Integrating Technology with Mathematical Modeling

Zhonghong Jiang
Florida International University

Abstract

This article discusses the issue of how we can effectively enhance the mathematical knowledge and problem-solving ability of the secondary school pre-service teachers. The reformed teaching featured by mathematical modeling and the use of technology is recommended for the mathematics teacher preparation programs. Several examples that embody the integration of mathematical modeling and technology are introduced to show how these problem explorations help the pre-service teachers take advantage of visual images of mathematical ideas, organize and analyze data in a more meaningful way, and develop a more coherent, flexible, and systematic understanding of the mathematical content.

Introduction

Based on my experience in working with pre-service teachers for many years, I noticed that many of them are weak in mathematics including school mathematics. The main aspects of the weaknesses are little understanding of basic mathematical concepts, limited ability of problem solving and mathematical reasoning, and poor mathematical communication. In a study to explore the Van Hiele levels of geometric thought in undergraduate pre-service teachers, Mayberry (1983) found that pre-service elementary teachers were not at the proper level to understand formal geometry. This was mainly due, according to the study, to the fact that pre-service teachers often do not understand properties of figures, and many of them do not understand and perceive class inclusions, relationships and implications. I have found in my ongoing research that the pre-service secondary mathematics teachers have similar weaknesses, despite the fact that they may have stronger background in mathematics than those in Mayberry's study.

The question is why these pre-service teachers are weak in mathematics. The answer to this question is not obvious. There are some student-related factors such as lack of motivation, lack of academic preparation, and lack of hard work. However, the essential reasons, in my view, include (1) the traditional teaching which features teacher-centered, lecture-oriented classroom, requires rote learning, and pays no attention to students' conceptual understanding, and (2) the school mathematics curriculum which covers too many pieces of content with the pieces at the surface level pursuing no depth.

The situation that our pre-service teachers are weak in mathematics must be fundamentally changed as quickly as possible. A teacher with a strong mathematics background does not necessarily make a good teacher since a good teacher should have sound knowledge of pedagogy, of learning, and of learners as well; but a teacher without a strong mathematics background is definitely not a good teacher. To be a qualified teacher, knowledge of mathematics and school mathematics is indispensable. Preparing future teachers to be effective in the standards-based reform climate depends in part upon teachers' experience of "qualitatively different and significantly richer understanding of mathematics than most teachers currently possess" (Schifter & Bastable, 1995).

Mathematical Modeling and the Use of Technology

The reformed teaching featured by mathematical modeling can effectively enhance the knowledge of the pre-service teachers when provided curriculum and instruction that is challenging and highly engaging. According to Wells, Hestenes, and Swackhamer (1995), the modeling approach brings instruction closer to emulating scientific practice, and it addresses serious weaknesses in traditional instruction. The approach can help students develop a more coherent, flexible, and systematic understanding of the content. It organizes the content around a small number of basic models, which describe basic patterns that appear ubiquitously in a variety of situations. Explicit emphasis on basic models focuses student attention on the structure of content knowledge as the basis for conceptual understanding. Reduction on the essential course content to a small number of models greatly reduces the apparent complexity of the subject. The mathematical modeling approach to learning has proven to be effective in countries such as Japan, which uses the modeling strategy and has outperformed the US in international mathematics and science competitions (Stigler & Hiebert, 1999).

The use of technology can effectively facilitate the pre-service teachers' learning of mathematics. The *Principles and Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM, 2000) indicates, " Electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving" (p. 24). In recent years, many research studies (Choi-koh, 1999; Jiang, 1993; O'Callaghan, 1998) have provided evidence supporting the belief that students could benefit by the use of technology.

