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$\overline{X}$ CONTROL CHART FOR SKEWED POPULATIONS BASED ON POWER TRANSFORMATIONS

Abstract:

This paper discusses the use of power transformations in setting up the control limits of the $\overline{X}$ chart for skewed populations. The power transformation is an effective and useful approach of transforming skewed data to achieve approximate normality. The idea is then to apply the standard 3σ Shewhart $\overline{X}$ control chart approach to the transformed data. The power transformation approach of normalizing a skewed data set is to take all the data values and raise each of them to some power λ, i.e., X^{λ}. Values of $\lambda < 1$ correspond to power transformations useful for normalizing positively (or right) skewed data while values of $\lambda > 1$ correspond to power transformations useful for normalizing negatively (left) skewed data. The more extreme the degree of skewness, the more extreme the value of λ chosen relative to the pivotal value of 1. A number of procedures are available to identify a possible value of λ for a skewed data set. However, we will only consider the Box-Cox procedure in this paper. A powerful statistical software, i.e., Minitab will be used to determine the value λ which is required for the power transformation based on a set of skewed data. Furthermore, if the skewed distribution is known, then its expected value, $E(X^{\lambda})$ and its variance, $Var(X^{\lambda})$ can be computed using Mathematica 4.0. With these information known, the standard Shewhart $\overline{X}$ control chart can be constructed by plotting the transformed values, X_i^{λ}, i = 1, 2,… against the $\overline{X}$ chart's control limits computed based on the transformed data. SAS version 6.12 will also be used to study the average run length properties of the $\overline{X}$ control chart based on skewed data. This research shows how computer technology using mathematical and statistical packages are used in the decision making and assessment of quality control procedures.