Teaching About the Graphs of Functions with Hand-held Technology

Dr. Brenda Lee
Wu Feng Institute of Technology
Blee@sun5.wfc.edu.tw

.

Abstract

Students have difficulties in understanding graphs of functions. We have observed that our night school students frequently have no idea what the graphs are representations for. What are the relationships between the graphs and (analytically defined) functions? Do the functions have anything to do with the real life situations? Do their graphs?

In this paper we present the results of a test designed to see if students were making these conceptual connections. We present their answers, and categorize their answers according to a certain framework intended to roughly measure their mathematical sophistication. Some discussion is given. We then discuss some calculator-aided activities which we hope will increase interest in learning to draw the graphs of functions and in turn will improve students' performance as measured by this type of test.

The Difficulties of Drawing the Graphs of Functions in Traditional Classroom

Learning mathematics is almost a nightmare for most of the night school students in Taiwan. Especially when doing the graphs of functions, many students have no ideas what the graphs are representations for. What are the relationships between the graphs and (analytical defined) functions? Do the functions have anything to do with the real life situations? Do their graphs?

Students are especially bored when drawing graphs by hand in the traditional classroom. There are several things that discourage them when doing this activity. First, when drawing the graphs of functions by hand, students are wondering how many points are enough in order to make the curve smooth. Secondly, computing the function values frequently requires some basic algebraic techniques which the students do not possesses. Thirdly, the underlying reasons for drawing the graphs are not apparent to them. The first two difficulties can be overcome with the use of hand-held technology. The third difficulty will require some new teaching strategies, and that is the main focus of the present paper.

The Hand-held Technology as Innovation in Our Classroom

Hand-held technology gives students marvelous opportunities to explore and discover relationships more efficiently than they can by using traditional pencil-and-paper methods. The technology often furnishes opportunities for extending the original content of the lesson to include insights that could not have been addressed otherwise (Cyrus & Flora, 2000).

In teaching about functions, we believe in the use of hand-held technology, with its capacity for quickly producing numerous accurate examples. In our classroom we use the Casio Fx9850 calculator to introduce the functions, and try to introduce some real life tie-ins for each type of function.

In order to test how well our students were doing, we presented them with five graphical diagrams (pictures in Figures 1-5 below) and asked them to write down how they think the functional graphs might apply to real life situations. Many of the answers were inaccurate, and interestingly enough to investigate, many students seemed to see the graphs purely as pictures. These phenomenon have been observed by many other researchers.

Misconceptions in Connecting Graphs with the Real World

Students may have difficulty distinguishing between the functional relationships of two variables and the visual stimuli received when observing the actual real world situations. The most common misconceptions here is graphs-as-a-picture confusion, where students do not see a graph as a relationship between variables but rather as one object (Dumham & Osborne, 1991; Mokros & Tinker, 1987; Lapp & Cyrus, 2000). Another is treat graph-shape and path-of motion as the same confusion. Also, students often believe that the shape of the graph should resemble the shape of the physical situation (Cyrus & Flora, 2000).

Monk and Nemirovsky (1994) suggest that students' misconceptions are not simply replaced by correct conceptions but that students refine their conceptions in a gradual and continuous way.

Transitioning between Graphs and Real life Situation

A vital skill in learning mathematics function is the ability to leap back and forth between a graph of function and the real life situation that the graph describes. The question is, how can we, in practice, help students make the leap from the real life situation to the graph and back? Bruner (1966) suggested a progression from enactive to iconic to symbolic representations, that is, the student moves from physical modeling the problem with materials (enactive) to diagramming or graphing (iconic) to putting the problem into an abstract mathematics form (symbolic).

Skemp identifies two levels of language: deep structures and surface structures. The vocabulary issues which the "surface structures" are used in classroom activities to transmit ideas as we engage students. We hope that classroom activities will lead students to the "deep structures" of mathematical concepts. We need to be sensitive to the language of mathematics we use in the classroom and the students' growth in fluency with them (Thompson & Rubenstain, 2000).

Two practices that offer promise in connecting graphs with physical events are prediction and duplication activities (Monk and Nemirovsky, 1994). Engaging students in activities that demonstrate the relationship among different types of graphs is beneficial. Letting students deal with different graphical representations of the same event can help develop understanding of how information is conveyed by various types of graphs.

In today's classroom, many students are strong visual learners thanks to video game practice, and they benefit when we support verbal learning with hand-held technology visual strategies. We can use pictures that are connected with written descriptions in the students' own words as a strategy to develop mathematical concepts.

