An Overview of Mathematica

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Mathematica is an integrated problem-solving environment that is powerful, general, consistent, and easy to use. It includes exact and arbitrary precision numerical computation, graphics, symbolic or algebraic manipulation, a high-level problem-oriented programming language, and interprocess communication. This hands-on tutorial is designed to get you started using Mathematica by highlighting its most important features.

■ 1. Getting Started

□ 1.1 Books on Mathematica

An up to date list of books about *Mathematica* is available at http://www.wolfram.com/bookstore. The following resources are particularly useful:

- The *Mathematica* Book: on-line
- *Mathematica* Standard Add-on Packages: on-line
- The *Mathematica* Journal: http://www.mathematica-journal.com
- Eric Weisstein's *World of Mathematics*: http://mathworld.wolfram.com

□ 1.2 Help Browser

To learn more about *Mathematica* The Help Browser is the primary reference. Choosing **Help...** from the **Help** menu at any time gives you access to *Mathematica*'s on-line documentation. An overview of the capabilities of *Mathematica* is presented in the Tour of *Mathematica* section, and it is also suggested that you browse through the Practical Introduction section.

□ 1.3 Exercises

All exercises in this hands-on tutorial look like

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1. What is your name?

There are two ways in which you will asked to supply answers to questions. Sometimes I will leave an answer field for you to write your solutions directly into the Notebook, *e.g.*,

Answer: Paul Abbott.

You just need to position your cursor after the : and start typing. Much of the time though you will just enter a *Mathematica* command in the appropriate place and the output will be automatically recorded in the Notebook.

□ 1.4 On-line Help

To find out what a particular command does, you can use the on-line help system. Choosing **Help...** from the **Help** menu at any time accesses *Mathematica*'s on-line documentation.

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2. Use **Help...** to find out what **FactorInteger** does.

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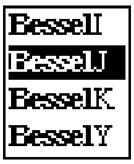
3. Alternatively, enter **? FactorInteger**. Also try **? Factor** and **? *Factor***.

□ 1.5 Command Completion

The long names can be difficult to enter, especially if you are a slow typist. Command completion increases the speed, correctness, and convenience of Notebooks.

Enter **Bess**, then select **Complete Selection** under the **Input** menu. The completions window,



will pop up and you can select which command you desire using the mouse, here **BesselJ**. This is advantageous for long commands.

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4. In a new cell, enter your own name with no spaces, e.g. **PaulAbbott** and evaluate the input. In a blank cell, enter the first two letters of your name and use command completion.

See, *Mathematica* knows a bit about you already.

□ 1.6 Templates

Templates work much like command completion. The **Make Template** command is also found under the **Input** menu.

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5. Use **Make Template** on **Plot3D** and produce a 3D plot of $\sin(x\,y)$ over $[0, \pi] \times [0, \pi]$.

⚠ Remember to create an *insertion point* in your Notebook *before* you enter an expression or use a palette.

Alternatively, you can use the Basic Calculations palette to find out more about how to use a function.

■ 2. Basic Calculations

Read the *Mathematica* as a Calculator section of the Tour of *Mathematica* and then attempt the following exercises. If it is not already open, you may find the Basic Input palette helpful. For more information on palettes see Using Palettes.

□ 2.1 Getting Started

You can use *Mathematica* as an enhanced scientific calculator. Let's start with a simple example.

45 + 78

123

The first line here is what you type in. The second line is the result.

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6. Use the Basic Input palette to type in the expression 3^{100} *below* this cell. Hit SHIFT-RET together to evaluate this expression. Note that you should get the *exact* answer.

⚠ You can position the cursor *anywhere* on the line for expression evaluation to work. Alternatively you can select the cell-bracket (or a range of cell brackets) that contains an expression (s) and select **Evaluate Cells** under the **Kernel/Evaluation** menu.

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7. Select your input line above (either by selecting its cell bracket or by scrolling over the expression with the mouse clicked) and make a copy using **Copy** under the **Edit** menu. **Paste** your expression below this box. Change the 100 to 1000 and evaluate the result.

You should have just evaluated 3^{1000}.

💡 An easy way to select an expression is to click on it as many times as is necessary to select the entire expression (this is a type of *scoping*).

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8. To get the result in the form you might get on a calculator, try $N[\%]$.

⚠ ***%*** indicates the previous expression. Similarly ***%%*** indicates the second-last expression and ***% n*** refers to **Out[*n*]**. Alternatively, expressions can be named using assignment (denoted using =).

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9. Enter pi = $N[\pi, 200]$ below this cell and evaluate it.

You should get the value of π to two hundred decimal places.

10. Enter $e^{\frac{\sqrt{163}\ \text{pi}}{3}}$ below this cell and evaluate it. What is special about this number?

Answer:

11. π has a run of six consecutive 9's in the first 1000 digits. Can you see where they are located? (One way to find the location is to convert the number to a string [using **ToString**], drop the leading **3.** [using **StringDrop**] and locating the position of the string **"999999"** [using **StringPosition**]).

Answer:

□ 2.2 More Functions

Mathematica knows about a big collection of mathematical functions — nearly all those you will find in any book of mathematical tables.

12. Compute $\sin(13\,\pi)$, $\log(2.1)$, and $e^{i\,\pi}$ by copying these expressions below this cell and entering them.

⚠ Note that e denotes the exponential e and i denotes $\sqrt{-1}$. You can these objects using the Basic Input palette.

