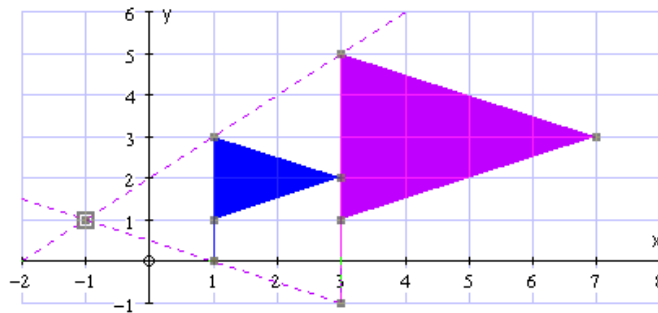
There are countless aspects of the secondary and college level mathematics curriculum for which dynamic software can be used. But the teachers need to practice making it effective, and training issues are a serious consideration. If the images proceed too quickly the learning can be lost. It is necessary to involve the class at each stage in the process, to get the students to anticipate what is about to happen, maybe to invite a student to draw on the screen first before the computer does it. This is a tried and tested technique, allowing the teacher to determine the pace, and if necessary to force errors from the class to help make good teaching points.

Autograph is particularly suited to this approach as it offers the chance for coordinate geometry objects to be created on screen. These can be manipulated dynamically by varying parameters or creating further dependent objects.

TRANSFORMING SHAPES

Here, the original objects can be moved around, and the parameter that created the second object can be animated.

It is instructive to ask a student to draw the transformation on the board to anticipate the result before the computer does it.

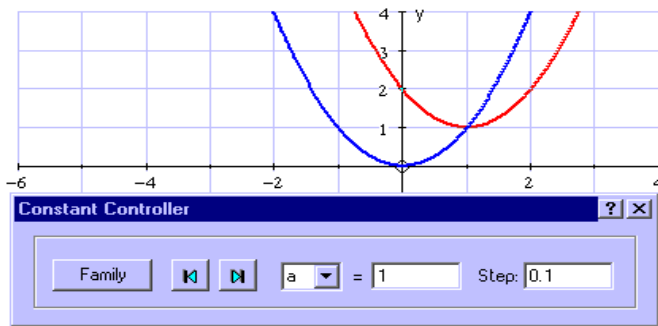


TRANSFORMING CURVES

Illustrated here is:

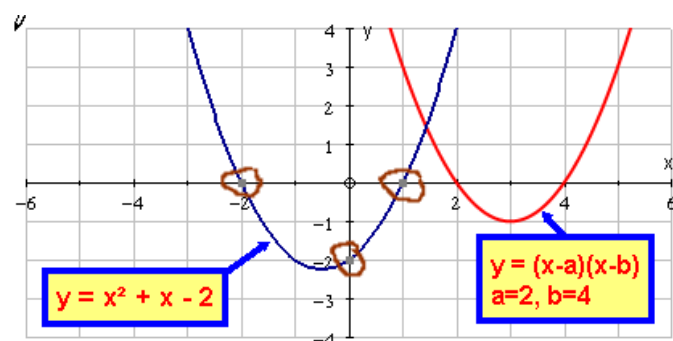
$y = x^2$, and $y - b = (x - a)^2$
 'a' and 'b' can be varied dynamically.

Using function definitions works well too, eg plot $f(x) = x^2$, then $y = f(ax + b) + c$; vary a, b, c. Then redefine $f(x)$ as $\sin x$.



