Analysis of a Buried Deep Point/Line Heat Source in a Cross-Anisotropic Porous Elastic Medium

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ABSTRACT

The long-term thermally elastic responses of a saturated elastic stratum containing a deep point/line heat source of constant strength are studied in this paper. To simulate the stratified earth medium, the soil mass is modeled as anisotropic with properties differing in the horizontal and vertical directions. On the basis of the fundamental solutions caused by a deep point heat source, analytic solutions of the ground deformation, pore water pressure distribution, effective stresses and temperature changes of the porous medium due to the deep line heat source are presented using the appropriate line integral techniques.
Keywords: Point heat source, Line heat source, Fundamental solution, Closed-form solution

INTRODUCTION

The thermally mechanical responses of the fluid saturated porous medium due to a deep buried point/line heat source of constant strength are studied in this paper. The heat source such as a canister of radioactive waste will cause a temperature rise in the soil and thus the solid skeleton and pore fluid will expand. This leads to an increase in pore water pressure and a reduction in effective stress because the volume increase of the pore water is greater than that of the voids of the solid matrix. Therefore, thermal failure of soil may occur as a result of the loss of shear resistance due to the decrease in effective stress.

Governing equations for a fluid-saturated poroelastic solid in an isothermal quasi-static state have been developed by Biot (1941, 1955). Booker and Savvidou (1984, 1985, 1989) have derived an extended Biot theory including the thermal effects and presented solutions of thermo-consolidation around the spherical and point heat sources. In their solutions, the flow or thermal properties are considered as isotropic or cross-anisotropic whereas the elastic properties of the soil are treated as isotropic. Moreover, the stratum is modeled as a full space to simulate the deep buried heat sources.

Soils in general are deposited through a process of sedimentation over a long period of time. Under the accumulative overburden pressure, soils display significant anisotropy on mechanical, flow and thermal properties. In order to describe the anisotropic nature of soils, it may be modeled as cross-anisotropic porous medium whose properties are symmetric about the vertical axis. If the heat source buried at a great depth, the effects of the half space boundary on thermally response can be neglected.

In this paper, the soil mass is modeled as a cross-anisotropic saturated elastic full space. Not only the permeability and conductivity but also the elastic properties are assumed to be cross-anisotropic for the soil mass. Long-term thermally elastic mechanical behaviors of the stratum are studied. On the basis of the derived deep point heat source induced fundamental solutions, closed-form solutions of the long-term ground deformation, effective stresses, temperature changes of the soil mass and excess pore water pressure due to a deep line heat source are obtained using the appropriate line integral techniques. Results are then reduced to an isotropic case to provide a better understanding of the thermally induced mechanical responses of the stratum.

POINT HEAT SOURCE INDUCED FUNDAMENTAL SOLUTIONS

Basic Equations

Figure 1 shows a point heat source buried deep in a cross-anisotropic porous stratum. The soil mass is considered as a homogeneous cross-anisotropic porous medium with a vertical axis of symmetry. The constitutive behavior of the elastic soil skeleton for linear axially symmetric deformation in the cylindrical coordinates (r, θ, z) can be expressed by

$$\sigma'_{rr} = A\frac{\partial u_r}{\partial r} + (A-2N)\frac{u_r}{r} + F\frac{\partial u_z}{\partial z} - \beta_r \vartheta , \tag{1a}$$

$$\sigma'_{\theta\theta} = (A-2N)\frac{\partial u_r}{\partial r} + A\frac{u_r}{r} + F\frac{\partial u_z}{\partial z} - \beta_r \vartheta , \tag{1b}$$

$$\sigma'_{zz} = F\frac{\partial u_r}{\partial r} + F\frac{u_r}{r} + C\frac{\partial u_z}{\partial z} - \beta_z \vartheta , \tag{1c}$$

$$\sigma'_{rz} = L\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) , \tag{1d}$$

where σ'_{rr}, $\sigma'_{\theta\theta}$, *etc.*, are the effective stress components; ϑ is the temperature change of the soil mass; u_r, u_z are the displacements in the radial and axial directions, respectively; *A*, *C*, *F*, *L*, *N* are the material constants of the cross-anisotropic medium defined by Love (1944);

$\beta_r = 2(A-N)\alpha_{sr} + F\alpha_{sz}$ and $\beta_z = 2F\alpha_{sr} + C\alpha_{sz}$ are the thermal expansion factors in the horizontal and vertical directions, respectively. The linear thermal expansion coefficients of the stratum in the horizontal and vertical directions, respectively, are denoted by α_{sr} and α_{sz}. The shear stress components $\sigma'_{r\theta}$ and $\sigma'_{\theta z}$ vanish by locating the vertical *z*-axis through the point heat source. For an isotropic medium, $A = C = \lambda + 2G$; $F = \lambda$; $L = N = G$; $\beta_r = \beta_z = (2G + 3\lambda)\alpha_s$; where λ, G, α_s are the Lame constant, shear modulus and linear thermal expansion coefficient of the isotropic porous matrix, respectively.

