Representation of knowledge of fraction in a computer environment by young children

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Traditionally teachers tended to use concrete material in providing alternative perspectives about fraction concepts and their applications. While the emphasis on the development of multiple representations for fractions has considerable support, there is also a need to ensure that children can relate parts of one representation with that of the others. Children who are able to connect different representations of fractions can be considered to have developed a deeper understanding of fractions as numbers.

In this study I examine the quality of understanding shown by a group of ten-year old children by analyzing the type of links they are able to construct between symbolic and diagrammatic representations of fractions. The diagrammatic representations provided in this study were constructed and modified within a dynamic *JavaBars* learning environment, a software that was designed to explore conceptual development of fractions. Results of the study showed that there were differences in the quality and quantity of links identified by the children. The low-achievers were unable to construct the relationship between the symbolic and diagrammatic representations of a given fraction. In contrast, the high-achieving children could go beyond interpretation of basic fraction numbers and test conjectures with the help of *Javabars*. I discuss these results in terms of schema theory. Tentative suggestions about ways to integrate *JavaBars* in teaching fractions are also provided.

Introduction

The study of fractions and related concepts is an important topic in most primary mathematics curriculum both in Australia and other countries. In the classroom, this topic is generally introduced to the students after they have developed an understanding of whole numbers. Children encounter fractions concepts not only in the formal classroom setting but also in real-life situations. A good understanding of what fractions are and how these can be used would help children make sense of a multitude of experiences in their daily lives. Fraction numbers are complex in character and provide important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic operations in later years of their school experience. Despite the critical conceptual link provided by the topic of fractions between mathematics strands such as space, money and measurement, this area continues to present difficulties for some young children in primary schools (Pitkethly & Hunting, 1996, Mack, 1990). While a number of studies have attempted to describe learning problems that are associated with fractions (Behr, Lesh, Post and Silver, 1983; Olive, 1999; Post, Cramer, Behr, Lesh and Harel, 1993) there is limited data on the nature of knowledge that drives children's understanding of and operations with fractions.

Developments in cognitive psychology have had significant effects on our understanding of processing of mathematical information by children. In particular, it is becoming clear that understanding of a mathematical concept is reflected by the representation of that concept. It is further suggested that the quality of representation constructed by a child is a function of the knowledge underpinning that representation (Anderson, 2000). This line of reasoning suggests that one useful way to examine children's understanding of fractions would be to analyse the nature of representations that they are able to construct. This strategy was adopted by Noble, Nemirovsky, Wright and Tierney (2001) who used the framework of representations in their study of children's understanding of one-half relations.

Although, children are exposed to numerous concrete material such as fraction strips and pattern blocks, computers provide a powerful and flexible environment within which to examine children's representations of fractions. Work on this issue would not only inform us about computer use in teaching but also help develop a better understanding of the link between children's understanding of fractions and their ability at representing fractions in different modes (Kaput, 1992; Norman, 1993).

Mathematics educators and teachers have invested considerable effort in exploring the instructional value of computers in helping children develop a better grasp of mathematical concepts including fractions. That computers could play a significant role in enriching the classroom experiences teachers provide has received further attention in major curricular documents such as the *Curriculum and Evaluation Standards* (National Council of Teachers of Mathematics, 2000). Indeed, it is now generally agreed that the appropriate use of computers in the instructional setting could promote the construction of deeper levels of conceptual understanding of fractions. This issue was taken up by Chinnappan (2000) in his study about the interaction between pre-service teachers' content and pedagogical knowledge of fractions while they were working within a fraction software called *JavaBars.* The results of this study showed that these teachers experienced difficulty in exploiting *JavaBars* in order to construct multiple representations for fractions which was argued to be an important aspect of effective teaching.

In the present study, I explored the above issue further by documenting Year 5 students' knowledge and understandings about part-whole relations in fraction numbers, and how they would utilise *JavaBars* in order to construct different representations for fractions. The aim was to identify potential links between teachers' and students' use of *JavaBars* in visualising the many facets of fractions, and explore implications for teaching.