Although linking a real life situation with its graph is important the student also needs to be flexible when interpreting graphs. For example, the same graph can be interpreting different physical events based on how the graph is visualized as position-time, velocity-time and acceleration-time graphs. Dealing with an apparent conflict between similar graphs arising from different real life situations can reinforce the way that information is obtained from each graph. Of course, the flexibility depends on an awareness of precisely what real life quantity is being measured along each axis. The vagueness of some of the responses to our questionnaire seems to indicate the students do not understand that the horizontal and the vertical axes must correspond to precisely understood and definitely measurable quantities.

The Project

We would like our students to have the ability to display graphical representations of data in real time and to have the ability to link the graph and the physical concepts. Bruner's progression is somewhat similar to, and suggested, our framework for categorizing students answers to our test and using Skemp' level of language to elicit students' understanding of the concept of function.

The Questionnaire

The test given to the students was as follows: What do you see when the following pictures are given? Use your own words to describe how these functional graphs might apply to real life situations.

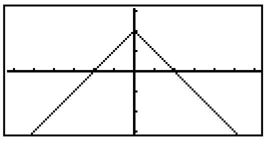


Fig. 1

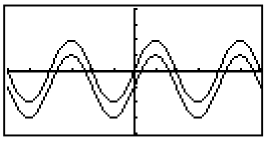


Fig. 2

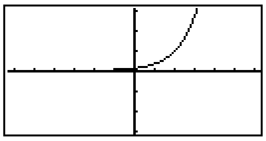


Fig.3

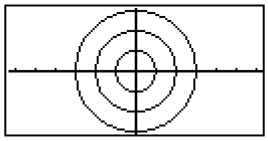


Fig. 4

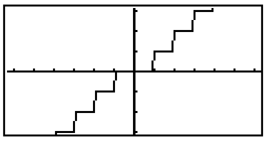


Fig. 5

The Framework

For some students they are graphs of functions or combinations of graphs of functions (symbolic). For some other students, they are pictures or diagrams (iconic) we see every day in our real life situations or some physical event phenomenon (enactive) they encounter everyday. Based on Bruner's progression, we designed the following criteria for our framework of categorizing the responses from our students' written works.

Criteria of the Framework

Category 0: Students presented a real life situation having no observable relation with the picture.
Category 1: Students presented a real life situation only seeing the graph purely as a picture.
Category 2: Students presented a real life situation with some vague mathematical content.
Category 3: Students presented a real life situation with some strong mathematical content.
Category 4: Students presented a real life situation comprising a mathematically correct answer.

The Results

In this paper, we do not analyze the results of students' responses. We simply present the students' responses and the categories of their responses based on our criteria of the framework.

First Graph.

Category 4: Upside-down absolute value, absolute value.

Category 3: linear function, number of customers in a restaurant from 2:00 to 10:00.

Category 2: corner of a baseball diamond, reach highest point then come down, stock market goes up and down (students tend to think growth laws should be linear), a person's mood during the day, a students' grade, equilateral triangle, Taiwan's economic miracle, human energy map, stress (on a job), mirror reflection, health status, like taking a math class (level of interest), working load.

Category 1: mountain, climbing mountain, pyramid, tent, road-sign, gun-sight, headlight pattern, measure angles, measure height, scaffolding.

Category 0: umbrella.

Second Graph.

Category 4: sine wave, motion of a swing.

Category 3: oscillations, stock market, price of a commodity, electrical current, sound wave.

Category 2: wave, pulse, two waves, a person's mood, brain wave, pain level of a toothache, Nielson rating, stress of homework.

Category 1: Two s-shapes, obstacle sign, bumpy road, path of a car with a drunk driver, river, race car road map, electrocardiogram, s-curve road, part of a snake's body, ribbon, permanent hair wave, two persons whose lives have no intersection.

Category 0: road map.

Third Graph.

Category 4: cumulative frequency graph, money in the bank, speed of running downhill, total salary if given a percentage raise every year, parabola.

Category 3: number of written words getting more and more, accelerating a car, cost of living (money spent), activity level rising after getting out of bed, $a^0=1$, index of good mood.

Category 2: "just a curve", stock market, "getting better and better", growth rate, desired growth of grades, flight of an airplane, inflation, rise of temperature, mood of a person (how do you quantify your mood?), accumulated money put in a slot machine (which actually shouldn't be exponential), people's age, performance on a job, life-style getting better, weight getting higher, blood pressure, production of a company.

