A Review of Research on Student Achievement and Attitude In Computer and Calculator Enhanced Mathematics Courses

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Abstract

This paper will briefly summarize the results from four previous reviews of research regarding the effect of calculator use on student achievement in mathematics from 1970 to 1995. Then a review of the research conducted on graphing technology during the 1990s will be provided. Suydam reviewed research on calculator use conducted in the 1970s. Hembree and Dessart analyzed studies comparing calculator and noncalculator groups that overlapped Suydam's review and also included research studies conducted during the early 1980s. The third review, by Smith, examined research studies reported from 1984 to 1995. Smith's critique included eight studies that implemented graphing calculators. The studies analyzed by Suydam, Hembree, and Smith included grades K - 12 in the United States. The fourth review, an analysis of US college level mathematics studies conducted from 1986 to 1995 regarding the effects of computer-enhanced instruction (including computer software and graphing calculators) on student achievement, was performed by King. Results of each of the above mentioned reviews of research that compared groups of students using calculators against groups that did not use calculators found the results from achievement measures favored the groups who used calculators.

Since the late 1980s graphing calculators and other graphing utilities have become more readily available in classrooms. During the 1990s, more than 60 research studies were published on the effects of graphing calculator use in mathematics courses. This paper provides an analysis of the comparison studies reported, between 1990 and 2000, investigating the use of graphing technology and computer algebra systems (CAS) in teaching mathematical topics found in algebra, trigonometry and calculus in secondary level schools and colleges. The 52 comparison studies examined in this paper were located through on-line computer searches from national and international databases. The majority of the studies located were doctoral dissertations or master degree theses. The other studies collected for review were articles published in refereed journals. Student achievement both procedural and conceptual along with attitude toward mathematics and technology are delineated in the report.

Results from the review of research are encouraging. Although the evidence supporting the use of graphing technology and computer algebra systems is not unanimous, it does strongly suggest that when used appropriately these technologies do assist in increasing conceptual understanding without adversely affecting procedural knowledge. More than two-thirds of the studies compiled for this paper reported better overall achievement for the treatment group (graphing technology and/or CAS) and 75% of the results on measures testing for conceptual understanding favored the treatment group while nearly two-thirds of the results on procedural knowledge indicated no significant difference between the control group and the treatment group. There was also evidence presented that use of graphing technology and/or CAS can help improve students' attitude and confidence in mathematics.

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Introduction

A major topic of interest concerning the future of mathematics education in recent years has been the use of calculators and computers as tools in the teaching and learning of mathematics. Starting in the late 1970s, studies were compiled in order to provide more information on the overall effect of calculator use in mathematics classrooms. In the 1970s and 1980s most of the calculator studies conducted in the United States were at the elementary grade level (K - 6) with a few studies on calculator use at the secondary level (7 - 12) (Hembree & Dessart, 1986; Suydam, 1976, 1980). Suydam's review of 75 studies from the late 1960s through the 1970s relating the effects of calculator use on mathematics education suggested the use of calculators did not adversely affect student achievement, and in many instances resulted in higher achievement than with noncalculator usage. Hembree and Dessart combined the information from Suydam and other studies comparing calculator-based instruction to traditional instruction and found the overall achievement for most grade levels were significantly and positively affected by the use of calculators for computation and problem solving even though many of the studies did not allow calculators on the exams.

Smith (1996) found more than 30 studies conducted from 1984 to 1995 in grades K - 12 that compared achievement of students who used calculators to students who did not use calculators in their mathematics course. Results on achievement in grade levels 3, 8, 9, and 10 were significantly higher for students who used calculators for problem solving, computation, and conceptual understanding compared to students who did not use calculators. No significant difference was demonstrated in the overall achievement of students in grades four, five, six and 11.

Beginning in the late 1980s graphing utilities such as graphing calculators became more widely available to mathematics teachers and students. Studies investigating the effect of the graphing technology soon began to appear. Smith (1996) located eight secondary school comparison studies involving the graphing calculator. One study of grade 12 students reported better overall achievement for the non-graphing calculator group. Analysis of the other studies showed no significant difference in achievement between students who used a graphing calculator to graph mathematical functions and those who did not use graphing calculators.