Fraction number knowledge in Grades 1-7

Several attempts have been made to capture the complexity of fraction numbers and children's construction of these numbers. The most detailed analysis of fraction numbers was undertaken by Kieren (1988). His analysis showed that the fraction number knowledge consists of many interwoven strands. He identified eight levels in his description of fraction number thinking. This is a hierarchical model in which the higher levels of thinking are based on developments at lower levels. An important outcome of this model is the specification of cognitive structures that provide the basis for the maturing of understanding of fraction numbers among young children. These structures or schemas which appear at levels three and four consists of what he referred to as *subconstructs*: partitioning, unit forming, quotients, measures, ratio and operations. These subconstructs among others play a key role in young children's interpretation of fractions. The tasks used in the present study involve partitioning and unit forming.

Although Kieren's framework is useful in describing the growth of fraction knowledge it does not show the links among the subconstructs. The proposed model (Figure 1) below not only disentangles fraction subconstructs but also draws out potential links among the constructs. These links are important for the analysis of the organisational quality of the knowledge that drive the representation of fractions. In the past discussions on fractions were limited to common fractions. In the proposed model, I have included decimal fractions within the fraction schema. A further advancement of the model is that it identifies the array of prior knowledge that young children bring to their constructions of formal representations of fractions. I argue that children’s domain knowledge base for fractions is better analysed with reference to this model as it details the structure as well as the relations among the subconstructs and their components.

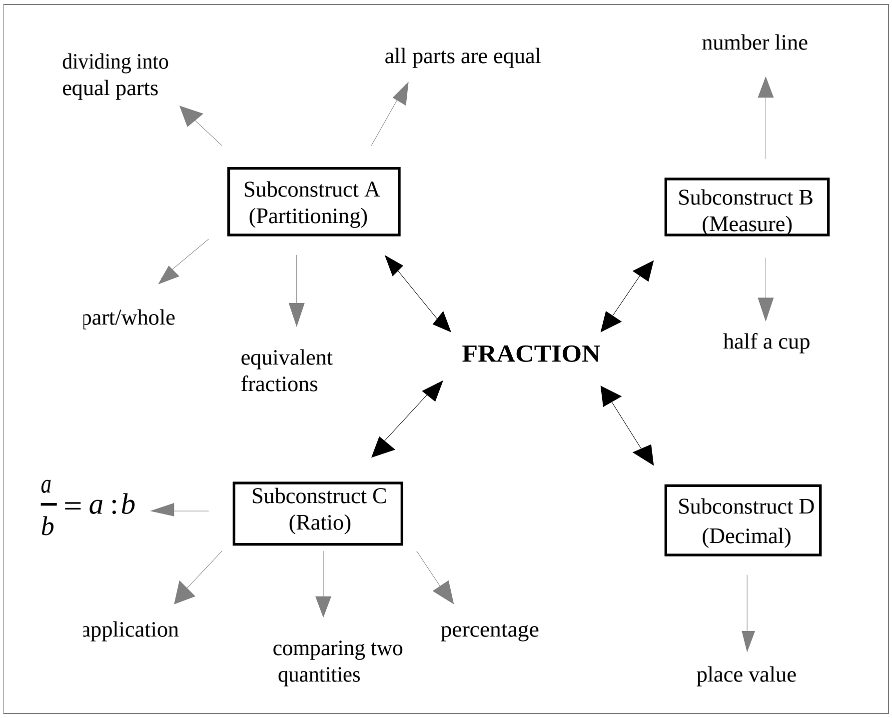

Figure 1: Fraction schema

Computers and representation of fraction concepts

The general recognition that fractional numbers are conceptually difficult and complex for young children to grasp has motivated some researchers to consider how best to use computers in order to assist children make the transition from whole numbers to fraction numbers. The assumption here is that a dynamic learning environment provided by computer software would assist children exhibit their understanding more flexibly. This issue was taken up by Hunting, Davis and Pearn (1996) who conducted a teaching experiment with 8- and 9-year old children with a software called Copycat. They reported that the environment was suitable for externalising children's prior understanding of whole numbers and fractions.

