

A Comparison between Two Pricing and Lot-Sizing Models with Partial Backlogging and Deteriorated Items

by

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Abstract

Recently, Abad (2003) studied the pricing and lot-sizing problem for a perishable good under finite production, exponential decay, partial backordering and lost sale. In this article, we extend his model by adding not only the backlogging cost but also the cost of lost goodwill. We then analytically compare the total profits between Abad's (2003) model (in which the cycle starts with an instant production to accumulate stocks, then stops production to use up stocks, and finally restarts production to meet the unsatisfied demands.) and Goyal and Giri's (2003) model (in which the cycle begins with a period of shortages, then starts production until accumulated inventory reaches certain level, and finally stops production and uses up inventory). In addition, we show that there is no dominant model. Furthermore, we provide certain conditions under which one is more profitable than the other. Finally, we give several numerical examples to illustrate the results.

Keywords: Inventory; Pricing; Partial Backlogging; Deteriorating Items

1. Introduction

Many researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic components and fashion goods. Ghare and Schrader (1963) were the first proponents for developing a model for an exponentially decaying inventory. They categorized decaying inventory into three types: direct spoilage, physical depletion and deterioration. Next, Covert and Philip (1973) extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Misra (1975) developed an economic order quantity (*i.e.*, EOQ) model with a Weibull deterioration rate for the perishable product but he did not consider backordering. Dave and Patel (1981) considered an EOQ model for deteriorating items

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with time-proportional demand when shortages were prohibited. Sachan (1984) then generalized the EOQ model to allow for shortages. Later, Hariga (1996) generalized the demand pattern to any log-concave function. Teng *et al.* (1999) and Yang *et al.* (2001) further generalized the demand function to include any non-negative, continuous function that fluctuates with time.

Abad (1996) established the optimal pricing and lot-sizing EOQ policies under conditions of perishability and partial backordering. Then Abad (2000) extended the optimal pricing and lot-sizing EOQ model to an economic production quantity (*i.e.*, EPQ) model. Balkhi and Benkherouf (1996) developed a general EPQ model for deteriorating items where demand and production rates are time varying, but the rate of deterioration is constant. Balkhi (2001) then further generalized the EPQ model to allow for time-varying deterioration rate. Recently, Abad (2003) studied the pricing and lot-sizing problem for a perishable good under finite production, exponential decay and partial backordering and lost sale. He assumed that customers are impatient and the backlogging rate is a negative exponential function of the waiting time. In addition, he assumed that the customers are served on first come first served basis during the shortage period. Then he provided a solution procedure to obtain the optimal price and lot-size that maximizes the average profit. However, he did not include the shortage cost for backlogged items and the cost of lost goodwill due to lost sales into the objective. If the objective does not include these two costs, then it will alter the optimal solution and overestimate the average profit. To correct them, in this paper, we add both the shortage cost for backlogged items and the cost of lost goodwill due to lost sales into the objective suggested by Abad (2003).

In Abad (2003), the production-inventory model starts with an instant production to accumulate stocks, then stops production to use up stocks, and finally restarts production to meet the unsatisfied demands. In fact, Abad's production-inventory model is similar to that in Balkhi and Benkherouf (1996). Lately, Goyal and Giri (2003) investigated a similar production-inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. However, pricing was not under consideration and the backlogging rate was assumed to be a constant fraction. They then proposed a new production-inventory model in which the cycle begins with a period of shortages, then starts production until accumulated inventory reaches a certain level, and finally stops production and uses up inventory. Finally, Goyal and Giri (2003) provided a numerical example to show that their model outperforms Balkhi and Benkherouf's model (1996) in terms of the least expensive total cost per unit time.

In this paper, we first extend Abad's (2003) pricing and lot-sizing model by adding not only the shortage cost for backlogged items but also the cost of lost goodwill due to lost sales into the objective. Next, we establish a new modeling approach as in Goyal and Giri (2003) to the same pricing and lot-sizing inventory problem. We then characterize the optimal solution to both distinct models, and prove that both two models provide the same profit if all parameters are constant. However, if any single parameter is varying with time, then the performances of these two models are varied. Furthermore, we obtain some theoretical results that show the conditions under which one model is more profitable than the other. Finally, we provide several numerical examples to illustrate the results, and conclusions are made.