According to Terzaghi's effective stress concept, the total stress τ_{ij} of a saturated porous material is given by $\tau_{ij} = \sigma'_{ij} + p\delta_{ij}$, in which the excess pore fluid pressure p is positive for compression, and δ_{ij} is the Kronecker delta. The total stress must satisfy the equilibrium relations $\tau_{ij,j} + f_i = 0$. By using equations (1a)-(1d) and Terzaghi's effective stress concept, the equilibrium equations for axially symmetric problem without body forces f_i can be expressed in terms of displacements u_i, temperature change of the soil mass ϑ, and excess pore fluid pressure p as follows:

$$A\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2}\right) + L\frac{\partial^2 u_r}{\partial z^2} + (F+L)\frac{\partial^2 u_z}{\partial r \partial z} - \beta_r \frac{\partial \vartheta}{\partial r} + \frac{\partial p}{\partial r} = 0, \tag{2a}$$

$$(F+L)\left(\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r}\frac{\partial u_r}{\partial z}\right) + L\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right) + C\frac{\partial^2 u_z}{\partial z^2} - \beta_z \frac{\partial \vartheta}{\partial z} + \frac{\partial p}{\partial z} = 0. \tag{2b}$$

A third and fourth relations between u_r, u_z, ϑ, and p can be obtained from the conservations of mass and energy:

$$\nabla \cdot \left[n\left(\boldsymbol{v}_f - \boldsymbol{v}_s\right)\right] + q_f = 0, \tag{3}$$

$$-\nabla \cdot \boldsymbol{h} + q_h = 0, \tag{4}$$

where n is the porosity of the porous medium; $\boldsymbol{v}_f$ and $\boldsymbol{v}_s$ are the velocities of fluid and solid, respectively; $\boldsymbol{h}$ is the heat flux vector; q_f and q_h are the internal or external fluid and heat sources, respectively.

Assuming that the anisotropic flow of pore water and thermal are governed by Darcy's law and Fourier's law, respectively, we have

$$n\left(\boldsymbol{v}_f - \boldsymbol{v}_s\right) = -\frac{k_r}{\gamma_f}\frac{\partial p}{\partial r}\boldsymbol{i}_r - \frac{k_z}{\gamma_f}\frac{\partial p}{\partial z}\boldsymbol{i}_z, \tag{5}$$

$$\boldsymbol{h} = -\lambda_{tr}\frac{\partial \vartheta}{\partial r}\boldsymbol{i}_r - \lambda_{tz}\frac{\partial \vartheta}{\partial z}\boldsymbol{i}_z, \tag{6}$$

where k_r and k_z denote the permeabilities in the horizontal and vertical directions, respectively; γ_f is the unit weight of pore fluid; λ_{tr} and λ_{tz} are the thermal conductivities in the horizontal and vertical directions, respectively.

Let us consider a point heat source of constant strength Q located at point $(0,0)$ and neglect the action of fluid source. Substituting (5) and (6) into (3) and (4), respectively, yield

$$\frac{k_r}{\gamma_f}\left(\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}\right)+\frac{k_z}{\gamma_f}\frac{\partial^2 p}{\partial z^2}=0, \tag{7}$$

$$\lambda_{tr}\left(\frac{\partial^2 \vartheta}{\partial r^2}+\frac{1}{r}\frac{\partial \vartheta}{\partial r}\right)+\lambda_{tz}\frac{\partial^2 \vartheta}{\partial z^2}+\frac{Q}{2\pi r}\delta(r)\delta(z)=0, \tag{8}$$

in which $\delta(x)$ is the Dirac delta function.

Eqs. (2a), (2b), (7) and (8) constitute the basic governing equations of the steady state axially symmetric thermoelastic responses of a saturated cross-anisotropic porous medium.

