

GEOMETRIC CONSTRUCTION AND ELEMENTARY ANALYSIS

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Abstract

The demonstration of smooth continuous relationships can be difficult to illustrate using some computer algebra systems. Very often, students can miss fundamental and continuous connections because of the discrete keyboard nature of such systems. A practical solution to such problems is the use of web based applets that allow users to smoothly change parameters. In particular, applets are constructed, using JavaSketchpad, to explore and illustrate some fundamental relationships of elementary analysis.

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1 Introduction

The use of computer technologies, in particular computer algebra systems and the web, have been the dominant driving force behind the re-thinking of mathematics education and presentation of mathematics courses in recent years. Such issues have been directly addressed at ATCM over a number of years now.

Changes in education in general have also occurred because of the many and varied demands of students who are no longer satisfied with the “one choice” option of traditional education. Many students require a flexible or open approach to their studies which better fit in with their needs and other commitments etc.

The changes in both presentation and content of mathematics courses at many education institutes in recent years have certainly been focused on the use of various computer technologies which in turn as a direct impact on the way students in general study. The use of computer algebra systems and the web provide an ideal framework to decentralise education in terms of time, location and teacher.

There exists many examples of mathematics course on the web which range from the rather dull posting of lecture notes² to fully interactive web based mathematics that exploit either a plug-in or Java.

Computer algebra systems are well known to be of great use and benefit if wisely used as part of a mathematics course but the use of such systems become very expensive for multi-user licenses. Student editions greatly help but this again can represent large additional expense to the presentation of courses. If, however, you are lucky enough to be using a computer algebra system which has an associated free plug-in that allows full interactivity with the mathematics engine (Mathcad does [3]), then this can greatly reduce the cost of course presentation.

Unfortunately, the vast majority of computer algebra system do not come with a free plug-in that works as a mathematics engine for computation on the web. This, in any case, would **almost** be the ideal solution for presenting interactive mathematics on the web.

²There still exists some technical problems for displaying mathematics on the web. The many GIF files created for even moderately used mathematics symbols will hopefully be superseded by something like MathML [1, 2]?

The ideal situation is to be able to construct interactive mathematics on the web which requires no plug-in and, therefore, can be viewed by any allowed user at anytime, anyplace and on any computer.

Such an approach is not only difficult but very time consuming. To enable some form of mathematics engine on the web without the use a plug-in requires the construction of Java like applets which indicate the need to construct Java programmes, which many educators will not have the time or patients to fully master.

For the non-expert Java programmer, what is therefore required is some form of “Java converter”. Such converters would ideally convert your interactive mathematics, constructed using some computer algebra or other mathematics system, to a form which is suitable for direct use on the web. That is, convert your computer algebra file to, say, Java.

In this paper, after a brief report on some experiences with various computer algebra system, a detailed account is given on the construction of Java applets using JavaSketchPad. In particular, some examples are shown that explore and illustrate some fundamental relationships of elementary analysis.

2 Computer algebra systems

Before discussing the construction of Java applets, it might be useful to discuss some support and limitations of a few popular computer algebra systems [4, 5] with regard to the presentation of mathematics on the web.

The purpose of computer algebra systems is to perform both symbolic and numerical calculations. Many of them have evolved into very sophisticated computational systems, most of which can be programmed and are actively used as indispensable research tools in many diverse fields.

In recent years, many computer algebra systems have developed easy-to-use point-and-click user interfaces which greatly help to reduce the learning curve of such systems. Together with these developments have been an increase in the numbers of optional output formats such as RTF, Word, \LaTeX , C, Fortran etc. Two output formats which are specific to web based presentation are html and MathML. Six popular computer algebra systems are listed below together with an indication of their web capabilities

in terms of html and MathML conversion.

Computer algebra system	html	MathML	Reader	Animation
Derive	×	×	×	×
Maple	✓	✓	×	✓
Mathcad	✓	✓	✓	✓
Mathematica	✓	✓	✓	✓
MATLAB	× (Java)	× (Java)	× (ASCII)	✓
Scientific WorkPlace	×	×	✓	×

Experience as shown that some of the above conversion capabilities are far from ideal and very often one needs to go into the conversion codes to manually tidy things up. In any case, such conversions presently only generate non-interactive documents (MathML can of course be used as input and output for some computer algebra systems, which is hoped to become more popular in the future [1, 2]).

A step towards web based interactivity is the use of a free plug-in or reader that enables computer algebra files to be read. Unfortunately, some of these free readers do not perform calculations and one is therefore left with an option that is not much better than html and/or MathML conversion.

One such worthy plug-in is Mathcad's **Mathcad Explorer** which is a fully functioning web browser that also acts as a mathematics engine for Mathcad files. This represents a very cost effective and straightforward way of presenting interactive mathematics on the web. Mathcad Explorer can be downloaded from [3] and can even be placed on your own academic website.

Another very useful capability of computer algebra systems is animation, which can be exported in various video or animated GIF and similar formats for direct use on the web, which are also indicated for the six popular computer algebra systems in the table above. Such animations are very useful for teaching and can illustrate specific concepts graphically, symbolically or numerically. The drawback, however is "what you see is what you get" and there is no scope to allow for full interactivity or to allow users to change parameters in the generated video or GIF files etc.