2. Assumptions and notations

The following assumptions are similar to those in Abad's (2003) model.

- (1) The planning horizon is infinite.
- (2) The initial and final inventory levels are both zero.
- (3) Shortages are allowed. However, the longer the waiting time, the smaller the backlogging rate.

Hence, we assume that the fraction of shortages backordered $B(\tau)$ is a decreasing and differentiable function of τ , where τ is the waiting time up to the next replenishment.

- (4) The demand rate is a decreasing function of the selling price and it is twice differentiable.
- (5) The production rate, which is finite, is higher than the demand rate.
- (6) A constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory. Hence, there is no salvage value for the deteriorated items.
- (7) The unit cost, the holding cost, the shortage cost for backlogged items, and the cost of lost goodwill due to lost sales are assumed to be functions of time.

In addition, the following notations are used throughout this paper.

- $I_i(t)$ on-hand stock level (or number of backorders) at time t in Phase i , $i = 1, 2, 3$, and 4 .
- R production rate for the item (units/unit time).
- K setup cost per setup.
- $v(t)$ unit cost, as a function of time t .
- $h(t)$ unit holding cost per unit time at time t .
- p unit selling price within the replenishment cycle, $p > v(t)$.
- $c_1(t)$ unit shortage cost per unit time for backlogged items at time t .
- $c_2(t)$ unit cost of lost goodwill due to lost sales at time t .
- $D(p)$ demand rate per unit time, which is a decreasing function of p . Note $R > D(p)$. We will use D and $D(p)$ interchangeably.
- σ decay coefficient, which is a constant (*i.e.*, exponential decay).
- T duration of positive inventory cycle.
- λ duration of negative inventory cycle (or shortage cycle).
- ψ duration of negative inventory before the start of production.
- β duration of positive inventory before the end of production.

In this paper, we assume WLOG that the vendor's objective is to maximize the average profit per unit time.

3. Mathematical formulations and theoretical results

In this section, we first establish Abad's modeling approach for the problem, then set up Goyal and Giri's modeling approach to the problem next, and finally compare the profits obtained from these two models.

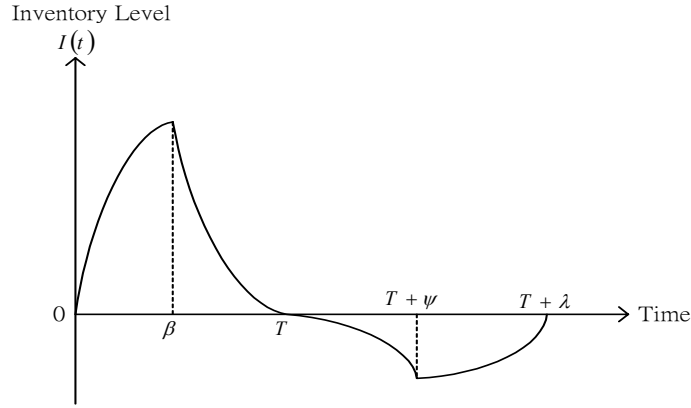


Fig. 1. A production-inventory cycle of Model 1

3.1 Model 1: Abad's (2003) approach

In this subsection, the behaviour of the inventory in a cycle is shown in Figure 1, as well as in Abad (2003). Consequently, the inventory cycle is described by the following four phases:

Phase 1. During the time interval $[0, \beta]$, the system is subject to the effect of production, demand and deterioration. Therefore, the change of the inventory level at time t , $I_1(t)$, is governed by

$$\frac{dI_1(t)}{dt} + \sigma I_1(t) = R - D, \text{ with the boundary condition } I_1(0) = 0. \quad (1)$$

Phase 2. In the time interval $[\beta, T]$, the system is affected by the combined the demand and deterioration. Hence, the change of the inventory level at time t , $I_2(t)$, is governed by

$$\frac{dI_2(t)}{dt} + \sigma I_2(t) = -D, \text{ with the boundary condition } I_2(T) = 0. \quad (2)$$

The solution to differential equation (1) is

$$I_1(t) = (R - D)(1 - e^{-\sigma t}) / \sigma, \quad t \in [0, \beta]. \quad (3)$$