Boundary Conditions

Since the point heat source is buried at a great depth, the effect of the point heat source must vanish at the infinity ($z \to \pm\infty$) derived the boundary conditions:

$$\underset{z\to\pm\infty}{Lim}\{u_r(r,z),u_z(r,z),p(r,z),\vartheta(r,z)\}=\{0,0,0,0\}. \tag{9}$$

Analytic Fundamental Solutions

With the help of the tools of Mathematica, the closed-form analytic fundamental solutions of the long-term thermally elastic response of ground deformations, effective stresses, temperature change of the soil mass, and excess pore water pressure due to a point heat source buried deep in a cross-anisotropic elastic full space can be obtained by using Hankel transform as follows:

$$u_r(r,z)=\frac{Q}{4\pi\lambda_{tz}}\left(a_1\frac{r}{R_1^*}+a_2\frac{r}{R_2^*}+a_3\frac{r}{R_3^*}\right), \tag{10a}$$

$$u_z(r,z)=\frac{Q}{4\pi\lambda_{tz}}\left(b_1\sinh^{-1}\frac{\mu_1 z}{r}+b_2\sinh^{-1}\frac{\mu_2 z}{r}+b_3\sinh^{-1}\frac{\mu_3 z}{r}\right), \tag{10b}$$

$$p(r,z)=0, \tag{10c}$$

$$\vartheta(r,z)=\frac{Q}{4\pi\lambda_{tz}}\frac{1}{\mu_3 R_3}, \tag{10d}$$

$$\sigma'_{rr}(r,z)=\frac{Q}{4\pi\lambda_{tz}}\left[A\left(a_1\frac{1}{R_1}+a_2\frac{1}{R_2}+a_3\frac{1}{R_3}\right)-2N\left(a_1\frac{1}{R_1^*}+a_2\frac{1}{R_2^*}+a_3\frac{1}{R_3^*}\right)\right.$$
$$\left.+F\left(b_1\frac{\mu_1}{R_1}+b_2\frac{\mu_2}{R_2}+b_3\frac{\mu_3}{R_3}\right)-\beta_r\frac{1}{\mu_3 R_3}\right], \tag{10e}$$

$$\sigma'_{\theta\theta}(r,z)=\frac{Q}{4\pi\lambda_{tz}}\left[A\left(a_1\frac{1}{R_1}+a_2\frac{1}{R_2}+a_3\frac{1}{R_3}\right)-2N\left(a_1\frac{\mu_1|z|}{R_1R_1^*}+a_2\frac{\mu_2|z|}{R_2R_2^*}+a_3\frac{\mu_3|z|}{R_3R_3^*}\right)\right.$$
$$\left.+F\left(b_1\frac{\mu_1}{R_1}+b_2\frac{\mu_2}{R_2}+b_3\frac{\mu_3}{R_3}\right)-\beta_r\frac{1}{\mu_3 R_3}\right], \tag{10f}$$

$$\sigma'_{zz}(r,z)=\frac{Q}{4\pi\lambda_{tz}}\left[F\left(a_1\frac{1}{R_1}+a_2\frac{1}{R_2}+a_3\frac{1}{R_3}\right)+C\left(b_1\frac{\mu_1}{R_1}+b_2\frac{\mu_2}{R_2}+b_3\frac{\mu_3}{R_3}\right)-\beta_z\frac{1}{\mu_3 R_3}\right], \tag{10g}$$

$$\sigma'_{rz}(r,z) = \mp\frac{Q}{4\pi\lambda_{tz}}\left[L\left(a_1\frac{\mu_1 r}{R_1 R_1^*} + a_2\frac{\mu_2 r}{R_2 R_2^*} + a_3\frac{\mu_3 r}{R_3 R_3^*} + b_1\frac{\mu_1|z|}{rR_1} + b_2\frac{\mu_2|z|}{rR_2} + b_3\frac{\mu_3|z|}{rR_3}\right)\right], \tag{10h}$$

in which the upper and lower signs of $\sigma'_{rz}(r,z)$ are for the conditions of $z \ge 0$ and $z < 0$, respectively; $R_i^* = R_i + \mu_i|z|$, $(i = 1, 2, 3)$; $R_i = \sqrt{r^2 + \mu_i^2 z^2}$, $(i = 1, 2, 3)$; and

$$a_1 = \frac{L\beta_r + [(F+L)\beta_z - C\beta_r]\mu_1^2}{CL\mu_1(\mu_1^2 - \mu_2^2)(\mu_1^2 - \mu_3^2)}, \tag{11a}$$