It is natural to integrate the use of technology with the mathematical modeling approach as the guiding idea of our strategies to prepare mathematically strong teachers. Several of the strategies are as follows:

1. While the advanced mathematical content courses such as abstract algebra and real analysis are important to the pre-service teachers, school mathematics content has more direct influence on their future teaching. Therefore, two or more (at least one!) courses concentrating on mathematical modeling at the school mathematics level should be designed and offered. The new course(s) should not simply review the mathematics that the pre-service teachers learned when they were secondary school students; rather, the course(s) should emphasize students' active investigations and explorations with the content and emphasize their conceptual understanding. Students should learn to identify problems, make assumptions and collect data, propose models, test the assumptions, refine the models as necessary, fit the models to data if appropriate, and analyze the underlying mathematics structure of the models (Dossey, Giordano, McCrone, & Weir, 2002). The course(s) should be strongly technology-oriented and the students will take full advantage of the power of technology to facilitate their mathematical learning. In this sense, I suggest that the new course(s) be titled something like *Mathematical Modeling with Technology*. Students' learning in the new course(s) and their learning of advanced mathematical subject matters should compensate and promote each other. While the former provides stronger bases and better preparations for learning more advanced mathematical content, the latter generates a higher level perspective in treating topics in school mathematics.

2. Even though two or more such courses could be established (not mentioning many places where it is not so easy to have even one such course), time still would not be enough to cover all topics included in school mathematics curriculum. In addition, even though we had enough time, we could not "take the old path" - touch everything without any emphasis. Therefore, we must implement the principle of "Less is More" and focus on the "big ideas" of school mathematics. The "Less is More" idea is simple but profound. If you teach less content but pay close attention to developing students' conceptual understanding, then the students will be able to transfer the ideas and skills they learn to many new situations. In this sense, they learn more mathematics. "Big ideas" refer to the most important ideas in secondary school mathematics curriculum that the pre-service teachers must grasp. The instructors of the new content course(s) and the methods course(s) should identify the "big ideas" according to the NCTM Standards and the students' actual situations.

3. "Rule of Four" should be one of the guiding principles in the content preparation of the pre-service mathematics teachers. Under a National Science Foundation grant, faculty at Harvard University and seven other institutions designed a reform-oriented calculus course emphasizing the principle "Rule of Three", which says that wherever possible topics should be taught graphically and numerically, as well as analytically (Hughes-Hallett, 1991). This principle is also significant in the content preparation of the pre-service mathematics teachers, but the graphical, numerical and analytical approaches should be integrated with the verbal representation and approach, which emphasize mathematical communication. The pre-service teachers should be asked to express their mathematical thinking processes and provide explanations for the validity of their mathematical models for various problems clearly and coherently. Technology has enabled us to implement "Rule of Four" much more effectively and efficiently than ever before. The use of technology puts students in a position to visually manipulate the mathematical objects and hence experience mathematics by seeing it happen or making it happen. It also helps link mathematical visualization to numerical and symbolic aspects of mathematics, and improves communications between the instructor and students, and among students.

Examples

In the following, I will briefly discuss a few mathematical modeling problems that are typically used to work with the pre-service teachers in their mathematical investigations and explorations related to school mathematics. The technologies used include the Geometer's Sketchpad (GSP) (Jackiw, 1995), Microsoft Excel spreadsheet and the Calculator-Based Laboratory.

Example 1. The "Street Parking" Problem

You are on the planning commission for Algebraville, and plans are being made for the downtown shopping district revitalization. The streets are 60 feet wide, and an allowance must be made for both on-street parking and two-way traffic. Fifteen feet of roadway is needed for each lane of traffic. Parking spaces are to be 16 feet long and 10 feet wide, including the lines. Your job is to determine which method of parking – parallel or angle – will allow the most room for the parking of cars and still allow a two-way traffic flow. (You may design parking for one city block (0.1 mile) and use that design for the entire shopping district.)