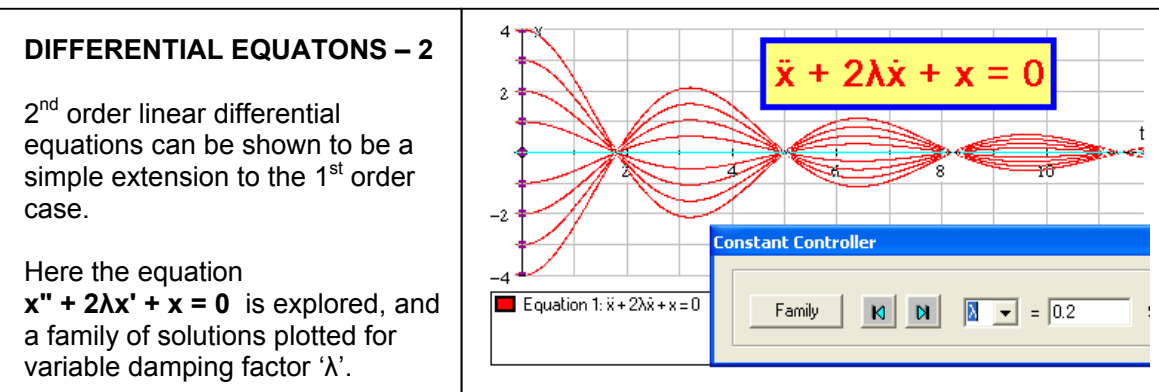
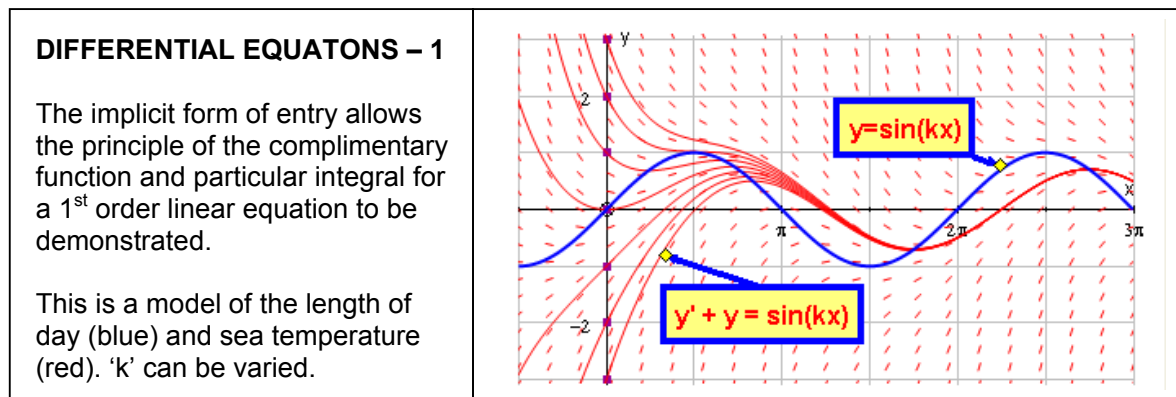
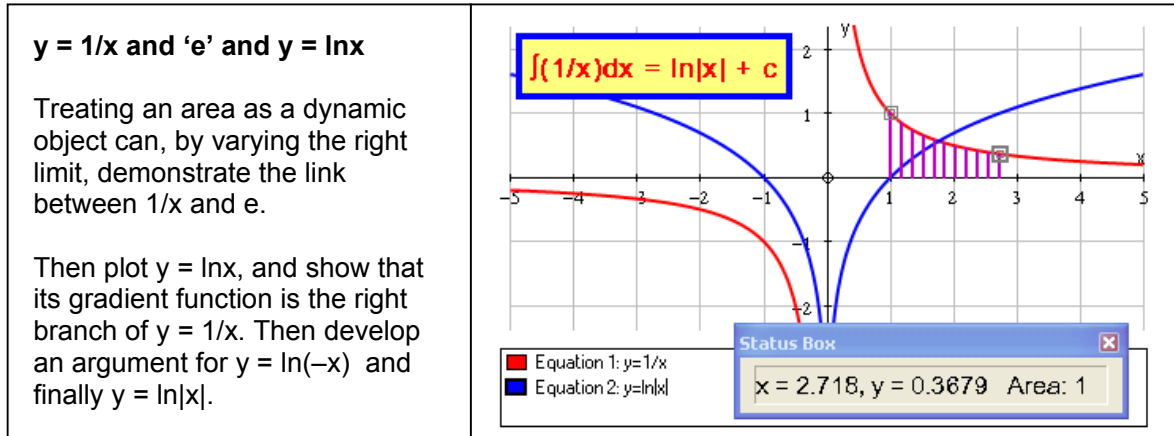
INVESTIGATING QUADRATICS