Setting $t = \beta$ into Eq.(3), we obtain the maximum positive inventory in a cycle is

$$I_1(\beta) = (R - D)(1 - e^{-\sigma \beta}) / \sigma. \quad (4)$$

Similarly, the solution to differential equation (2) is

$$I_2(t) = D(e^{\sigma(T-t)} - 1) / \sigma, \quad t \in [\beta, T]. \quad (5)$$

Equating expressions (3) and (4) at $t = \beta$, we have

$$I_1(\beta) = I_2(\beta) = (R - D)(1 - e^{-\sigma \beta}) / \sigma = D(e^{\sigma(T-\beta)} - 1) / \sigma, \quad (6)$$

Solving Eq.(6) for β , we have

$$\beta = \frac{1}{\sigma} \ln \left[\frac{R - D + D e^{T\sigma}}{R} \right]. \quad (7)$$

Phase 3. For $t \in [T, T + \psi]$, similar to Abad's (2003) model, the backlogging rate $B(\tau)$ is a negative exponential function of the waiting time τ . Therefore, we have

$$B(\tau) = k_0 e^{-k_1 \tau}, \quad k_0 \leq 1, \quad 0 \leq k_1. \quad (8)$$

Since customers are served on first come first served basis during shortage period, we know from Figure 1 that the waiting time is given by $\tau = T + \psi - t - I_3(t) / R$, for $t \in [T, T + \psi]$. Therefore, the number of backorders at time t , $I_3(t)$, satisfies the following differential equation:

$$\frac{dI_3(t)}{dt} = -DB(T + \psi - t - I_3(t)/R) = -Dk_0 e^{-k_1(T+\psi-t-I_3(t)/R)}, \quad (9)$$

with the boundary condition $I_3(T) = 0$.

The solution to (9) for $t \in [T, T + \psi]$ is

$$I_3(t) = -\frac{R}{k_1} \left[\ln(Dk_0 e^{k_1(t-T-\psi)} + R - Dk_0 e^{-k_1\psi}) / R \right]. \quad (10)$$

Phase 4. For $t \in [T + \psi, T + \lambda]$, the waiting time is given by $\tau = -I_4(t)/R$. Therefore, the number of backorders at time t , $I_4(t)$, satisfies the following differential equation:

$$\frac{dI_4}{dt} = R - DB(-I_4(t)/R) = R - Dk_0 e^{-k_1(-I_4(t)/R)}, \text{ with the boundary condition } I_4(T + \lambda) = 0. \quad (11)$$

The solution to (11) for $t \in [T + \psi, T + \lambda]$ is

$$I_4(t) = -\frac{R}{k_1} \left[\ln((R - Dk_0)e^{-k_1(t-T-\lambda)} + Dk_0) / R \right]. \quad (12)$$

Given the condition $I_3(T + \psi) = I_4(T + \psi)$, we get

$$\psi = \frac{1}{k_1} \left[\ln(Dk_0 + e^{k_1\lambda}(R - Dk_0)) / R \right]. \quad (13)$$

Applying Eq.(13) into Eq.(12), we can rewrite the Eq.(12) as follows:

$$I_4(t) = -\frac{R}{k_1} \left[\ln(R e^{-k_1(t-T-\psi)} - Dk_0 e^{-k_1(t-T)} + Dk_0) / R \right]. \quad (14)$$

Next, the average profit per unit time consists of the following six elements:

(a) Revenue is given by

$$R_1 = pDT + pR(\lambda - \psi). \quad (15)$$

(b) The set up cost is given by

$$SC_1 = K. \quad (16)$$

(c) The production cost is given by

$$PC_1 = \int_0^\beta v(t)R dt + \int_{T+\psi}^{T+\lambda} v(t)R dt. \quad (17)$$

(d) The inventory holding cost is given by

$$\begin{aligned} HC_1 &= \int_0^\beta h(t)I_1(t)dt + \int_\beta^T h(t)I_2(t)dt \\ &= \int_0^\beta h(t)(R - D)(1 - e^{-\sigma t})/\sigma dt + \int_\beta^T h(t)D(e^{\sigma(T-t)} - 1)/\sigma dt. \end{aligned} \quad (18)$$