More recently, a team of researchers from the University of Georgia has been investigating children's understanding of fraction with the aid of a computer software known as *Javabars* (Olive, 2000). The software has been primarily developed to examine the type of representations of fractions constructed by children. The software provides children with menus that help them draw bars of different shapes that can be modified in a number of ways. For instance, a given bar can be divided into equal or unequal parts that in turn could either be filled with different colours or isolated from the parent bar. This facility is analogous to children's manipulation of counters in learning about parts and wholes of fractions. Children are, therefore, able to transfer their skills with concrete objects into the computer environment. Since its development a number of studies have used the software successfully in order to examine its effects on children's learning of fractions (Tirosh, 2000; Tzur, 1999). It was thus decided that this software would be useful to examine knowledge of fraction numbers and its representation by the participants in the present study.

Method

Participants

Nine children in Year 5 (10 years of age) from a regular suburban school in Australia participated in the study. The sample contained approximately equal number of boys and girls. The children have studied whole numbers and fractions within the Number Strand of the K-6 mathematics curriculum in the previous two years of primary school. At the time of the present study, the children have completed the topic on fractions in Year 5. The participants were assigned to two achievement groups on the basis of classroom tests and teacher ratings. There were four and five children in the High-achieving (HA) and Low-achieving (LA) group respectively.

Tasks and procedure

The aim of study was to assess knowledge that children have built in the area of fractions and examine how this knowledge could be activated and used during a problem-solving situation within *JavaBars.* Two fraction partition tasks were developed for the purposes of assessing children's knowledge about fractions. The tasks focused on children's understanding of part-whole relationship, and how this understanding was mapped in *JavaBars.*

For the purposes of the first task (Problem 1), two bars were presented on the computer screen (Figure 2). The first bar on the left was not partitioned, representing a unit. The second bar was a copy of the first bar except that it had two features: a) it was divided into four equal segments, b) two of the segments were coloured in grey. Students were required to write the fraction for the grey part, and talk about it.

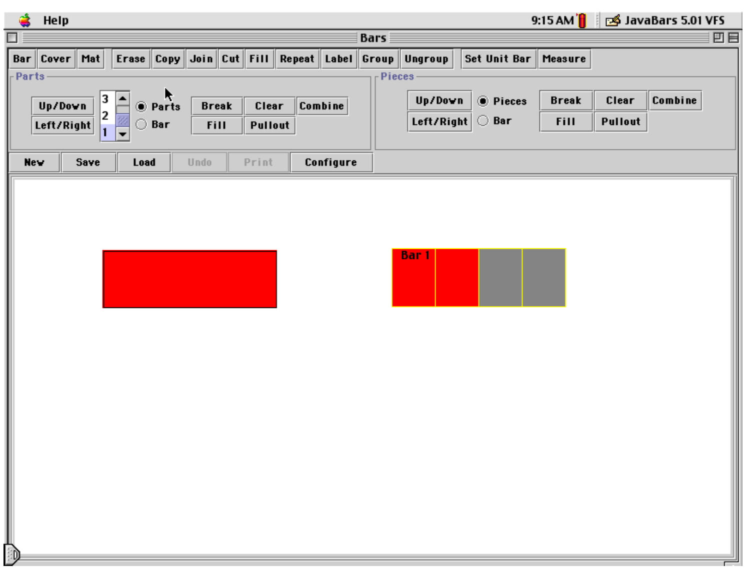

Figure 2: *Javabars* screen for Problem 1

The second task (Problem 2) was similar to Problem 1. Again two bars were provided on the screen. The first bar on the left was the unit bar (Figure 3). The second bar (on the right) was identical to the first except that it was segmented into two unequal parts. The smaller part (blue) was one-sixth the size of the unit bar. Students were asked to name two fractions that might be represented by the blue part. Further, each student was asked to test their responses by using menus on *JavaBars*. Students could activate the 'break' button on the menu and separate the smaller of the two segments. They could subsequently move this blue bar and align it along the unit bar or superimpose it on the unit bar. Alternatively, the students could carry out similar comparisons with the larger segment (red). The 'break', and 'copy' buttons on the computer screen could be used for the above moves, and students showed facility with these and related moves during the training session (see below) with an unrelated problem.