Category 1: climbing mountain, flying a kite, roller coaster, turning left, go uphill (based on the slope), a children's slide, shape of thread of flying kite, rocket flight, path of airplane, path of a released balloon, ran from the bottom to the top of a slide.

Category 0: rising sun, promotions, no job then finding a job.

Fourth Graph.

Category 4: concentric circles.

Category 3: sound waves, supersonic waves.

Category 2: radar, indicators of a mountain's height (contour map).

Category 1: growth rings of a tree, target, multiple vision (as when dizzy), racetrack, water ripples, hula hoops, expanding yourself (by walking out of the circle you are in), smoke rings, planetary orbits, spiral, examining things locally at first then from broader and broader points of view (as in making a business decision).

Category 0: seeing stars (as in cartoons when knocked on the head), clock, cycle of a washing machine, typhoon, donuts, car, wheel.

Fifth Graph.

Category 4: step-function.

Category 3: ages, yearly growth of tuition, postage, electricity bill, income tax.

Category 2: grades, getting promoted each stage of the way, growth rate, temperatures, progress of the homework, interest rate, different people's income.

Category 1: Stairs, climbing the stairs, merit pay, ages of classmates.

Category 0: different stages of life.

Discussion

We note that a large number of answers landed in Category 1, which indicates that many students were interpreting the graphs in purely pictorial terms. Evidently these students have not learned to think of a graph as a relationship between variables, for the very context of the test should have set them to looking for functional relationships.

Some answers were very interesting and imaginative even when they were not mathematically sound. We especially liked the answer about the number of customers in a restaurant (for Figure 1) (The linearity of the graph may not be quite plausible, but it is very reasonable that the high point occurs at 6:00.) Some answers gave psychological or sociological interpretations: for instance the interpretation of three concentric circles in terms of expansion of one's personality. These answers show a capacity for abstraction, but in different direction from mathematics. We did not get a lot of answers in Category 0, which we are inclined to find encouraging. Almost always, the students tried to make some sense out of the graph, and gave answers which had some clear relationship to what they were seeing. There were few instances in which the answers were incomprehensible to us. In most cases, however, a strong link to the mathematical language of equations and functions were not there.

For the third graph, only a couple of students made explicit mention of the exponential function. However, many gave the real life situations that are frequently used as examples when teaching the exponential function. These many students could look at that graph and think of compounded interest on a saving account, but perhaps could not explicitly identify the graph as an exponential function. A really good answer for Figure 4 would have to involve the equation of a circle. Ideally, the students should be aware that a circle is the union of two function graphs. Nobody gave such an answer, but many interesting applications of circle were given.

We believe we see that the "surface structures" (in Skemp's terminology) have communicated themselves fairly well, but the "deeper structures" are not really grasped by many of the students. Students remember the real life tie-ins used to introduce the functions but are not familiar with the functions themselves. We noted that increasing functions are generally perceived as linearly increasing and there are not many answers of different kinds of growth. Many students do not seem to understand that horizontal and vertical axes have to represent clearly defined and measurable quantities.

Although the students' notion of functional graphs is based on the lower categories, still it reveals an education practical value. We can verify that the use of technology can reinforce the connection between the real life situations and the analytical expression of functions. For the present study, the data clearly show that the students are not good at identifying the analytical expression of functions. They exhibit the graph as-a-picture confusion, treat graph-shape and path-of motion as the same confusion and believe that the shape of the graph should resemble the shape of the physical situation. However, they surely learn to make the connection between the real life situations and the functional graphs. With the help of technology, we hope to explain that mathematics is not just for the elite but for all, because it is around us and it is fun.

We hope to return in later work to the results of this test and other like it, perhaps with more rigorous examination of data and possibly some refinement of our simple five-category scale for measuring responses (many of which seemed to fall on the boundaries of categories).

Most of the students in night school environment will find the notion of functions intimidatingly abstract. The new hand-held technology, with its capacity for quickly generating a variety of examples, can greatly help in getting the function concept across. Functions can be presented in three representations; by equations, by graphs, and by tables of values. Each type of presentation makes connections with real life situations in different ways. Hand-held technology can help students tie these different forms of presentation together as a single concept. We turn now to a discussion of some calculator-aided suggestions that we hope will help students build the deep structures for understanding and working with function graphs.