When exploring this problem, a considerable number of students (i.e., pre-service teachers, the same hereafter if not specified) may intuitively conjecture that angle parking is better than parallel parking (because they mostly experience angle parking). The parallel parking situation is quite simple, and students are able to quickly determine how many cars can be parked in one city block. However, most of the students would have difficulty formulating a mathematical solution for the angle-parking situation, other than the intuitive conjecture. I will present a constructed GSP sketch (Figure 1) for students to investigate the situation. By working with the sketch, the students can clearly see that in order not to block the traffic on the right side of the street, the left side of each parking space has to be "dragged down" so that the yellow rectangle does not intersect the lane of traffic (or at most "touches" the traffic lane at one point only). Doing so will make the curb space very long, resulting in the fact that much more space is wasted and fewer vehicles can be parked in one city block. This visual feedback will help students correct their misconceptions. Moreover, their experience with the situation enhances the change that they will be able to identify key factors in the situation. Using similar-triangle relationships and the Pythagorean theorem, students can finally arrive at a mathematical solution and understand why parallel parking is often used in downtown areas where streets may be narrow. The solution is a system of equations: $x/10=(16+y)/15$ and $y^2+10^2=x^2$, where x is the length of the curb space, and y, x, and 10 are the lengths of the three sides of a right triangle.

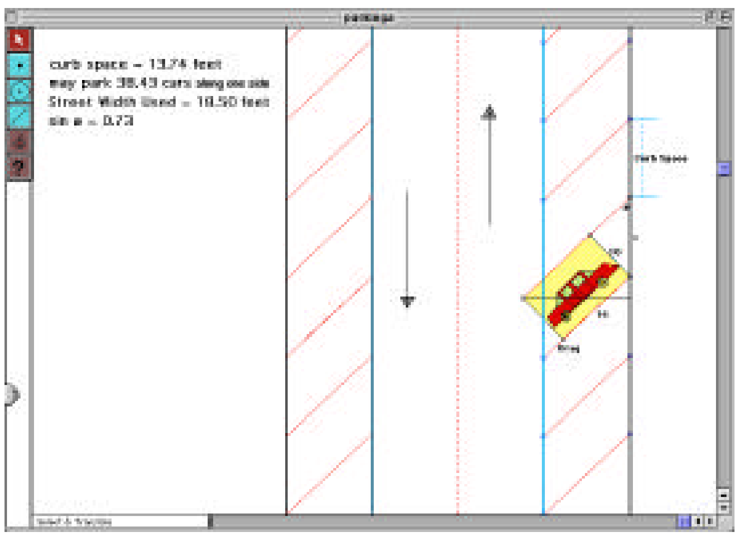

Figure 1. A GSP sketch representing the angle-parking situation

Example 2. The "Investment" Problem

You plan to invest part of your paycheck to finance your children's education. You want enough money in the account to be able to draw $1,000 a month, every month for 8 years beginning 20 years from now. The account pays 0.5% interest each month. (a) How much money will you need 20 years from now to finance one child' education? Assume you stop investing when your first child begins college. (b) How much you must deposit each month over the next 20 years?

Many students would have difficulty formulating a mathematical solution when they are first given this problem. I will use a spreadsheet to help them understand the problem situation, identify variables, and find the relationships among the variables as well as constants given in the problem.

Through group discussions, the students would agree that it is a good idea to analyze the situations regarding deposit, interest earning, and money drawing month by month until they find the pattern, which can be easily handled using formulas in a spreadsheet. Their understanding of the situations of the first three months are reflected in Figure 2:

	A	B	C	D	E
1	Month no.	Beginning of month	End of month		monthly deposit
2	1	=E$2	=B2*0.005+B2		200
3	=A2+1	=C2+E$2	=B3*0.005+B3		
4	=A3+1	=C3+E$2	=B4*0.005+B4		

Figure 2. The formulas for calculating the transactions for the first three months