King (1997) compiled college level mathematics studies that investigated the effect of computer-enhanced instruction on student achievement. Technology use included teacher demonstration using a single computer, student use of a graphing or programmable calculator, or students (singly or in pairs) using microcomputers in a laboratory setting. Thirty studies were collected from dissertations and journal articles published from 1986 to 1995. A statistically significant positive effect on overall achievement was found when the computer or graphing calculators were used while no significant difference was found between technology and control groups on procedural achievement. However, a significant favorable effect on procedural achievement was found when the experimental group was allowed to use technology during testing.

Since Smith's (1996) review and King's (1997) analyses were completed, more than 60 studies investigating the impact of graphing utilities on mathematics instruction have been conducted. Hence, the purpose of this paper is to review and summarize the findings of studies that examined the effect of graphing technology (including computer algebra systems CAS) in mathematics instruction for courses on algebra through calculus during the past decade (1990 to 2000). Topics discussed in this paper include student overall achievement, conceptual understanding, procedural knowledge, and attitude toward mathematics and toward the technology. Criteria for inclusion in the review included: (a) comparison of experimental and control groups on achievement measures, (b) more than 10 students participated in each of the treatment and comparison groups, (c) at least part of the treatment must include computer or graphing calculator use, (d) the mathematical content must be topics found in courses of algebra through calculus, and (e) the study must have gone through a refereeing process (e.g., a refereed article, a defense of doctoral dissertation or master degree thesis with subsequent committee approval, etc.)

Eight studies for this review came from Smith (1996), sixteen appropriate studies came from King (1997), and 28 more were gathered from on-line computer searches of Dissertation Abstracts International, Education Abstracts, British Education Abstract, ERIC, and Humanities and Social Science Abstracts. Fifty-two studies were found to meet all the criteria for this review; 5 at the beginning algebra level, 4 high school Algebra II, 9 high school precalculus, 3 high school calculus, 4 college level elementary or intermediate algebra, 14 college algebra, 5 college precalculus (including trigonometry), and 8 college calculus studies. The studies included 40 dissertations, 3 master theses, 7 journal articles, and 2 proceedings articles.

As a matter of note, unlike many countries that have an integrated mathematics curriculum where topics in geometry, algebra, and statistics may all be taught in one course, the US curriculum typically separates mathematics courses by topics. For example, a secondary level student may take one year to study beginning algebra, one year to study geometry, another year to study algebra (called Algebra II which includes a 45-50% overlap of what was covered in beginning algebra). The material for the precalculus course again overlaps about 45% of the Algebra II material. College level intermediate algebra is very similar to Algebra II taught in secondary schools, but the college course takes only one-third the time. College algebra, college precalculus and high school precalculus are all quite similar in their content except the college courses complete the material in one-third the time. Since US students are not typically required to take math each year in high school, a number of them take remedial or developmental math courses in college such as Beginning Algebra, Intermediate Algebra or College Algebra.

For purposes of this review, treatment and /or experimental group is defined to be the group in which at least part of the treatment included the use of graphing technologies or CAS. Control and/or comparison group is defined to be the group that was typically taught in a traditional manner and did not use the graphing technology.

Results for Overall Achievement

One question of major interest concerning the use of technology in mathematics courses is how the overall achievement of students who use graphing technology or CAS as an aid to learning compared with students not using the technology. To address this issue, 46 of the studies located for this review contained information on overall achievement of the students in the treatment groups and comparison groups. Twenty-nine of the studies found statistically significant overall achievement favoring the treatment group while only one study found statistically significant overall achievement in favor of the comparison group. Thirteen of the studies found no significant differences in overall achievement between the technology enhanced courses and the control courses. See Table 1.

Table 1. Overall Achievement Results

Course	Num. of Studies	Treatment Higher	No Sign. Difference	Control Higher	Not Reported	Interaction
MS/HS Begin. Alg.	5	4	1			
HS Algebra II	4	3	1			
HS Precalculus	9	6	3			
HS Calculus	3		1		2	
Coll. Elem./Int. Alg.	4	2	2			
College Algebra	14	6	2	1	2	3
College Precalculus	5	4	1			
College Calculus	8	4	2		2	
TOTAL	**52**	**29**	**13**	**1**	**6**	**3**