(e) The shortage cost for backlogged items is given by

$$\begin{aligned} BC_1 &= \int_T^{T+\psi} c_1(t)[-I_3(t)] dt + \int_{T+\psi}^{T+\lambda} c_1(t)[-I_4(t)] dt \\ &= \frac{R}{k_1} \left\{ \int_0^\psi c_1(t+T) \left[\ln(Dk_0 e^{k_1(t-\psi)} + R - Dk_0 e^{-k_1\psi}) / R \right] dt \right. \\ &\quad \left. + \int_\psi^\lambda c_1(t+T) \left[\ln(R e^{-k_1(t-\psi)} - Dk_0 e^{-k_1 t} + Dk_0) / R \right] dt \right\}. \end{aligned} \quad (19)$$

(f) The cost of lost goodwill due to lost sales is given by

$$LC_1 = \int_T^{T+\psi} c_2(t)[1 - B(T + \psi - t - I_3(t)/R)]D dt + \int_{T+\psi}^{T+\lambda} c_2(t)[1 - B(-I_4(t)/R)]D dt$$

$$= \int_0^\lambda c_2(t+T) \left(1 - \frac{R k_0 e^{k_1(t-\psi)}}{R - D k_0 e^{-k_1\psi} + D k_0 e^{k_1(t-\psi)}} \right) D dt. \quad (20)$$

Given the above, the profit during time-span $[0, T + \lambda]$ is

$$\begin{aligned} F_1(p, T, \lambda) &= R_1 - SC_1 - PC_1 - HC_1 - BC_1 - LC_1 \\ &= [pDT + pR(\lambda - \psi)] - K - \left[\int_0^\beta v(t)R dt + \int_{T+\psi}^{T+\lambda} v(t)R dt \right] \\ &\quad - \left[\int_0^\beta h(t)(R-D)(1-e^{-\sigma t})/\sigma dt + \int_\beta^T h(t)D(e^{\sigma(T-t)} - 1)/\sigma dt \right] \\ &\quad - \frac{R}{k_1} \left\{ \int_0^\psi c_1(t+T) \left[\ln(Dk_0 e^{k_1(t-\psi)} + R - Dk_0 e^{-k_1\psi})/R \right] dt \right. \\ &\quad \left. + \int_\psi^\lambda c_1(t+T) \left[\ln(R e^{-k_1(t-\psi)} - Dk_0 e^{-k_1 t} + Dk_0)/R \right] dt \right\} \\ &\quad - \int_0^\lambda c_2(t+T) \left(1 - \frac{R k_0 e^{k_1(t-\psi)}}{R - D k_0 e^{-k_1\psi} + D k_0 e^{k_1(t-\psi)}} \right) D dt. \end{aligned} \quad (21)$$

Hence, the average profit per unit time is

$$\Pi_1(p, T, \lambda) = F_1(p, T, \lambda)/(T + \lambda), \quad (22)$$

where $F_1(p, \lambda, T)$ is given by Eq. (22). As a result, the problem faced by the vendor is

$$(P1) \quad \max. \quad \Pi_1(p, T, \lambda) \quad (23a)$$

$$\text{subject to } \psi = \frac{1}{k_1} \left[\ln(Dk_0 + e^{k_1\lambda}(R - Dk_0))/R \right], \quad (23b)$$

$$\beta = \frac{1}{\sigma} \ln[(R - D + D e^{T\sigma})/R], \quad (23c)$$

$$0 < \psi < \lambda, \quad (23d)$$

$$0 < \beta < T, \quad (23e)$$

$$v \leq p. \quad (23f)$$

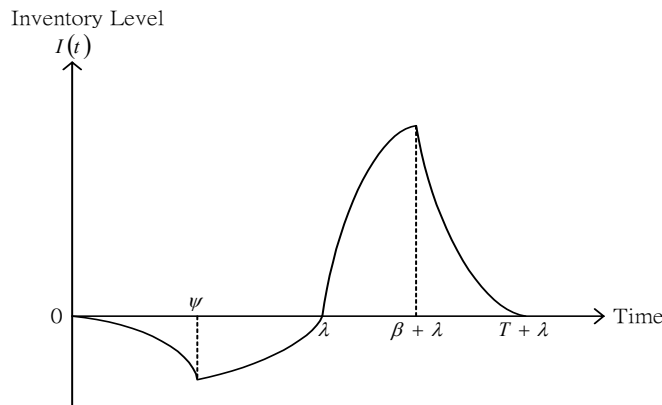


Fig. 2. A production-inventory cycle of Model 2

3.2 Model 2: Goyal and Giri's (2003) approach

In this subsection, the behaviour of the inventory in a cycle is depicted in Figure 2, as well as in