$$a_2 = \frac{L\beta_r + [(F+L)\beta_z - C\beta_r]\mu_2^2}{CL\mu_2(\mu_2^2 - \mu_1^2)(\mu_2^2 - \mu_3^2)}, \tag{11b}$$

$$a_3 = \frac{L\beta_r + [(F+L)\beta_z - C\beta_r]\mu_3^2}{CL\mu_3(\mu_3^2 - \mu_1^2)(\mu_3^2 - \mu_2^2)}, \tag{11c}$$

$$b_1 = \frac{L\beta_z\mu_1^2 + (F+L)\beta_r - A\beta_z}{CL(\mu_1^2 - \mu_2^2)(\mu_1^2 - \mu_3^2)}, \tag{11d}$$

$$b_2 = \frac{L\beta_z\mu_2^2 + (F+L)\beta_r - A\beta_z}{CL(\mu_2^2 - \mu_1^2)(\mu_2^2 - \mu_3^2)}, \tag{11e}$$

$$b_3 = \frac{L\beta_z\mu_3^2 + (F+L)\beta_r - A\beta_z}{CL(\mu_3^2 - \mu_1^2)(\mu_3^2 - \mu_2^2)}; \tag{11f}$$

μ_1, μ_2 must satisfy the characteristic equation

$$CL\mu^4 - [AC - F(F+2L)]\mu^2 + AL = 0, \tag{12}$$

and $\mu_3 = \sqrt{\lambda_{tr}/\lambda_{tz}}$. From the solutions presented, the excess pore fluid pressure disappears under the steady state. With the help of the tools of Mathematica, fundamental solutions of an isotropic soil mass can be obtained from (10a)-(10h) by taking the limit $\mu_1 = \mu_2 = \mu_3 = 1$ and using L'Hospital's rule. Carrying out the procedure, we obtain

$$u_r = \frac{Q\alpha_s(1+\nu)}{8\pi\lambda_t(1-\nu)}\frac{r}{R}, \tag{13a}$$

$$u_z = \frac{Q\alpha_s(1+\nu)}{8\pi\lambda_t(1-\nu)}\frac{z}{R}, \tag{13b}$$

$$p = 0, \tag{13c}$$

$$\vartheta = \frac{Q}{4\pi\lambda_t}\frac{1}{R}, \tag{13d}$$

$$\sigma'_{rr} = -\frac{QG\alpha_s(1+\nu)}{4\pi\lambda_t(1-\nu)}\left(\frac{1}{R} + \frac{r^2}{R^3}\right), \tag{13e}$$

$$\sigma'_{\theta\theta} = -\frac{QG\alpha_s(1+\nu)}{4\pi\lambda_t(1-\nu)}\frac{1}{R}, \tag{13f}$$

$$\sigma'_{zz} = -\frac{QG\alpha_s(1+\nu)}{4\pi\lambda_t(1-\nu)}\left(\frac{1}{R} + \frac{z^2}{R^3}\right), \tag{13g}$$

$$\sigma'_{rz} = -\frac{QG\alpha_s(1+\nu)}{4\pi\lambda_t(1-\nu)}\frac{rz}{R^3}. \quad (13h)$$

LINE HEAT SOURCE INDUCED THERMOELASTIC BEHAVIORS

The deep horizontal line heat source, shown in Figure 2, may induce thermally elastic behaviors can be derived from the deep point heat source induced fundamental solutions. In the Cartesian coordinates system (x, y, z), the fundamental solutions in (10a)-(10h) or (13a)-(13h) can be expressed as

$$u_x(x,y,z) = u_r \cos\theta - u_\theta \sin\theta, \quad (14a)$$

$$u_y(x,y,z) = u_r \sin\theta + u_\theta \cos\theta, \quad (14b)$$

$$u_z(x,y,z) = u_z, \quad (14c)$$

$$p(x,y,z) = p, \quad (14d)$$

$$\vartheta(x,y,z) = \vartheta, \quad (14e)$$

$$\sigma'_{xx}(x,y,z) = \sigma'_{rr}\cos^2\theta + \sigma'_{\theta\theta}\sin^2\theta + \sigma'_{r\theta}\sin 2\theta, \quad (14f)$$