I may need to explain the Excel formulas and syntax used in this spreadsheet. The “200” in cell E2 is a tentative number, representing the student’s initial guess about how much “you” should deposit each month. The formula “=E$2” in cell B2 represents $200 “you” deposit during the first month of the investment. The “$” sign in this cell is to guarantee that the money amount represented is always the one in the second cell in the E column no matter how you copy the formula in the B column. Why “E$2” is referenced in cells such as B2 is for flexibility—one only needs to manipulate the number in sell E2 to adjust the content of the whole spreadsheet to get the desired result. The formula “=B2*0.005+B2” in cell C2 simply expresses the balance at the end of the first month (i.e., interest + principal). The formula “=C2+E$2” in cell B3 represents the balance at the beginning of the second month (including the second deposit). The content in the rest of the cells seems to be self-explanatory. Based on the analysis embedded in this spreadsheet, students will be ready to use the revealed pattern and copy the formulas in the third row or those in the fourth row down to all rows until the 241st row, which gives the 240th month (the last month of the 20 years from now) and the balances at the beginning and end of that month. If the number in cell E2 is the correct amount of money for the monthly deposit, then the number in cell C241 represents the money that you will need 20 years from now to finance one child’s education (assuming “you” stop investing when “your” first child begins college).

Starting from the 242nd row, the students will discuss the situations in which "your" first child draws $1,000 from the account every month during the next 8 years. Let's look at the formulas used in the 241st row and the 242nd row shown in the following spreadsheet (Figure 3).

	A	B	C	D	E
1	Month no.	Beginning of month	End of month		monthly deposit
2	1	=E$2	=B2*0.005+B2		200
3	=A2+1	=C2+E$2	=B3*0.005+B3		
240	=A239+1	=C239+E$2	=B240*0.005+B240		
241	=A240+1	=C240+E$2	=B241*0.005+B241		
242	=A241+1	=C241-1000	=B242*0.005+B242		
243	=A242+1	=C242-1000	=B243*0.005+B243		

Figure 3. The difference between the formula in cell B241 and that in cell B242

It is clear that the formula used in cell B242 is different from that used in cell B241. The formula "=C241-1000" indicates that "your" first child has begun college and drawn money from the account, and at the same time, "you" have stopped investing. The formula in cell C242 remains the same as the one in cell C241 (in terms of the mathematical relationship represented by the formula). This is because the account still pays interest before all money in the account is drawn.

Figure 4 shows the numerical results of applying the corresponding formulas. The results provide the answers to the questions raised by the problem: The money you will need 20 years from now to finance one child' education is $76,475.70 assuming you stop investing when your first child begins college, and you must deposit $164.70 each month over the next 20 years. The 337th row gives the balance of the account during the last month of 28 years from now. In that month, "your" child draws the last $1,000 from the account and the balance becomes 0. This is why cell C 337 is empty.

	A	B	C	D	E
1	Month no.	Beginning of month	End of month		monthly deposit
2	1	$164.69	$165.52		***$164.69369***
3	2	$330.21	$331.86		
4	3	$496.56	$499.04		
5	4	$663.73	$667.05		
238	237	$74,475.93	$74,848.31		
239	238	$75,013.00	$75,388.07		
240	239	$75,552.76	$75,930.53		
241	240	$76,095.22	***$76,475.70***		
242	**241**	**$75,475.70**	**$75,853.07**		
243	**242**	**$74,853.07**	**$75,227.34**		
244	**243**	**$74,227.34**	**$74,598.48**		
245	**244**	**$73,598.48**	**$73,966.47**		
246	**245**	**$72,966.47**	**$73,331.30**		
333	**332**	**$3,950.50**	**$3,970.25**		
334	**333**	**$2,970.25**	**$2,985.10**		
335	**334**	**$1,985.10**	**$1,995.03**		
336	**335**	**$995.03**	**$1,000.00**		
337	**336**	**$0.00**			

Figure 4. A numerical solution to the problem

Because the spreadsheet activity described above helps students examine every important specific case, find the patterns, and explore the relationships that are necessary for building a mathematical model, it will be not very hard for the students to come up with a mathematical solution to the problem. For example, if the money that you must deposit each month over the next 20 years is represented by x, then following the same strategy used in the spreadsheet activity, the students may construct a table such as Table 1 to solve the problem via an inductive approach.