Six of the studies (Austin, 1996; Fox, 1998; Monticelli, 1996; Ottinger, 1993; Pankow, 1994; Quesada, 1994) specifically mentioned comparing teaching mathematics courses using graphing calculators with courses using only scientific calculators. Three of the six found no significant difference in overall achievement. Two of the studies found significant differences favoring the treatment group. The other study found an interaction between instructor and calculator type; therefore the researcher gave separate results for the two instructors (Austin, 1996). Results showed one instructor had no significant difference between scientific calculator and graphing calculator groups on overall achievement while results for the other teacher were statistically significant favoring the graphing calculator group (neither were included in Table 1). Interactions were found in two other studies where additional treatments (besides graphing calculator versus non-graphing calculator) were investigated (Adams, 1993, Coston, 1994). Because of the interactions, the researchers were unable to obtain an overall effect concerning achievement when using graphing technology. Six other studies did not report overall achievement, but did provide achievement measures for conceptual understanding and/or procedural knowledge.

Conceptual Understanding and Procedural Knowledge Results

One of the prevalent claims for the use of graphing technology in mathematics courses is the improvement of conceptual understanding and visualization of mathematical concepts. Thirty-two studies investigated conceptual understanding and/or spatial visualization of mathematical concepts. Eighty-eight different results were provided concerning conceptual understanding (including problem solving and visual thinking). There were 66 statistically significant results favoring the experimental / treatment group while one study reported two results on conceptual understanding that favored the control group. Twenty results indicated no significant difference between the experimental group and the comparison group on conceptual understanding.

Another area of paramount interest when technology is used in mathematics courses is the effect it may have on students' ability to acquire the paper and pencil skills often referred to as procedural knowledge. Twenty-eight studies examined procedural knowledge or skills acquisition.

There were 51 different results given. Nine results (from 7 studies) favored the treatment group, nine results (5 studies) favored the control group and 33 results (18 studies) found no significant difference in symbolic manipulation of algebra or calculus procedures. See Table 2.

Table 2. Results of Conceptual and Procedural Assessment Measures

	Procedural Knowledge			**Conceptual Knowledge**		
Course	Treatment Greater	No Sign. Diff.	Control Greater	Treatment Greater	No Sign. Diff.	Control Greater
MS/HS Beg. Algebra	1	1,1		1,2,3,3 *		
HS Algebra II				1,1		
HS Precalculus	2	3	2,2	1,1,1,2,2,2,2	1	
HS Calculus		1,2,4	1,2	1	1,2	
Coll. Elem/Inter Alg.	1,1	1		2,3		
College Algebra	1,1	1,1,2,9	2	1,1,2,2,2,3,	1,1,13	2
College Precalculus		1,1,1		1,1,1,2		
College Calculus	1	1,1,1,1		1,1,1,5	1	
TOTAL RESULTS	**9**	**33**	**9**	**66**	**20**	**2**

*****Each number in Table 2 represents a study; its value represents the number of results reported within the study. E.g., 1, 2, 3, 3 means there were four studies in that category with respectively, 1, 2, 3, and 3 results reported.

If the descriptive statistics supplied by the 52 comparison studies are considered at face value, the benefits of the use of technology on student achievement are evident. Sixty-seven percent of the 43 studies, reporting overall achievement results without interactions, found the treatment groups performed better than the comparison groups who did not have computer or graphing calculator enhanced instruction (using the vote-count method the effect size is 0.13). Only 2% of the studies (one study) found the control classes had better overall achievement results than the treatment classes. If secondary and college level are considered separately, the results are quite similar. Sixty-eight percent of the overall achievement results for secondary level mathematics favored the experimental group (effect size 0.14) and 32% of the results found no significant difference in the overall achievement between the two groups. The college level studies reporting comparisons on student overall achievement found 67% of the results favored the experimental group (effect size 0.12), 29% of the results found no significant difference between the groups and about 4% of the results favored the control group.

When analysis was refined to distinguish between procedural knowledge and conceptual understanding, results again strongly support the computer and calculator enhanced instruction. Seventy-five percent of the conceptual understanding results favored the experimental group (effect size 0.19) while only 2% of the results favored the control group and 23% of the results indicated no significant difference between the two groups on conceptual understanding. Results concerning procedural knowledge also reinforce the claim that algebraic skills and symbolic manipulation are not adversely affected when graphing technology is utilized in the mathematics classroom. About 65% of the results indicated no significant difference between the two groups. Approximately 18% of the results favored the control group (5 studies) and 18% favored the treatment group (7 studies).