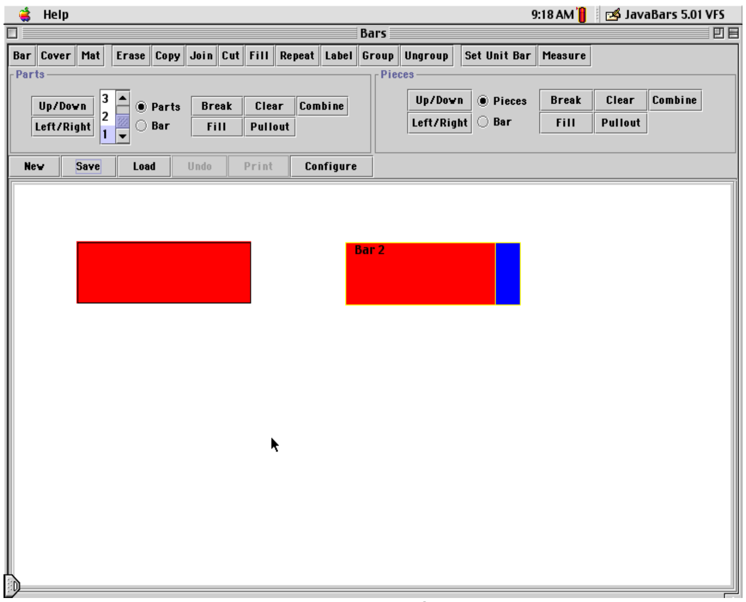

Figure 3: *Javabars* screen for Problem 2

Each child was met individually for 90 minutes. During the first half of the interview the investigator introduced *JavaBars* to the children and showed some of the basic features such as constructing a bar, colouring, partitioning (vertically/horizontally), and moving bars around the screen. Comments from the participating children suggested that they found the activity enjoyable and easy to work with. The children were given time to experiment with *JavaBars* by clicking the various buttons on the screen, and raise questions. All the children were able to perform the basic functions within the first 30 minutes. In the second part of the interview, each child was asked to attempt to complete the two target problems. Children's responses were also probed to generate more complete data about their knowledge of fractions, and how they used *JavaBars*. The interview sessions were audio-taped and transcribed for subsequent analysis. The use of the above techniques has been argued to provide rich data about interconnections that exists between knowledge units, and their structure (Royer, Cisero & Carlo, 1993).

Scoring system

The scoring system that was developed for this study aimed at capturing students' representations of the two problems. In Problem 1, the main interest was on characterizing students' basic knowledge of part-whole relations with respect to two-fourths. Four components of the solution process were analysed: expressions used to label two-fourths (Language), writing two-fourths in the symbolic form (Symbols, a/b), mapping the bar with the a/b notation for two-fourths (Part/Whole) and final answer (Solution, 2/4). A score of 0 was given for incorrect/no response, 1 for partially correct response and 2 for correct responses for all the components. Thus, the range of scores for the four process components were 0-2.

The component processes for Problem 2 were somewhat different but related as the solution involved partitioning and activating other features of *JavaBars* in order to verify students' responses. The solution outcome (Solution) and part-whole analysis (Part/Whole) were used in the same way as in Problem 1 except that students could obtain a maximum score of 4 for the Solution score as they were required to generate two solution possible answers. Students were expected to make a guess about the proportion (Estimation) for the relative size of the blue part in Bar 2 (Figure 3), and test their ideas (Testing conjectures). As in Problem 1, the scores ranged from 0 to 2 for these three dimensions of children's cognitive processing.