Place 2 points on the x-axis and one on the y-axis and ask for a quadratic. Now move the points around. The shape of the curve and its (factorizable) equation can be usefully compared to entering an equation in the form $y = (x-a)(x-b)$ and varying 'a' and 'b'.



THE TEACHING OF CALCULUS

Bright pupils by and large can see what is going on with calculus, but the average students need some visual hooks to hang on to. Calculus is all about change, so is ideally suited to a dynamic approach. Newton developed the unimaginable ideas of dividing two zero-width lengths (differentiation) and an infinite number of zero-width strips (integration). Computer animations can get quite close to this with powerful zooming features, so students can more easily visualise the processes involved.



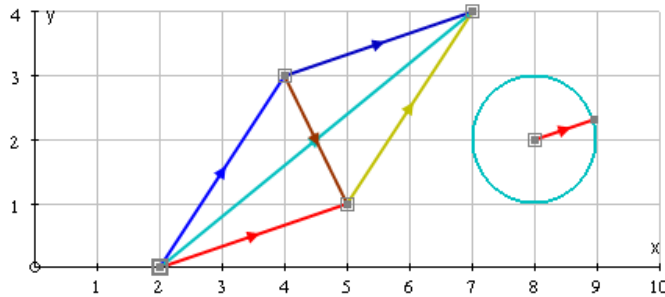
THE TEACHING OF VECTORS in 2D and 3D

Many students find the concept of vectors difficult, to the point even of declaring that they hate them! Vectors were not discovered, like 'e' or 'π'. They are man made, so need to have their own set of rules. A graphical environment that enables vectors to be entered, translated, copied and added can allow teachers to define the zero vector and the negative vector, and then establish subtraction and the two forms of multiplication, all in a logical and visually stimulating way.

Applications in kinematics, mechanics and coordinate and Euclidean geometry abound, so students can be given a wide variety of situations to practice on.

FUN WITH VECTORS in 2D

The principle of copying, adding and subtracting vectors can be explored dynamically. Also scalar multiples (vector equation of a line). The difficult concept of a unit vector can be explored by constructing a unit circle over it and varying the parent vector.



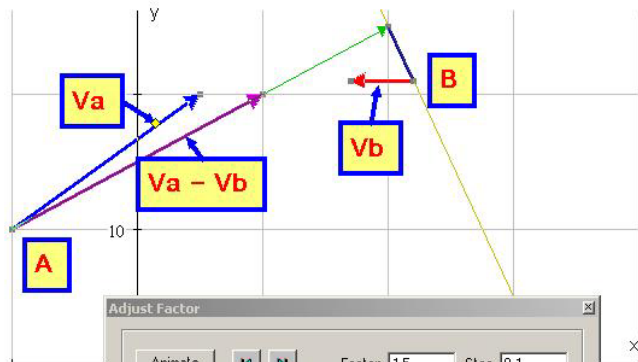
RELATIVE VELOCITY in 2D

At noon two ships A and B have positions (\mathbf{r} km) with velocities (\mathbf{v} kmh⁻¹) given by

$$\mathbf{r}_A = (-10\mathbf{i} + 10\mathbf{j}) \quad \mathbf{v}_A = (15\mathbf{i} + 10\mathbf{j})$$

$$\mathbf{r}_B = (22\mathbf{i} + 21\mathbf{j}) \quad \mathbf{v}_B = (-5\mathbf{i} + 0\mathbf{j})$$