Student Attitude Results

A fourth area of interest regarding the impact of implementing graphing technology in mathematics instruction is the possible effect it may have on student attitude toward mathematics and toward the use of technology. Twenty-one studies examined the effect the experimental courses had on student attitude toward mathematics. Forty-three percent of the studies investigating student attitude found that the experimental group had an improved attitude toward mathematics (or better attitude than the control group) at the end of the treatment period. Fifty-seven percent of the studies found no change in the attitude of the experimental group (or no difference between the experimental group's and the control group's attitude toward mathematics) at the conclusion of the study. See Table 3.

Table 3. Attitude Toward Mathematics and Technology at the End of the Study

	Attitude Toward Mathematics			**Attitude Toward Technology**		
Course	Treatment Greater	No Sign. Diff.	Control Greater	Treatment Greater	No Sign. Diff.	Treatment Lower
MS/HS Beg. Algebra		2				
HS Algebra II	2**			1		
HS Precalculus						
HS Calculus						
Coll. Elem/Inter Alg.	1	2		1	1	
College Algebra	4	5		3	2	1
College Precalculus	1	1		2		
College Calculus	1	2		2		
TOTAL	**9**	**12**	**0**	**9**	**3**	**1**

**The numbers in the table represent the number of studies that fit in the specific category.

Thirteen studies surveyed students concerning the benefit of the technology in learning the concepts of the course. Sixty-nine percent of the studies that examined attitude toward the technology found that the experimental group had a positive attitude about using the technology (or a more positive attitude than students who did not use the graphing technology) and felt it had helped them learn mathematics. Twenty-three percent of the studies found no change in the experimental group's attitude toward the technology at the conclusion of the treatment (or no difference between treatment group and comparison group). Eight percent of the studies (one actual study that used computers in a lab setting requiring extra work of the treatment group students) showed a drop in attitude toward the computer as an instructional tool. See Table 3.

Although the evidence that use of graphing calculators or computer software improves the attitude students have toward mathematics is not overwhelming, none of the studies found a decline in attitude toward mathematics for those students who were in the experimental group using the technology. There is substantial evidence that students using the graphing technology believed that it helped them to learn the concepts in the mathematics course. It was found that using the technology also allowed the students to work more realistic and challenging problems. The one study that reported a negative response to the technology was a computer lab setting where the computer materials were viewed as add-ons instead of an integral part of the course.

Discussion of Research Themes

Before discussing the themes found in the studies, it should be noted that some of the studies suffered from design, procedural, or assessment flaws that may lessen the validity of their conclusions. For instance, possible teacher effect was often not considered (including instructional training, teacher knowledge, or teacher attitude toward a new curriculum or toward technology) and validity and reliability of the instruments were infrequently established. Also, there were problems with some researchers not reporting potentially useful information, such as failure to mention the specific nature of the instructional treatment including the amount of time and context of computer or calculator usage. Also, the type of instruction and set of classroom activities were sometimes not specified, making if more difficult to sort out effective teaching approaches which utilized the technology as a tool.

Some of the studies were more carefully designed and thus provided more detailed and reliable information on student learning in specific areas. However, the statistical results from these studies were not substantially different from the other less carefully designed studies. Because the results were similar in nature, the whole group of studies was examined in order to better identify recurring and consistent themes associated with the use of graphing calculators and graphing utilities. A few of the more frequently occurring themes will be addressed.

Functions and Graphing

More than half of the studies in this review examined the topic of functions and their graphs. A variety of approaches were used in investigating students' understanding of functions. Six studies used or developed an assessment instrument to measure graphical understanding and the relationship of functions and their graphs (Browning, 1990; Devantier, 1993; Hollar, 1999; O'Callaghan, 1998; Pankow, 1994; Taylor, 1991). Five of the six studies found the experimental group performed better on the assessment instrument and were functioning at significantly higher levels of graphical understanding. The other study by Pankow was the only one of the six that included efforts to establish validity and reliability for the assessment instruments. She also found that the treatment group scored higher than the control group on the first and second unit tests and on the posttest, however, the results for the posttest were not statistically significant.

Several studies examined students' ability to relate functions and graphs and found that the experimental groups were better able to understand the relationship between a function and its graphical representation (Alexander, 1993; Boer-Van Oosterum, 1990; Browning, 1990; Caldwell, 1994; Chandler, 1992; Estes, 1990; Hollar, 1999; Mayes, 1995; Norris, 1994; O'Callaghan, 1998; Rich, 1991; Ruthven, 1990; Taylor, 1991; Tolias, 1993). Students with access to graphing technology were often better able to consider different aspects of graphs and discuss global features of the graphs including domain, increasing and decreasing behavior, and asymptotic and end behaviors (Melin-Conjeros, 1992; Mustafa, 1997; Norris, 1994; Rich, 1991).