Suggestions for Classroom Use

Try to see the following scenarios. If the students were asked to plan for a camping trip and they need to build a tent. What shape does the tent have? If they need to draw the graph of the tent, what function do they need? Will the angle of the tent change the shape of the tent? Will the height of the tent change the shape of the tent?

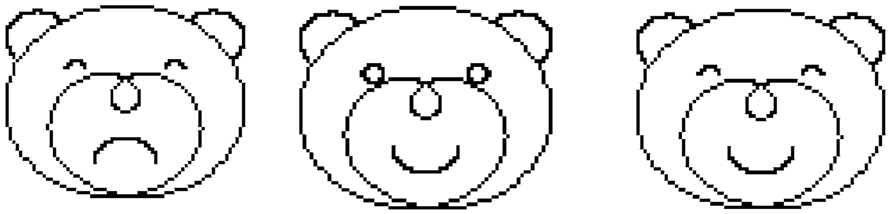
If the students were asked to design a s-shape road for the roller skaters, what function should they use? If this S-shape road is designed for less than 10 years old or older than 10 years old, do you think the function should be the same? Why is the design the same or why is it not? If the students were asked to show the demand and supply relationship, what functions should they use? How are the shift, the phase, and the height of the sine functions related to the economical cycles?

If the students were asked to design a postage rate for different types of letters, what function should they use to describe these situations. Do the registered mails differ from the airmails? Are the stamps for the registered mails and the airmails the same? Does the weight of the letters affect the stamps we put on the letters?

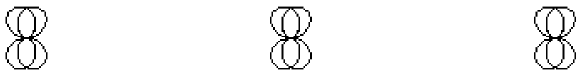
The Fun Parts of Mathematics Functional Graphs

Happy Bear

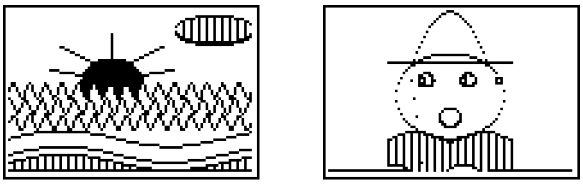
If the students were asked to draw the shape of Teddy Bear, what functions do they need to use? What do we need to change in order to make the Teddy Bear smile, look astonished or look grouchy? In order to see the Teddy Bear in the screen of the calculator, what window setting should be used?



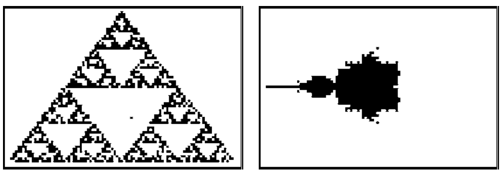
The Number Eight



Sun Rising and the clown



Other fun graphs drawn by hand-held technology



The first eight figures are composed of graphs of functions, and the students should try to identify the functions used for various parts of these pictures. The last two figures, well-known in the area of fractals, are of course not graphs of functions, but can be used to illustrate the notion of iteration of functions for students who have a more advanced understanding of the concept of functions or the ability to design their own programs. These drawings of fun graphs might create a reason for students to pay more interest in learning the notion of functions.

References

Bruner, Jerome S. (1966). Toward a Theory of Instruction. New York: W. W. Norton & Co.

Cyrus, Vivian F. & Flora, Ben V. (2000). Don't Teach Technology, Teach with Technology. Mathematics Teacher 93, p.564-567.

Dumham, Penelope H. & Osborne, Alan (1991). Learning How to See: Students' Graphing Difficulties. Focus on Learning Problems in Mathematics 13, p.35-49.

Lapp, Douglas, A. & Cyrus, Vivian F. (2000). Using Data-Collection Devices to Enhance Students' Understanding. Mathematics Teacher, Vol. 93 #3, p.463-511.

Mokros, Janice R. & Tinker, Robert F. (1987). The Impact of Microcomputer-based Labs on Children's Ability to Interpret Graphs. Journal of Research in Science Teaching 24, p.369-383.

Monk, Stephen and Nemirovsky, Ricardo (1994). The case of Dan: Students Construction of a Functional Situation through Visual Attributes. In Research in Collegiate Mathematics education I, edited by Dubinsky, Ed; Schoenfeld, Alan & Kaput, James, p139-168.

Steward, Ian. (1990). Change. In Steen, L. A. (Ed) On the shoulders of giants, New approaches to Numeracy. Washington, D. C.: National Academy. Press, p.183-217.

Thompson, Denisse R. & Rubenstain, Rheta N. (2000). Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies. Mathematics Teacher 93, p.568-574.