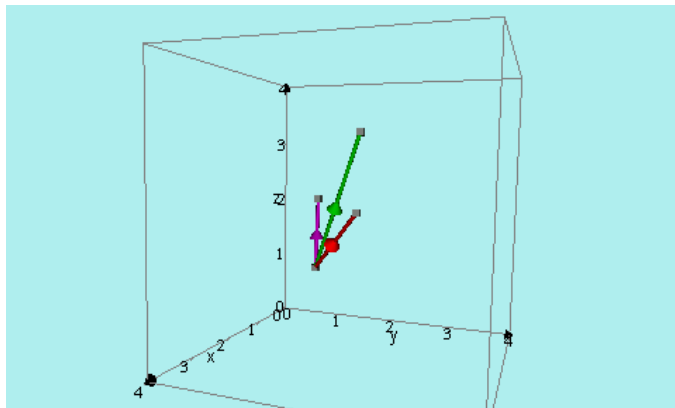
If the ships continue with these velocities, what is the *closest distance* between the two ships?



FUN WITH VECTORS in 3D

The ability to create vectors in 3D and to rotate the frame, opens up new ways to explain how vectors add, subtract and multiply.

It is particularly valuable to be able to show that any linear combination of 2 vectors in 3D creates a third vector in the same plane.



LINES, PLANES and SURFACES in 3D

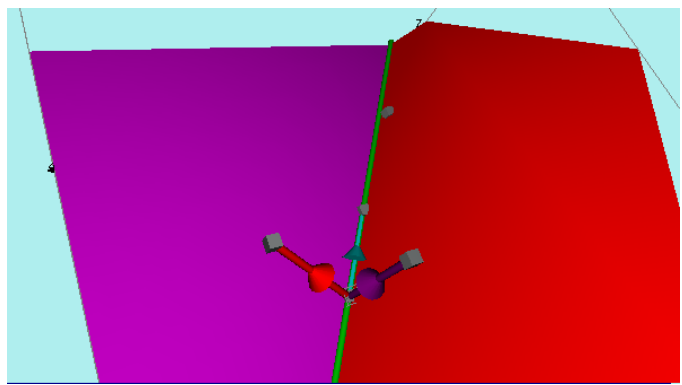
Lines can be represented in 3D by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} d \\ f \\ g \end{pmatrix}$ and planes by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$;

lines can also be created from 2 points or planes from 3 points or one point and a vector, etc, as well as the standard equations of the form $ax + by + cz = d$. It is important for students to understand the geometrical interpretation of all these forms, all of which can be explored dynamically on screen.

LINE OF INTERSECTION (3D)

Illustrated here: two planes and the line of intersection; a point on the line and the normal vector to each plane; finally the cross product of the two normal vectors.

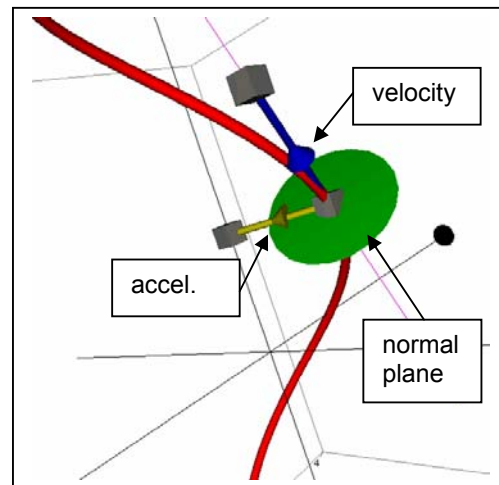
This is a useful visual link to the classic development of the line of intersection of two planes.



PARAMETRIC COORDINATES in 3D

Illustrated here: a line entered in parametric form: $x = \sin t$, $y = \cos t$, $z = t$ and a moveable point added to it.