$$\sigma'_{yy}(x,y,z) = \sigma'_{rr}\sin^2\theta + \sigma'_{\theta\theta}\cos^2\theta - \sigma'_{r\theta}\sin 2\theta, \quad (14g)$$

$$\sigma'_{zz}(x,y,z) = \sigma'_{zz}, \quad (14h)$$

$$\sigma'_{xy}(x,y,z) = (\sigma'_{rr} - \sigma'_{\theta\theta})\cos\theta\sin\theta + \sigma'_{r\theta}\cos 2\theta, \quad (14i)$$

$$\sigma'_{xz}(x,y,z) = \sigma'_{yz}(x,y,z) = \sigma'_{rz}, \quad (14j)$$

in which $u_\theta = 0$ and $\sigma'_{r\theta} = 0$. The symbol r in (10a)-(10h) or (13a)-(13h) denotes the horizontal component of the distance between the point heat source and any location of the stratum. Figure 3 presents the horizontal component of the distance from an elementary heat source at point (0, s, 0). Consider the elementary length ds of the line heat source, the thermal strength of the length is equal to qds, and this can be treated as a point heat source. To determine the thermally mechanical response due to the elementary heat source at a point (x, y, z), we can substitute qds for Q and $r = \sqrt{x^2 + (y-s)^2}$ for $r = \sqrt{x^2 + y^2}$. With the help of the tools of Mathematica, the thermo-mechanical behavior at a point (x, y, z) in the xz-plane due to the entire line heating source may now be obtained by integration with respect to the symbol s from $-\infty$ to ∞ and can be given by

$$u_x = \frac{q}{4\pi\lambda_{tz}}(h_1 + h_2 + h_3), \quad (15a)$$

$$u_y = 0, \quad (15b)$$

$$u_z = \frac{q}{4\pi\lambda_{tz}}(h_4 + h_5 + h_6), \quad (15c)$$

$$p = 0, \quad (15d)$$

$$\vartheta = \frac{q}{4\pi\lambda_{tz}}h_7, \quad (15e)$$

$$\sigma'_{xx} = \frac{q}{4\pi\lambda_{tz}}(Ah_8 + Fh_9 - \beta_r h_7), \quad (15f)$$

$$\sigma'_{yy} = \frac{q}{4\pi\lambda_{tz}}\left[(A-2N)h_8 + Fh_9 - \beta_r h_7\right], \tag{15g}$$

$$\sigma'_{zz} = \frac{q}{4\pi\lambda_{tz}}\left(Fh_8 + Ch_9 - \beta_z h_7\right), \tag{15h}$$

$$\sigma'_{xy} = 0, \tag{15i}$$

$$\sigma'_{xz} = \frac{q}{4\pi\lambda_{tz}} Lh_{10}, \tag{15j}$$

$$\sigma'_{yz} = 0, \tag{15k}$$

where $h_i(i=1,2,\cdots,10)$ can be expressed as following:

$$h_1 = -a_1\left[x\ln\left(x^2+\mu_1^2 z^2\right) + \mu_1 z\tan^{-1}\frac{2\mu_1 xz}{\mu_1^2 z^2 - x^2} + \mu_1|z|\right], \tag{16a}$$

$$h_2 = -a_2\left[x\ln\left(x^2+\mu_2^2 z^2\right) + \mu_2 z\tan^{-1}\frac{2\mu_2 xz}{\mu_2^2 z^2 - x^2} + \mu_2|z|\right], \tag{16b}$$

$$h_3 = -a_3\left[x\ln\left(x^2+\mu_3^2 z^2\right) + \mu_3 z\tan^{-1}\frac{2\mu_3 xz}{\mu_3^2 z^2 - x^2} + \mu_3|z|\right], \tag{16c}$$

$$h_4 = -b_1\left[x\tan^{-1}\frac{2\mu_1 xz}{x^2-\mu_1^2 z^2} + \mu_1 z\ln\left(x^2+\mu_1^2 z^2\right)\right], \tag{16d}$$

$$h_5 = -b_2\left[x\tan^{-1}\frac{2\mu_2 xz}{x^2-\mu_2^2 z^2} + \mu_2 z\ln\left(x^2+\mu_2^2 z^2\right)\right], \tag{16e}$$

$$h_6 = -b_3\left[x\tan^{-1}\frac{2\mu_3 xz}{x^2-\mu_3^2 z^2} + \mu_3 z\ln\left(x^2+\mu_3^2 z^2\right)\right], \tag{16f}$$