Other studies found students using technology could better model problem situations (Alexander, 1993; Boers-Van Oosterum, 1990; Hollar, 1999; Mayes, 1995; O'Callaghan, 1998; Paschal, 1994; Trout, 1993) and were more aware that solving problems can be done by graphical methods as well as by algebraic manipulation (Mayes, 1995; Rich, 1991; Stick, 1997; Tolias, 1993). Both studies that investigated the concept of variable found richer understandings for the concept of variables and the use of variables by students in the experimental groups (Boers-Van Oosterum, 1990; Ottinger, 1993). Also, not only did the experimental groups often demonstrate a better understanding of variables, functions, and equations, they were also better able to apply these concepts (Boers-Van Oosterum, 1990; Ottinger, 1993; Tolias, 1993**.** Students using graphing technology typically demonstrated higher ability than the comparison students on items where graphs could be utilized in their solution (Austin, 1996; Dunham, 1993; Pankow, 1994; Ruthven, 1990). Several researchers noted that the use of technology provided students access to a greater variety of approaches for solving and checking their work (Austin, 1996; Connors, 1995; Dyer, 1994; Estes, 1990; Fox, 1998; Mayes, 1995; Ruthven, 1990; Stick, 1997).

One study found no significant difference between the graphing technology students' and the other students' ability to link functions and their graphs (Giamati, 1991). There were also indications in Giamati's study that students not using graphing technology had better understanding of specific transformations including shrinks, stretches, and vertical and horizontal translations. However, Pankow (1994) and Chandler (1992) found just the opposite to be true that the experimental groups were better able to translate graphs of functions and find their equations especially when given only the graph of the original function and its equation. Hence, benefits of one of the more frequently mentioned uses for graphing technology (transformations of functions and their graphs) remain inconclusive.

Visualization

Another area of focus for eleven studies was the effect of the graphing technology on visual thinking and spatial skills. Drottar (1998) stated that the greatest benefit related to graphing calculator use was to enhance students' visualization of algebra concepts. Chandler (1992) noted there was a positive increase in students' understanding and achievement when they were able to visualize the concepts and problems to be solved. Stick (1997) concluded that teachers should encourage visual displays before introducing analytical method in dealing with algebra and calculus concepts (inequalities, asymptotes, derivatives, concavity, integrations, transcendental functions, and series). Others found the experimental group showed significant gains in spatial visualization skills (Vasques, 1991; Shoaf-Grubbs, 1992). Shoaf-Grubbs further stated the control group did not experience the "positive momentum" in level of understanding and spatial skills exhibited by the experimental group throughout the research period. Castillo (1997) found use of the graphing calculator by the experimental group enhanced their visualization of three-dimensional points and surfaces. Mayes (1995) found the experimental group was better able to solve a problem from a visual approach. Alexander (1993) found the experimental group showed a better understanding of the algebra concepts and of modeling real world problem solving applications through the use of concrete visualization. Merckling (1999) noted that the tactual/visual learning preference students were especially positively affected on achievement measures when learning algebra concepts using the calculator. Also, it is interesting to note that the tactual/visual preference students demonstrated a significantly better working knowledge of the graphing calculator than those who did not have that preference of learning style. Norris (1994) gathered qualitative data that revealed strong support for the graphing calculator as a visual aid for the teaching and learning of precalculus. Ruthven (1990) found that the graphics calculator enhanced students' development of spatial skills and found a notable correlation between graphing calculator use and spatial visualization skills in senior secondary students, particularly among females. Results of these studies suggest that graphing calculators can assist in the development of spatial skills and in visualizing math concepts.