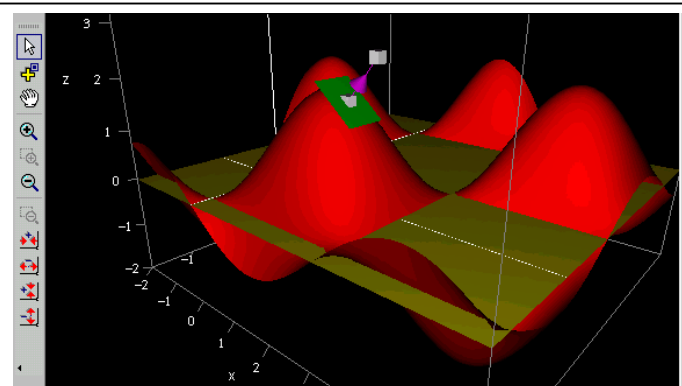
A normal plane can be added, together with velocity and acceleration vectors, to give a rich understanding of the dynamic behaviour of parametric functions as the point moves along the curve.



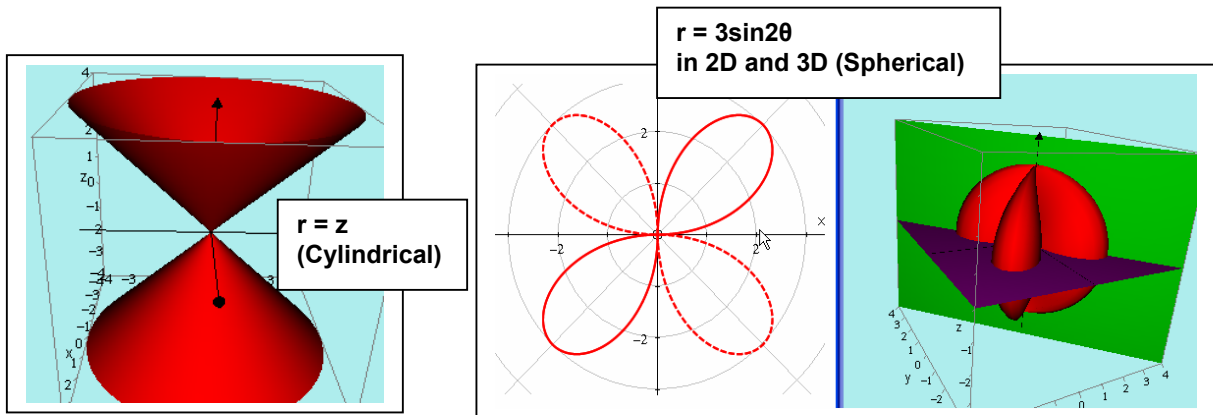
SURFACES in 3D

Most students who are familiar with the definition of slope in 2D need to associate slope in 3D with the normal vector at a point, and the tangent plane.

Shown here, the surface $y = a \sin x \cos y$.



Cylindrical and Spherical coordinate systems: again, there are opportunities for a real insight into the geometrical properties of these systems on screen:



CONCLUSION

Mathematicians have been teaching the subject with nothing more than a writing surface and a piece of chalk for centuries, and to good effect, certainly with the more able – these are the students who can, by and large, visualise the subject for themselves. The real beneficiaries in the new, visual approach, are the next ability level down. The rewards to them can be considerable, and there is every chance that this approach can help the numbers who want to take the subject further to rise.

There is plenty of anecdotal evidence too that the incidental use of computer generated images in the classroom can, in the right hands, make teaching both more effective and more efficient, and above all more enjoyable. So, more time for review, more secure understanding and more fun - the gains can be dramatic!

REFERENCES:

A collection of good resources is maintained by the author on:

www.argonet.co.uk/oundlesch/mlink.html

"Waldo's Maths Pages", by Ron Barrow (UK)

www.waldomaths.com

"The Cross Product", by Syracuse University (USA)

www.phy.syr.edu/courses/java-suite/crosspro.html

"Manipula Maths" (Japan)

www.ies.co.jp/math/java

"Regular Polyhedra", by Gian Marco Todesco (Italy)

www.divideo.it/personal/todesco/java/polyhedra

"Autograph" v. 2.10 and 3.00, by Douglas Butler (UK)

www.autograph-math.com