$$h_7 = -\frac{1}{\mu_3}\ln\left(x^2+\mu_3^2 z^2\right), \tag{16g}$$

$$h_8 = -a_1\ln\left(x^2+\mu_1^2 z^2\right) - a_2\ln\left(x^2+\mu_2^2 z^2\right) - a_3\ln\left(x^2+\mu_3^2 z^2\right), \tag{16h}$$

$$h_9 = -b_1\mu_1\ln\left(x^2+\mu_1^2 z^2\right) - b_2\mu_2\ln\left(x^2+\mu_2^2 z^2\right) - b_3\mu_3\ln\left(x^2+\mu_3^2 z^2\right), \tag{16i}$$

$$h_{10} = a_1\mu_1\tan^{-1}\frac{2\mu_1 xz}{\mu_1^2 z^2 - x^2} + a_2\mu_2\tan^{-1}\frac{2\mu_2 xz}{\mu_2^2 z^2 - x^2} + a_3\mu_3\tan^{-1}\frac{2\mu_3 x|z|}{\mu_3^2 z^2 - x^2}$$
$$+\frac{b_1\mu_1 z}{x}\ln\left(x^2+\mu_1^2 z^2\right) + \frac{b_2\mu_2 z}{x}\ln\left(x^2+\mu_2^2 z^2\right) + \frac{b_3\mu_3 z}{x}\ln\left(x^2+\mu_3^2 z^2\right). \tag{16j}$$

Proceeding in a similar manner, we can also determine the solutions of an isotropic soil mass due to the deep horizontal line heat source as below:

$$u_x = -\frac{q(1+\nu)\alpha_s}{4\pi\lambda_t(1-\nu)} x\ln\sqrt{x^2+z^2}, \tag{17a}$$

$$u_y = 0, \tag{17b}$$

$$u_z = -\frac{q(1+\nu)\alpha_s}{4\pi\lambda_t(1-\nu)} z\ln\sqrt{x^2+z^2}, \tag{17c}$$

$$p = 0, \tag{17d}$$

$$\vartheta = -\frac{q}{2\pi\lambda_t}\ln\sqrt{x^2+z^2}\ , \tag{17e}$$

$$\sigma'_{xx} = \frac{q(1+\nu)G\alpha_s}{2\pi\lambda_t(1-\nu)}\left(\ln\sqrt{x^2+z^2} - \frac{x^2}{x^2+z^2}\right), \tag{17f}$$

$$\sigma'_{yy} = \frac{q(1+\nu)G\alpha_s}{\pi\lambda_t(1-\nu)}\ln\left(x^2+z^2\right), \tag{17g}$$

$$\sigma'_{zz} = \frac{q(1+\nu)G\alpha_s}{2\pi\lambda_t(1-\nu)}\left(\ln\sqrt{x^2+z^2} - \frac{z^2}{x^2+z^2}\right), \tag{17h}$$

$$\sigma'_{xy} = 0, \tag{17i}$$

$$\sigma'_{xz} = -\frac{q(1+\nu)G\alpha_s}{2\pi\lambda_t(1-\nu)}\frac{xz}{x^2+z^2}, \tag{17j}$$

$$\sigma'_{yz} = 0\ , \tag{17k}$$

in which the displacement component u_y and shear stress components σ'_{xy}, σ'_{yz} vanish by locating the *y*-axis through the deep line heat source.

CONCLUSIONS

With the help of the tools of Mathematica, closed-form solutions of the long-term thermoelastic responses due to a constant point/line heat source buried in a cross-anisotropic elastic full space have been obtained using suitable integral techniques. The results have been checked by reducing the solutions of cross-anisotropic thermally elastic behaviors into the isotropic case. All field quantities are functions of the distance from the heat source and are proportional to the linear thermal expansion coefficient, but inversely proportional to the thermal conductivity. For the isotropic cases, the shear modulus does not have influence on displacements and temperature change of the soils.