Goyal and Giri's (2003) model. Based on the assumptions in Section 2, and from Figure 2, we know that the inventory is also described by the following four phases:

Phase 1. For $t \in [0, \psi]$,

$$\frac{dI_1(t)}{dt} = -Dk_0 e^{-k_1(\psi-t-I_1(t)/R)}, \text{ with the boundary condition } I_1(0) = 0. \quad (24)$$

Phase 2. For $t \in [\psi, \lambda]$,

$$\frac{dI_2(t)}{dt} = R - Dk_0 e^{-k_1(-I_2(t)/R)}, \text{ with the boundary condition } I_2(\lambda) = 0. \quad (25)$$

Phase 3. For $t \in [\lambda, \beta + \lambda]$,

$$\frac{dI_3(t)}{dt} + \sigma I_3(t) = R - D, \text{ with the boundary condition } I_3(\lambda) = 0. \quad (26)$$

Phase 4. For $t \in [\beta + \lambda, T + \lambda]$,

$$\frac{dI_4(t)}{dt} + \sigma I_4(t) = -D \text{ with the boundary condition } I_4(T + \lambda) = 0. \quad (27)$$

The solutions of the above ordinary differential equations are given as follows.

$$I_1(t) = -\frac{R}{k_1} \ln \left[\left(Dk_0 e^{k_1(t-\psi)} + R - Dk_0 e^{-k_1\psi} \right) / R \right], \quad t \in [0, \psi], \quad (28)$$

$$I_2(t) = -\frac{R}{k_1} \ln \left[\left(R e^{-k_1(t-\psi)} - Dk_0 e^{-k_1 t} + Dk_0 \right) / R \right], \quad t \in [\psi, \lambda], \quad (29)$$

$$I_3(t) = (R - D) \left(1 - e^{-\sigma(t-\lambda)} \right) / \sigma, \quad t \in [\lambda, \beta + \lambda], \quad (30)$$

and

$$I_4(t) = D \left(e^{\sigma(T+\lambda-t)} - 1 \right) / \sigma, \quad t \in [\beta + \lambda, T + \lambda], \quad (31)$$

respectively. Solving the boundary conditions $I_1(\psi) = I_2(\psi)$ and $I_3(\beta + \lambda) = I_4(\beta + \lambda)$, we obtain the following equations which are the same as Eqs. (7) and (13), respectively.

$$\psi = \frac{1}{k_1} \left[\ln \left(Dk_0 + e^{k_1\lambda} (R - Dk_0) \right) / R \right] \text{ and } \beta = \frac{1}{\sigma} \ln \left[\left(R - D + D e^{T\sigma} \right) / R \right]. \quad (32)$$

Therefore, the average profit per unit time consists of the following elements.

(a) Revenue is given by

$$R_2 = pDT + pR(\lambda - \psi). \quad (33)$$

(b) The set up cost is given by

$$SC_2 = K. \quad (34)$$

(c) The production cost is given by

$$PC_2 = \int_{\psi}^{\lambda} v(t)R \, dt + \int_{\lambda}^{\beta+\lambda} v(t)R \, dt. \quad (35)$$

(d) The inventory holding cost is given by

$$HC_2 = \int_{\lambda}^{\beta+\lambda} h(t)(R - D) \left(1 - e^{-\sigma(t-\lambda)} \right) / \sigma \, dt + \int_{\beta+\lambda}^{T+\lambda} h(t)D \left(e^{\sigma(T+\lambda-t)} - 1 \right) / \sigma \, dt. \quad (36)$$

(e) The shortage cost for backlogged items is given by

$$BC_2 = \frac{R}{k_1} \left\{ \int_0^{\psi} c_1(t) \ln \left[\left(Dk_0 e^{k_1(t-\psi)} + R - Dk_0 e^{-k_1\psi} \right) / R \right] dt \right. \\ \left. + \int_{\psi}^{\lambda} c_1(t) \ln \left[\left(R e^{-k_1(t-\psi)} - Dk_0 e^{-k_1 t} + Dk_0 \right) / R \right] dt \right\}. \quad (37)$$

(f) The cost of lost goodwill due to lost sales is given by

$$LC_2 = \int_0^\lambda c_2(t) \left(1 - \frac{R k_0 e^{k_1(t-\psi)}}{R - D k_0 e^{-k_1\psi} + D k_0 e^{k_1(t-\psi)}} \right) D dt. \quad (38)$$