13. Lookup cos in the on-line help and determine closed-form expressions for $\cos(\frac{\pi}{5})$.

Answer:

Mathematica can do many kinds of exact computations with integers.

14. Evaluate FactorInteger[70612139395722186]. What is the meaning of this output?

Answer: **FactorInteger** produces a set of pairs of numbers which are …

15. Recombine the numbers in the above output to show how they form 70612139395722186.

□ 2.3 Simple Expressions

Mathematica can deal with mathematical formulas in algebraic form.

16. Enter the expression $(x^2 + 2\,x + 1)^{20}$. Expand this expression using **Expand[%]**. Factor the expanded result.

17. Re-do the steps in the above computation using the Basic Calculations palette.

18. Evaluate **TrigReduce[sin(*x*) cos(*y*) – cos(*x*) sin(*y*)]** and apply **TrigExpand** to the result. What can you say about the relationship between these functions?

Answer:

□ 2.4 Basic Plots

The above sections have guided you through each calculation. Now you are expected to use palettes such as the Basic Calculations palette or on-line help to work out how to achieve each of the following tasks.

19. Produce a plot of $\sin(x^2)$ over $x \in [0, 5]$.

20. Produce a phase-space plot of $\cos(x)$ versus $\sin(3x)$ for over $x \in [0, 2\pi]$. *Hint*: lookup **ParametricPlot**. Do you recognise this figure?

Answer:

21. Produce a contour plot and density plot of $x e^{-x^2-y^2}$ for $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

□ 2.5 Calculus

You can do calculus. Try a simple integral:

22. Compute $\int \frac{x}{x^3-1}\, dx$. Check by differentiating the resulting expression with respect to x and then using **Simplify**.

You can also get exact solutions to many definite integrals.

23. Compute $\int_0^\infty \frac{\sin^2(x)\cos^3(x)}{x}\, dx$.

Many integrals do not have a simple closed form. If you try such a definite integral it will be returned unevaluated. You can still use *N* to get a numerical answer.

24. Try $\int_0^1 \sin(\sin(x))\, dx$. Compute the numerical value of this expression using *N*[%].

⚠ Note that *Mathematica* can find a closed form for $\int_0^\pi \sin(\sin(x))\, dx$. Try it.

□ 2.6 Solving Equations

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25. Use **Solve** to obtain the roots of the quadratic equation $a\,x^2 + b\,x + c == 0$, and call the set of solutions ***s***. Check the solutions by back-substitution using the syntax $\boldsymbol{a\,x^2 + b\,x + c\ /.\ s}$. You will need to **Simplify** the result.

⚠ Note that **==** denotes *equality*, not assignment (which is **=**).

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26. Solve the cubic equation $x^3 - x + 1 == 0$. Evaluate this expression numerically using ***N***. Does your answer satisfy the requirement that roots of equations with real coefficients must come in *conjugate pairs* (a conjugate pair is a set of two numbers of the form $a \pm b\,i$)?

Answer:

□ 2.7 Matrices

■ Entering matrices

Matrices can be entered using List brackets ({}). Alternatively, they can be entered using the Basic Input palette or from the **Create Table/Matrix/Palette...** entry under the **Input** menu.

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27. In a new cell, enter the 2 × 2 matrix, **mat** = $\begin{pmatrix} \mathbf{3.5} & \mathbf{7.2} \\ \mathbf{-2.4} & \mathbf{6.4} \end{pmatrix}$. Compute the inverse and eigenvalues of **mat** — you should be able to guess the names of the appropriate *Mathematica* commands.

■ Linear algebra

Mathematica also does linear algebra on symbolic matrices.

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28. Enter **mat** = $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. This overwrites your previous value for the matrix **mat**. Compute the inverse of **mat** and assign (using **=**) the result to **minv**. Compute **minv.mat** and simplify the result.

⚠ The **.** (*i.e.*, **Dot**) in ***a.b*** indicates that the usual dot product and is used both for vectors *and* matrices. This makes sense because each element in the resulting matrix is formed by dot product multiplication of rows of ***a*** with columns of ***b***. Without the **Dot** one gets the direct (*i.e.*, element-by-element) product. The direct product is useful for general matrix operations such as *masking* and *convolution*.

29. Generate a random real 3×3 matrix and call it **rand** (*Hint*: use **Random** and **Table**). Replace the {2, 2} entry of **rand** with the symbolic parameter x (*Hint*: look-up **Part**). Compute the (symbolic) inverse of **rand** and determine the numerical value of the inverse when $x \to 1$.

□ 2.8 Curve Fitting

30. Use the command **Table[*i*!, {*i*, 1, 10}]** to compute the first 10 values of the factorial function and assign the answer to the variable called **is**.

We could plot this table but the dynamic range is so large that it would be difficult to produce a meaningful plot. Let us take the logarithm of these values.

31. Take the log of values using **log(is)** and numerically evaluate the result. Save this result as a named variable called **data**.

Now that we have a table of values we can plot it using the **ListPlot** command.

32. Use **ListPlot** to plot **data** and save your graphic as a named variable called, e.g., p_1.

33. Compute the best (least-squares) third-order polynomial fit to this data using **Fit**. Obtain a plot of this fit and call it p_2. You need to choose and appropriate range for the **Plot** command (see **PlotRange**). Show p_1 and p_2 superimposed using **Show**.

⚠ You can, in fact, do fits using any linear combination of functions but **Fit** presently does not allow exponent optimization — you need to use the `Statistics`NonlinearFit`` package for this.