Results

Tables 1 and 2 provide the results of analysis of students' problem solving attempts for Problem 1 and 2 respectively. The pattern of results in Table 1 shows that all the students in HA group were able to name the fraction that was provided in the form of a bar that was divided into four equal sections two of which were shaded (2/4). These students also showed how the numerator and denominator were linked to the unit bar (Part/Whole) thus providing another correct representation for two-fourths. In contrast, the low-achievers experienced some difficulty in all the four components of the solution process. Two students in the LA group could not name the fraction number for two-fourths and write the appropriate symbol for the numerator and denominator. One of the LA students could recognize that two parts were shaded out of four. However she could not translate this information to a fraction. The other interesting feature of this problem was that compared to the HA students the LA students made no reference to the equality of the parts in identifying the fraction, an important characteristic of part-whole relationship– Sub-construct A (Figure 1).

Table 1: Component processes for solution of Problem 1

	Low Achievers		High Achievers	
	Mean	SD	Mean	SD
Language	1.20	1.10	2.00	0.00
Symbol	1.40	0.89	2.00	0.00
Part/Whole (P/W)	1.00	1.00	2.00	0.00
Solution	1.20	1.10	2.00	0.00

Table 2 shows the results of students' solution attempts for Problem 2. Compared to Problem 1, this problem required the activation of relatively advanced knowledge of fractions and the use of estimation skills. The solution processes are identified under four components. All the HA students were able to estimate the relative size of the smaller segment to that of the larger one (see Figure 3). Most of their estimations were in the range of 6 to 8. These students were then able to express this proportion as a fraction (Part/Whole). That is, once they have estimated the relative size of the smaller part, they deduced that the blue part was $1/7^{th}$ of the unit bar. The Solution component with the mean score of 1.56 (Maximum score = 4) indicates that the HAs were generally successful in producing at least possible fraction.

Table 2: Component processes for solution of Problem 2

	Low Achievers		High Achievers	
	Mean	SD	Mean	SD
Estimation	0.40	0.55	2.00	0.00
Testing conjectures	0.80	1.09	1.50	1.00
Part/Whole (P/W)	0.40	0.55	1.75	0.50
Solution	0.40	0.55	1.56	1.59

Testing Conjectures was an important phase of the solution path for Problem 2. This component refers to students checking their estimate of the relative proportions of the smaller segment with respect to the unit bar. All the students were required to demonstrate this aspect of their solution. The scores (Mean = 1.50) show that all the HA students were able to separate the small segment from the larger one, and make a visual comparison by aligning the smaller part either along the larger part or the unit bar. This action helped them verify their earlier estimation and adjust it if necessary. The corresponding mean score for the LA students (0.80) shows that these students experienced difficulty in testing estimations. This difficulty is also reflected in the mean score for Part/Whole analysis and solution outcomes.

Discussion and Implications

The nature of the problems presented in this study were such that in order to respond correctly students needed the ability to map four representations of fractions: part/whole relations, language, symbols and bars (unit and partitioned). Students who have developed a deeper understanding of fractions will have built up information that is relevant to these four representations. Additionally, students will also be able to show the links among these representations when asked to illustrate their understanding of fractions. That is, the integration of the different representation constitutes a key characteristic of the quality of knowledge that supports students’ understanding and interpretation of fractions (Figure 1). This feature of their knowledge base that emphasises connectedness or organization has been argued to facilitate better use of prior knowledge of mathematics (Schoenfeld, 1992; Prawat, 1989; Chinnappan, 1998). A more structured knowledge base for fractions is also necessary for further developments. For example, a recent study conducted by Mack (2001) about operations involving fractions led her to suggest that failure to transfer symbolic understanding of fractions to the concept of partitioning could impede students’ ability to perform multiplication and divisions operations involving fractions. Here one could see the conceptual value of establishing links between partitioning fractions and their symbols, one of the key issues in this study.