The table suggests the following equation: $y*(1+0.005)^{95}-1000\{[(1+0.005)^{96}-1]/0.005\}=0$ (where $y = x*\{[(1+0.005)^{241}-1]/0.005 -1\}$), which is a mathematical model for this problem.

Table 1

Month no.	Beginning of month	End of month
1	x	x (1+0.005)
2	x*(1+0.005) + x	x*$(1+0.005)^2$ + x*(1+0.005)
3	x*$(1+0.005)^2$ + x*(1+0.005) + x	x*$(1+0.005)^3$ + x*$(1+0.005)^2$ + x*(1+0.005)
. . .	. . .	. . .
240	x*$(1+0.005)^{239}$ + x*$(1+0.005)^{238}$ … + x*(1+0.005) + x	x*$(1+0.005)^{240}$ + x*$(1+0.005)^{240}$ + … + x*(1+0.005) = x*{[$(1+0.005)^{241}$-1]/0.005 -1}
241	y-1000 where y = x*{[$(1+0.005)^{241}$-1]/0.005 -1}	y(1+0.005)-1000(1+0.005)
242	y*(1+0.005)-1000(1+0.005)-1000	y*$(1+0.005)^2$-1000*$(1+0.005)^2$-1000*(1+0.005)
243	y*$(1+0.005)^2$-1000*$(1+0.005)^2$-1000*(1+0.005)-1000	
. . .	. . .	. . .
396	y*$(1+0.005)^{95}$-1000*$(1+0.005)^{95}$-… -1000*(1+0.005)-1000 = y*$(1+0.005)^{95}$-1000*{[$(1+0.005)^{96}$-1]/0.005}	

Example 3. The "Shortest Path" Problem

A boy at location B (see Figure 5) wants to go first to the river to water his horse and then go home (location H). What is the shortest path for him to do so?

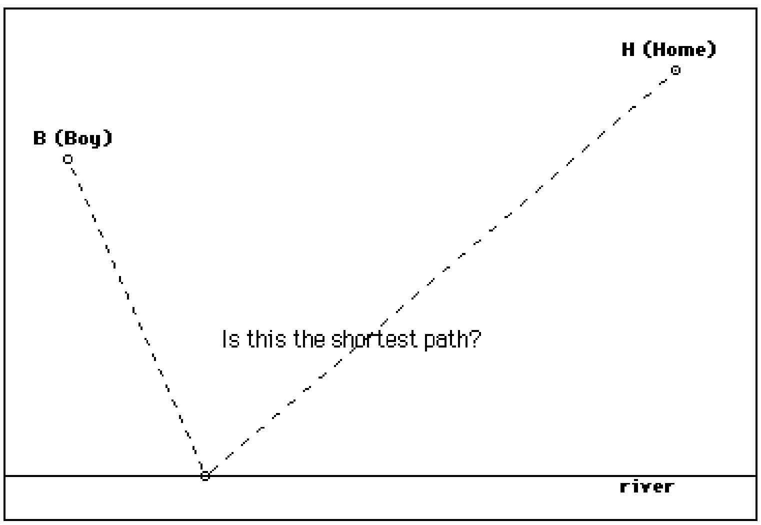

Figure 5. The "Shortest Path" problem

To help students construct a mathematical solution for this problem with conceptual understanding, it seems appropriate for them to do an experiment using the Calculator-Based Laboratory (CBL). For this experiment, two CBL equipment combinations are required. Each combination consists of a CBL unit, a motion detector and a TI-83 graphing calculator. This experiment also requires the SHORTPATH.83P program, which will be developed. Students may simulate the problem situation and come up with a mathematical model as follows:

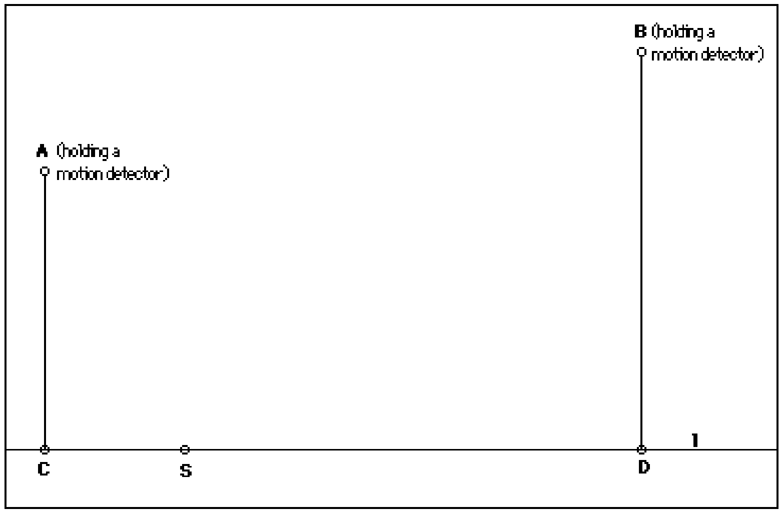

Figure 6. Experiment Setup

1) Mark a line segment on the floor in the classroom (like segment l in Figure 6). Two students stand respectively at points A and B. Each student holds a motion detector. Mark segments AC and BD, which are perpendicular to segment l. The third student (S) stands on segment CD. He is ready to walk along segment l.

2) Turn on the CBL unit and the TI-83 calculator, which are connected. The CBL system is now ready to receive commands from the calculator. Specifically, the motion detector (also connected to the CBL unit) will detect the movement of the third student walking along segment CD.

3**)** The distances between point A (or B) and the walking student S will be stored in L_2, while their corresponding time values are stored in L_1 (for both TI-83 calculators).

4) After the data collection is completed, link the two TI-83 calculators only, and send the L_2 values in one calculator to the other, and store them in L_3 for the latter.

5) Use the calculator with the L_2 and L_3 values discussed above to do the analysis.

6) Store the sums of the corresponding values in L_2 and L_3 in L_4 (L_2+ L_3 -> L_4).

7) Find the minimum value of L_4 (Min(L_4)).

8) Use the sequence number of the minimum value to determine the location of S on segment CD so that AS + SB is the shortest path. For example, if L_4(25) is the minimum value, then if the sample size is 60, we will have m(CS) = (25/60) * m(CD), i.e., m(CS):m(SD) = 25:35.

9) Place a mirror at point S whose location is determined by Step 8, and the mirror plane should be along segment CD, and perpendicular to the floor. The student at point A can use the laser flashlight to aim at the mirror (a dark room is recommended for this part of the experiment). The student at point B will see the light beam. (Students will also find ASC and BSD are congruent.) Through this activity, students will know that if AS is the path for the light beam from A, then SB is the path of the reflection light. In other words, the location of "minimum point" S can be determined by finding the reflection image of point A (or point B), and then

connect this image and point B (or point A). This gives a mathematical solution to the problem, which is illustrated in Figure 7, in which A' is the reflection image of point A.

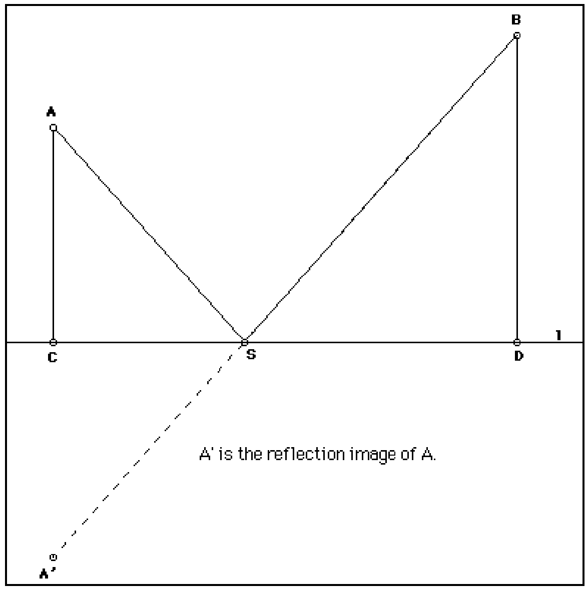

Figure 7. Determine the shortest path by reflection

The same program and configuration of CBL system can also be used to find the solutions for other shortest path situations such as the following: *A road will be constructed from Town A to Town B, which are separated by a river (see Figure 8). Therefore a bridge is necessary. Suppose the bridge is perpendicular to the riverbanks, what is the shortest path for the road?*