As mentioned earlier, the ideal situation would be a direct conversion from ones favourite computer algebra system to a form which is suitable for direct use on the

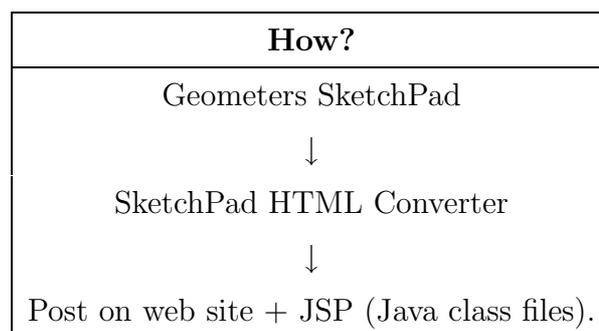
web, without a plug-in or reader, that generates interactive mathematics. A brilliant and important step in this direction is discussed in the next section.

3 Java applets and Geometers SketchPad

Geometers SketchPad [6] has a long tradition in schools, which has been discussed and illustrated in workshops at ATCM in previous years. However, there seems not to be much emphasis on using Geometers SketchPad in the teaching of higher mathematics, which is presumable due to its lack of symbolic and heavy duty numerical capabilities.

In this section, it is demonstrated that Geometers SketchPad is very useful as a teaching aid for university level mathematics, as well as a relatively straightforward tool for generating interactive Java applets for direct use on the web. The use as a teaching aid will be demonstrated with some specific examples but first we consider the details of using Geometers SketchPad as a tool for constructing interesting and interactive applets for use on the web.

Geometers SketchPad, together with the SketchPad HTML converter (β - version), creates webpages for use with **JavaSketchPad** [7]. This represent a nice and relative straightforward way of putting fully interactive Java applets on the web (no plug-in required).



The steps above represent the direct way of converting SketchPad sketches to a form which preserves most of its interactive capabilities for use on the web without the need of a plug-in (however, you need to post JSP Java class files on your website but the user of your website does not need to load these files). In particular [7]

- The **Sketchpad HTML Converter** is a program that converts your desktop Sketchpad sketch files into HTML pages suitable for publication on the web. Eventually, desktop Sketchpad will support HTML output directly with a "Save As HTML" option in the File menu. Until that time, you must use a separate program that simply translates an existing sketch into an HTML page. With DR3, HTML Converters are available for both Macintosh and Windows versions of The Geometer's Sketchpad.
- The **JavaSketchpad applet** is a directory of files that must reside in the same directory as your web page (i. e. your HTML file) so that your readers' web browsers have access to the computational engine that makes interaction with your sketches possible. (If you understand Java security rules and HTML AP-LET tags, you can store the applet elsewhere. In fact, eventually, you may not have to store the applet anywhere...your web pages can refer to the applet as located somewhere on the Internet—for instance, on my computer rather than yours. However, for the time being, the Sketchpad HTML Converter assumes that the applet's subdirectory can be found in the same directory as your HTML files.)

It perhaps should be pointed out that the html converter is a β -version and some tidying up of the raw converted code may be necessary (such as background, point and line colour etc.). Indeed, the construction grammar is fully available from [7] which means that you can code directly in the plain text of an html file. That is, you can specify constructions manually by typing them into your html editor. The applets here, however, were all created using Geometers SketchPad with some manual touching up of the construction grammar (mainly to improve presentation).

Examples

The examples illustrated at the conference include the following.

- For a function $f(x)$, the definition of its derivative (assuming the limit exists) is given by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

This limit is illustrated for various classes of functions. Users have full control of the limit h approaching zero and there are additional animated capabilities for users to experiment with.

- The graph of a function $f : X \rightarrow Y$ with domain X and codomain Y of course depends on its domain. The importance of the domain X within the definition of a function is illustrated and stressed. Users can change the domain and experiment with its consequences.
- The image set or range of a function $f : X \rightarrow Y$ with domain X and codomain Y is a subset of Y defined by $\{f(x) : x \in X\}$. Applets are shown that illustrate the image set of a class of functions.
- Inverse functions are illustrated as the reflection about the line $y = x$ (assuming the inverse function uniquely exists). Applets are shown which allow students to experiment with the inverse of a class of functions, as well as functions with no unique inverse. Students can discover here that the inverse of a function can be made to exist if its domain is appropriately restricted.
- A nice spin off from the geometric construction of inverse functions is a geometric understanding of the natural log function $f(x) = \ln x$ for $x > 0$. Students have little difficulty understanding the exponential function $f(x) = \exp x$ (the number e raised to the power x) but have great difficulty in understanding logarithmic functions. This example illustrates the function $f(x) = \ln x$ as the inverse of the exponential function $\exp x$.

The examples here, and more, can be found at [8].

4 Conclusion

It is hoped that this article has clearly demonstrated that Geometers SketchPad is not only a useful teaching tool in higher mathematics in its own right, but is also a very useful editor for creating interactive applets for direct use on your website.

The examples mentioned above illustrate geometric relationships, which is the primary purpose of Geometers SketchPad, but creative educators can construct applets to

illustrate other mathematics such as symbolic or numerical ideas. For example, rules of differentiation and integration for various classes of functions can be constructed, sequences and series can be illustrated together with the important notion of *radius of convergence*, difficult $\epsilon > 0$ type definitions of convergence and continuity can be reinforced and the many ideas of graph theory can be easily and clearly illustrated [8].

The use of Geometers SketchPad together with SketchPad html converter represents a neat and straightforward way of generating interactive applets for direct use on the web. The benefit of this approach is it allows educators to remain focused on creative teaching ideas and methodologies without being immersed, too much, in the details of construction codes, etc.

It perhaps should be finally noted that Cabri Geometry also offers similar teaching and Java capabilities as presented here, details of which can be found at [9].

References

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