Overall, all the students in the present study had developed some understanding of fractions but one could detect a number of gaps in their knowledge. This was particularly the case with the students in the LA group. A number of these students experienced problems in transferring their understanding of fractions to the bars. Performance in Problem 1 suggest that most of these students could write down the correct symbolic representation (2/4) and say ‘two out of four’. Although these actions show that they could count two grey parts out of the four, all except one of the students attempted to show the relations between the parts and the unit bar provided on the screen (see Figure 2). This lack of comparison could be interpreted as suggesting that a few of the low-achievers had developed only a partial understanding of the relationship between the numerator and denominator of a fraction expressed in the form a/b. A further gap in the fraction schema for the low-achievers in the present was the concept of fraction equivalence. ‘Two out of four’ is equivalent to one half of the given bar. Although this fact was visually presented on the screen none of the students from this group could ‘see’ it as a half of the unit bar. It is also of note that none of the LA students could express the fraction in Problem 1 as ‘two-fourths’.

In contrast, all the students in the high-achieving group could exploit the dynamic bar features provided by *Javabars* in a number of conceptually powerful ways. Firstly, they could align the partitioned bar with the unit bar in order to make judgments about the relative size of the partitions and produce the correct numerical forms. They showed a degree of comfort in breaking and assembling parts to make a whole (unitizing), and interpreting the result in terms of fractions. This was particularly in evidence during their solution of Problem 2. These students could shift from one representation for ‘two out of four’ to another without too much effort. This apparently seamless transfer among representations is indicative of the robustness of their fraction schemas. In their analysis of mathematical understanding, English and Halford (1995) argued that the mapping of elements of one representation with elements in a different representation induces cognitive load, and that one way to reduce this load would be to improve the strength of the links among knowledge components in the schema. Thus it would seem that the robustness of the schema of the high-achievers in the present study helped them decrease the cognitive load associated with the mapping process. The reduction in cognitive load also would account for the ease with which these students could move across representations.

Difficulties that some students in the low-achieving group had with Problem 2 (which require that they visually estimate the relative size of the segments and write the appropriate fraction) may be due to their inability to see the link between the two segments and the unit bar. This is a two-step process in which they have to determine the number of small segments that would make the larger one and then map this information into their understanding of part-whole relationship. Contrary to expectation these students also failed to exploit the ‘break’ buttons in order to test their estimations. The HA students, on the other hand, were able to ‘tinker’ with the bars on the computer screen, and test their conjectures about the fraction. Thus, these students were able to stretch their mathematical understanding with the aid of the given software. The dynamic learning environment provided by *Javabars* can be argued to facilitate experimentation by students in the present study. The advantage conferred by the use of software was highlighted by Wiest (2001) in his analysis of the role of computers in teaching and learning K-12 mathematics.

The use of *Javabars* by teachers’ (Chinnappan, 2000) showed that they could not fully exploit the environment in analyzing fractions. Results of the present study indicate that students, especially the low-achievers failed to make important links between symbolic, linguistic and diagrammatic representations of elementary fraction numbers. Taken together, these results suggest that mathematics teacher training programs need to focus on helping future teachers a) develop their understanding of students’ fraction knowledge and b) how to utilize computer environments to design effective lessons.

Implications

The use of computers is increasingly being accepted as a viable alternative to the traditional paper and blackboard approaches to teaching mathematics. This view is based on the assumption that students will be working from a sufficiently developed knowledge about the subject-matter and the software. Results from this study suggest that students who have poorly developed understanding of fractions can be expected to experience difficulty in exploiting a learning environment provided by *JavaBars.* While it is too early to generalise on the basis of this study, the results do seem to suggest that teachers need to become familiar with the state of level of children’s knowledge of fractions before a software such as *Javabars* can be introduced either as a learning or teaching tool. Such an approach should aim to generate learning activities in which students could explore the interrelations between their own mathematical knowledge and how that knowledge could be transformed within the computer environment.

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