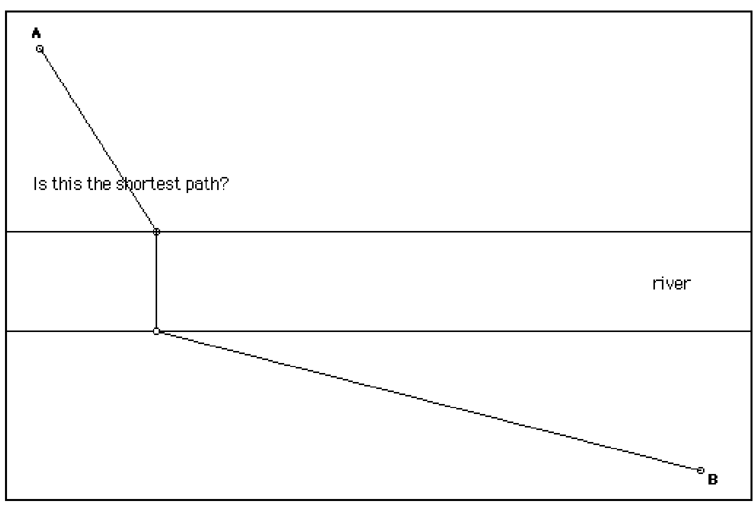

Figure 8. Another shortest path situation

Conclusion

I believe that the integration of technology and mathematical modeling will effectively help our pre-service teachers reach a better understanding of mathematical concepts and develop stronger problem solving abilities. To provide evidence to this belief, research is necessary. I have proposed a research study to seek the answers to the following research questions: (1) Do the pre-service teachers with rich modeling experience perform better academically than those with little or no modeling experience? (2) What is the impact of the modeling approach on the pedagogical knowledge of the pre-service teachers? Experimental research will be the main research method used for the study. A pretest and posttest experimental design will be implemented to groups of pre-service teachers. Analyses such as ANOVA will be implemented to analyze the data collected. In addition to the quantitative research, the study will also use in-depth interviews of pre-service teachers to collect qualitative data. A constant comparative approach will be used to analyze these data.

References

Choi-koh, S. S. (1999). A student's learning of geometry using the computer. *The Journal of Educational Research*, 92, 301-311.

Dossey, J., Giordano, F., McCrone, S. & Weir, M. (2002). *Mathematics methods and modeling for today's mathematics teachers*. Pacific Grove, CA: Brooks/Cole.

Hughes-Hallett, D. (1991). Visualization and calculus reform. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in Teaching and Learning Mathematics* (pp. 121-126). Washington, DC: Mathematical Association of America.

Jackiw, N. (1995). *The Geometer's Sketchpad* (Software). Berkeley, CA: Key Curriculum Press.

Jiang, Z. (1993). *Students' learning of introductory probability in a mathematics microworld*. Unpublished doctoral dissertation, University of Georgia, Athens.

Mayberry, J. (1983). The Van Hiele levels of geometric thought in undergraduate pre-service teachers. *Journal for Research in mathematics Education*, 14, 58-69.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: The Council.

O'Callaghan, B. R. (1998). Computer-Intensive Algebra and students' conceptual knowledge of functions. *Journal for Research in Mathematics Education*, 29, 21-40.

Schifter, D & Bastable, V. (1995, April). *From the teachers' seminar to the classroom: The relationship between doing and teaching mathematics, an example from fractions*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Stigler, J. & Hiebert, J. (1999). The teaching gap: Best ideas from the world's teachers for improving education in the classroom. New York: Free Press.

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics*, 63(7), 606-619.