Hence, the profit during time-span $[0, T + \lambda]$ is

$$\begin{aligned} F_2(p, T, \lambda) &= R_2 - SC_2 - PC_2 - HC_2 - BC_2 - LC_2 \\ &= [pDT + pR(\lambda - \psi)] - K - \left[\int_\psi^\lambda v(t)R dt + \int_\lambda^{\beta+\lambda} v(t)R dt \right] \\ &\quad - \left\{ \int_\lambda^{\beta+\lambda} h(t)(R - D)(1 - e^{-\sigma(t-\lambda)})/\sigma dt + \int_{\beta+\lambda}^{T+\lambda} h(t)D(e^{\sigma(T+\lambda-t)} - 1)/\sigma dt \right\} \\ &\quad - \frac{R}{k_1} \left\{ \int_0^\psi c_1(t) \ln \left[\frac{D k_0 e^{k_1(t-\psi)} + R - D k_0 e^{-k_1\psi}}{R} \right] dt \right. \\ &\quad \left. + \int_\psi^\lambda c_1(t) \ln \left[\frac{R e^{-k_1(t-\psi)} - D k_0 e^{-k_1t} + D k_0}{R} \right] dt \right\} \\ &\quad - \int_0^\lambda c_2(t) \left(1 - \frac{R k_0 e^{k_1(t-\psi)}}{R - D k_0 e^{-k_1\psi} + D k_0 e^{k_1(t-\psi)}} \right) D dt. \end{aligned} \quad (39)$$

The average profit per unit time is

$$\Pi_2(p, T, \lambda) = F_2(p, T, \lambda)/(T + \lambda), \quad (40)$$

where $F_2(p, \lambda, T)$ is given by Eq. (40). Consequently, the problem faced by the vendor is

$$(P2) \quad \max. \quad \Pi_2(p, T, \lambda) \quad (41a)$$

$$\text{subject to } \psi = \frac{1}{k_1} \left[\ln \left(\frac{D k_0 + e^{k_1\lambda} (R - D k_0)}{R} \right) \right], \quad (41b)$$

$$\beta = \frac{1}{\sigma} \ln \left[\frac{(R - D + D e^{T\sigma})}{R} \right], \quad (41c)$$

$$0 < \psi < \lambda, \quad (41d)$$

$$0 < \beta < T, \quad (41e)$$

$$v \leq p. \quad (41f)$$

3.3 A comparison between two models

Now, we compare the above two models, and identify which model has more profit than the other under what conditions. From the above results, we can obtain the following theorems.

Theorem 1.

- If all time-varying parameters are constant (i.e., $v(t) = v$, $h(t) = h$, $c_1(t) = c_1$, and $c_2(t) = c_2$), then $\Pi_1(p, T, \lambda) = \Pi_2(p, T, \lambda)$
- If the holding cost $h(t)$ is non-decreasing with t , and the other parameters are constant (i.e., $v(t) = v$, $c_1(t) = c_1$, and $c_2(t) = c_2$), then $\Pi_1(p, T, \lambda) \geq \Pi_2(p, T, \lambda)$. On the other hand, if the holding cost $h(t)$ is non-increasing with t , and the other parameters are constant, then $\Pi_1(p, T, \lambda) \leq \Pi_2(p, T, \lambda)$.
- If the shortage cost $c_1(t)$ is non-increasing with t , and the other parameters are constant (i.e., $v(t) = v$, $h(t) = h$ and $c_2(t) = c_2$), then $\Pi_1(p, T, \lambda) \geq \Pi_2(p, T, \lambda)$. Conversely, if the shortage

cost $c_1(t)$ is non-decreasing with t , and the other parameters are constant, then $\Pi_1(p, T, \lambda) \leq \Pi_2(p, T, \lambda)$.

- (d) If the cost of lost goodwill $c_2(t)$ is non-increasing with t , and the other parameters are constant (*i.e.*, $h(t) = h$, $v(t) = v$ and $c_1(t) = c_1$), then $\Pi_1(p, T, \lambda) \geq \Pi_2(p, T, \lambda)$. In contrast, if the cost of lost goodwill $c_2(t)$ is non-decreasing with t , and the other parameters are constants, then $\Pi_1(p, T, \lambda) \leq \Pi_2(p, T, \lambda)$.