Negative Results

There were five studies that found topics where the experimental group did not perform as well as the control group (Giamati, 1991; Hinerman, 1997; O'Neill, 1995; Rich, 1991; Upshaw, 1993). As already mentioned, Giamati found the treatment group did not do as well on translation of functions with stretches and shrinks, but there were other studies that found the opposite to be true. Rich found the experimental group did not do as well as the control group on paper and pencil procedures for finding the slope of linear functions, they did however, demonstrate better understanding of more difficult concepts involving connections between an algebra equation and its graph. Rich also found the control group scored significantly higher on trigonometry identities. Although Upshaw found that the calculus treatment group did not perform as well as the control group on a particular graphical problem, it is interesting to note that the level of significance used seems unusually high. Hinerman found that the calculus treatment group did not score as high as the control group on tests about integrals and area. There was no significant difference on a test concerning the shell method (for volume of a solid of revolution) or on a test about position, velocity and acceleration. Hinerman suggested a possible reason why the treatment group scored lower on the first two exams. At the beginning of the treatment, the technology group was not familiar with the graphing calculators so time was taken during class to teach the students how to use the graphing calculator while the control group was learning about calculus. By the third and fourth tests, the treatment group had caught up with the control group. This indicates that a short treatment with graphing calculators may not be as effective with students who are unfamiliar with the general use of the technology. O'Neill was the only one who reported a negative effect on overall achievement for the technology group. O'Neill stated that it was the initial experience for the teachers using graphing calculators and although it was a somewhat negative experience, all the teachers felt the graphing calculator should continue to be used in the course. O'Neill further concluded the instructors needed more training on the use of graphing calculators in their teaching and that teaching competency should be a factor in selecting instructors to teach college algebra.

Discussion and Conclusions

Given the positive results from previous reviews of research on the effect of calculator use in grades K-12 and the favorable results found in examining computer and calculator enhanced instruction in college mathematics combined with the results found in this review, (67% of the studies reported better overall achievement for the graphing technology or CAS groups, 75% of the measures of conceptual understanding favored the treatment group, and 65% of the results on procedural knowledge found no significant difference in the two groups), constitute strong evidence that the use of calculators, graphing utilities, and CAS have been effective in the teaching and learning of mathematical concepts.

Results of the studies reported to date indicate students using graphing technology understood the relationship between a function, its properties, and its graphical representation better than the non-technology students. The graphing technology groups also demonstrated a wider variety of approaches to solving problems and were better at modeling problem situations and solving real world applications particularly through the use of concrete visualization. Further, students in classes where technology was utilized demonstrated more conceptual understanding of algebra and calculus concepts than students not using technology.

It is interesting to note that a few of the studies attempting to isolate the effect of technology on student achievement found the graphing technology group demonstrated better conceptual understanding of the topic being tested. Yet, the majority of the studies seeking to isolate the technology variable, by controlling curriculum, text, homework, exams, and teacher variables, did not find a significant difference in overall achievement between the treatment group and the control group. These findings suggest that simply having access to technology does not insure it will be used to enhance learning. From the results of his study, Ruthven (1990) suggested the impact of the technology in the secondary classroom might depend as much on the ways in which the technology is used to mediate mathematics in the classroom as on simple access to the technology. Dunham and Dick (1994) also noted the mere presence of graphing technology most likely does not account for the positive results that have been found in studies. Rather, the combination of changes in curriculum and instruction with the use of graphing technology should be examined.

Regarding teaching practices, Stick (1997) noted that when implementing technology in teaching college mathematics, those instructors who regularly put some emphasis on class discussion had fewer adjustments to make than those who used a lecture-only format. Space restrictions will not permit elaboration on teaching strategies facilitated with access to graphing technology. Suffice it to say that approaches to teaching and learning which emphasize problem solving, foster visualization and exploration of concepts**,** student participation, and which allow students to actively construct meaning for the mathematics they encounter, find in graphing technology a natural and mathematically powerful partner.

On that note, at the ICME-9 recently held in Japan, individuals from several countries made comments about teaching and learning mathematics that can be connected to the findings provided in this paper. Wee Heng Tin from Singapore, discussed the need for a teaching paradigm shift, moving from a teacher-centered environment to a learner-centered focus. Shlomo Vinner from Israel commented that, for many students, learning mathematics is often the memorization of ritualistic procedures. Vinner suggested concerted teaching efforts be made to move away from meaningless rituals toward a more meaningful understanding of mathematics. Suat Khoh Lim-Teo from Singapore, spoke about the struggle to teach teachers to teach for meaning and understanding; for they were use to doing mathematics in a procedural manner and had great difficulty in coming to grips with the concepts. Lim-Teo also believed Singapore's high performance on TIMSS was due to the fact that the students are good test takers and not necessarily because of teaching or curriculum. Nozaki Akihiro from Japan, also speaking about the TIMSS study stated, "Our students are strong in computation skills, but are not as strong on understanding." Gilah Leder from Australia commented that in teaching mathematics, "Too much focus is placed on achievement and not on other richer information." (Quotes are from notes taken at ICME-9 in August 2000.)