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SYMBOLS

a_1, a_2, a_3	parameters defined in equations (11a)-(11c)
A, C, F, L, N	elastic constants of the cross-anisotropic porous medium defined by Love(1944)
b_1, b_2, b_3	parameters defined in equations (11d)-(11f)
b_r, b_θ, b_z	body forces
G	shear modulus of the isotropic porous medium
$\boldsymbol{h}$	heat flux vector
$h_1, \cdots, h_{10}$	functions defined in equations (15a)-(15j)
$\boldsymbol{i}_r, \boldsymbol{i}_z$	unit vector parallel to the radial/vertical direction
k_r, k_z	Horizontal/vertical permeability
n	porosity of the porous medium
p	excess pore fluid pressure (positive for compression)
q	strength of the line heat source
q_f, q_h	internal/external fluid and heat sources
Q	strength of the point heat source
(r, θ, z)	cylindrical coordinates system
R	parameter defined as $R = \sqrt{r^2 + z^2}$
R_1, R_2, R_3	parameters defined as $R_i = \sqrt{r^2 + \mu_i^2 z^2}$, $(i = 1,2,3)$
R_1^*, R_2^*, R_3^*	parameters defined as $R_i^* = R_i + \mu_i \lvert z \rvert$, $(i = 1,2,3)$
u_r, u_θ, u_z	radial/tangential/axial displacement of the porous medium
u_x, u_y, u_z	displacements of the porous medium expressed in Cartesian coordinates system
$\boldsymbol{v}_f$, $\boldsymbol{v}_s$	velocity of fluid/solid
(x, y, z)	Cartesian coordinates system
α_s	linear thermal expansion coefficient for solid skeleton of the isotropic porous medium
α_{sr}, α_{sz}	linear thermal expansion coefficient of the skeletal material in the horizontal/vertical direction
β_r, β_z	thermal expansion factor in the horizontal/vertical direction

γ_f	unit weight of pore fluid
$\delta(x)$	Dirac delta function
δ_{ij}	Kronecker delta
ϑ	temperature change of the porous medium
λ	Lame constant of the isotropic porous medium
λ_t	thermal conductivity of the isotropic porous medium
λ_{tr}, λ_{tz}	horizontal/vertical thermal conductivity of the cross-anisotropic porous medium
μ_1, μ_2	characteristic roots defined in equation (12)
μ_3	characteristic root, $\mu_3 = \sqrt{\lambda_{tr}/\lambda_{tz}}$
ν	Poisson's ratio for the isotropic porous medium
σ'_{ij}	effective stress components of the porous medium
τ_{ij}	total stress components of the porous medium

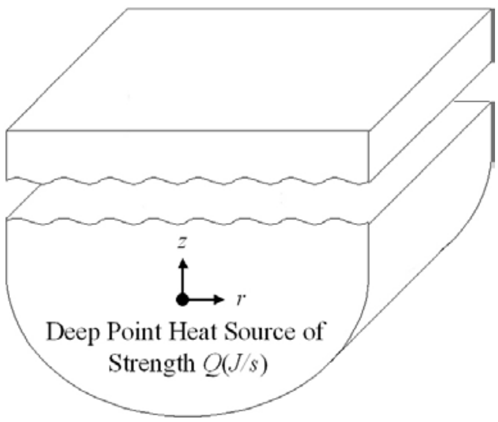

Figure 1. Point heating problem

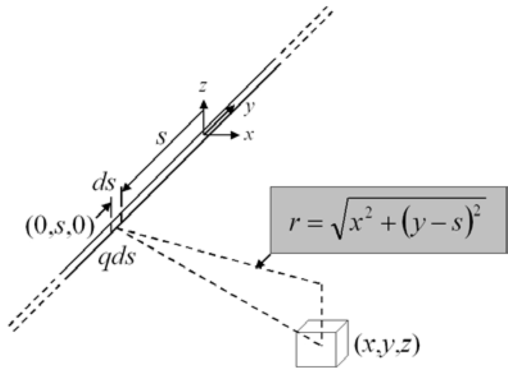

Figure 3. The horizontal component of the distance from an elementary heat source at point (0, *s*, 0)

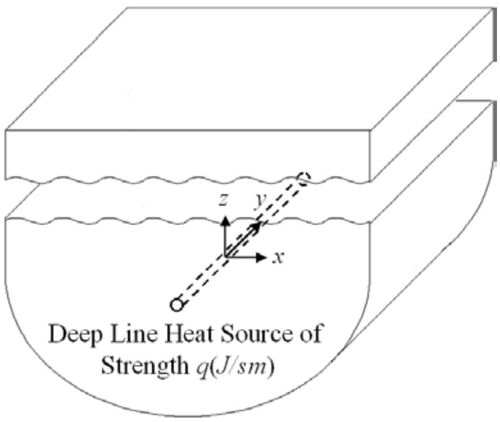

Figure 2. Horizontal line heating problem