Proof: It is trivial to show this theorem.

Theorem 2.

If the unit cost $v(t)$ is time-varying, and the other parameters are constant (*i.e.*, $h(t) = h$, $c_1(t) = c_1$, and $c_2(t) = c_2$), then we obtain

$$\Pi_1(p, T, \lambda) \geq \Pi_2(p, T, \lambda) \text{ if and only if, } \int_0^\beta [v(t) - v(t + \lambda)] dt + \int_\psi^\lambda [v(t + T) - v(t)] dt \leq 0. \quad (42)$$

Proof: It is trivial to show this theorem.

In order to find the optimal values of p , λ and T , we have to solve the complex, nonlinear equations $\partial \Pi_i(p, \lambda, T) / \partial p = 0$, $\partial \Pi_i(p, \lambda, T) / \partial \lambda = 0$, $\partial \Pi_i(p, \lambda, T) / \partial T = 0$, and some additional complementary conditions, for $i = 1$, and 2. Although it is difficult to solve the problem analytically, the reader can follow the solution procedure proposed by Abad (2003) with proper software to solve the problem numerically.

4 Numerical examples

In this section, we use software MATHEMATICA version 4.1 to obtain the optimal solutions for both (P1) and (P2).

Example 1. To understand the effect of adding the shortage cost $c_1(t)$, and the cost of lost goodwill $c_2(t)$ to the average profit per unit time, we adopt the same example in Abad (2003). Therefore, we suppose $D(p) = 1600000 p^{-3}$, $R = 1000$ units/week, $v(t) = \$10/\text{unit}$, $h(t) = \$1/\text{unit/week}$, $K = \$1000/\text{production run}$, $\sigma = 0.3$, $k_0 = 0.9$, and $k_1 = 0.6$. However, we add $c_1(t) = \$8/\text{unit}$, and $c_2(t) = \$5/\text{unit}$. We obtain the computational results as shown in Table 1. Comparing with the computational results in Abad (2003), we know that the optimal price would be lower while the average profit per unit would be higher if we do not include the shortage cost and the cost of lost goodwill into the model. Table 1 also verifies Part (a) of Theorem 1.

Table 1. The optimal values for Example 1.

Model(i)	ψ	λ	β	$\beta + \lambda$	T	$T + \psi$	$T + \lambda$	p	Π_i
1	0.1650	0.2669	0.6602	0.9271	1.3329	1.4979	1.5998	15.3142	1039.02
2	0.1650	0.2669	0.6602	0.9271	1.3329	1.4979	1.5998	15.3142	1039.02

Example 2. To see the effect of the unit cost on the average profit per unit time, let us assume that the unit cost is as below, and the rest parameters are the same as in Example 1.

$$v(t) = \begin{cases} \$9 + e^{-(t-\beta)} & , 0 \leq t \leq \beta \\ \$10 & , \beta \leq t \leq \beta + \lambda \\ \$9 + e^{(t-\beta-\lambda)} & , \beta + \lambda \leq t \leq T + \lambda \end{cases} \quad \text{/unit.} \quad (43)$$

Consequently, we know from Theorem 2 that Model 2 is more profitable than Model 1, which is shown in Table 2.

Table 2. The optimal values for Example 2.

Model(<i>i</i>)	ψ	λ	β	$\beta + \lambda$	T	$T + \psi$	$T + \lambda$	p	Π_i
1	0.1833	0.2560	0.4396	0.6957	1.1757	1.359	1.4317	16.8683	876.19
2	0.2597	0.3700	0.5008	0.8708	1.2469	1.5066	1.6169	16.4814	999.11

5 Conclusions

If we omit the shortage cost and the cost of lost goodwill into the production-inventory model with many lost sales, then we alter the results, and overestimate the profits. In this paper, we not only extend Abad's (2003) model by adding the shortage cost and the cost of lost goodwill into his model, but also compare his modeling approach (as well as in Balkhi and Benkherouf (1996)) and Goyal and Giri's (2003) approach. We analytically prove that both models provide the same average profit per unit time if all parameters are constant. Otherwise, under certain conditions Abad's model is more profitable than Goyal and Giri's approach, and vice versa. In short, there is no dominant modeling approach.

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