The above statements made by education leaders suggest there is a growing desire within the international mathematics community to seek for more student-active learning and conceptual understanding. If this is the case, findings reported in this paper suggest a possible avenue to pursue to assist students in developing better reasoning ability. By drawing on the rich information available when graphing calculators and computer technology are utilized appropriately, mathematics concepts can be made more concrete and meaningful to the learner. It may be possible to argue that use of graphing technology is not required to bring about a student-active shift in curricular emphasis or for more conceptual understanding to take place, but the research has shown technology can clearly serve as a catalyst in leading to a deeper understanding of mathematical concepts. Perhaps through "teacher lesson study" practiced in several countries, experimentation and research efforts may be pursued to determine if the results of technology use found in US and UK classrooms transfer to students in other countries with different cultural situations. It is hoped that the information gleaned from the studies reviewed in this paper will help direct research efforts as well as the teaching of mathematics in the 21^{st} century.

References

Adams, T. L. (1993). The effects of graphing calculators and a model for conceptual change on community college algebra students' concept of function. (Doctoral dissertation, University of Florida, 1993). Dissertation Abstracts International, 55(7), 1859A. *****

Alexander, M. P. (1993). The effective use of computers and graphing calculators in college algebra. (Doctoral dissertation, Georgia State University, 1993). Dissertation Abstracts International, 54(6), 2080A. *****

Austin, J. S. (1996). The effect of graphing calculator use on student achievement in college algebra: Gender and age-related differences. (Doctoral dissertation, University of Missouri, 1996). Dissertation Abstracts International, 57(8), 3433A. *

Boers-Van Oosterum, M. A. M (1990). Understanding of variables and their uses acquired by students in traditional and computer-intensive algebra. Dissertation Abstracts International, 51(5), 1538A. (University Microfilms No. DA 9030857). *

Browning, C. (1990). The computer/calculator precalculus (C^2PC) project and levels of graphical understanding. In F. Demana & B. K. Waits (Eds.), Proceedings of the conference on technology in collegiate mathematics. Reading, MA: Addison-Wesley Publishing Company. *

Caldwell, F. W. Jr. (1994). Conceptual and procedural Achievements: Effect of a graphics calculator as a learning tool on college students' learning of mathematical functions and graphs. (Doctoral dissertation, Clemson University, 1994). Dissertation Abstracts International, 56(2), 483A. *

Castillo, T. F. (1997). Visualization, attitude, and performance in multivariable calculus: Relationship between use and nonuse of graphing calculator. (Doctoral dissertation, The University of Texas at Austin, 1997). Dissertation Abstracts International, 59(2), 438A. *

Chandler, P.A. (1992). The effect of the graphing calculator on high school students' mathematical achievement. (Doctoral dissertation, University of Houston, 1992). Dissertation Abstracts International, 53(11), 3832A. *

Connors, M.A. (1995). An analysis of student achievement and attitudes by gender in computer-integrated and non-computer-integrated first year college mainstream calculus courses. (Doctoral dissertation, University of Massachusetts, 1995). Dissertation Abstracts International, 56(3), 835A. *

Coston, Y. M. (1994). The effects of graphics calculator-enhanced instruction, and cooperative learning on college algebra students' understanding of the function concept, achievement of algebraic skills, and attitudes toward mathematics. (Doctoral dissertation, North Carolina State University, 1994). Dissertation Abstracts International, 55(8), 2310A. *

Devantier, A. L. (1993). The impact of graphing calculators on the understanding of functions and graphs. (Master's Thesis, Central Michigan University, 1992). Dissertation Abstracts International, 31, 535A*

Dunham, P. H. (1993). Teaching with graphing calculators: A survey of research on graphing technology. In L. Lum (Ed.), Proceedings of the Fourth International Conference on Technology in Collegiate Mathematics. Reading, MA: Addison-Wesley Publishing Company. *

Dunham, P. H. and T. Dick (1994). Research on graphing calculators. Mathematics Teacher, 87(6), 440-445.

Drottar J. F. (1998). An analysis of the effect of the graphing calculator on student performance in algebra II. (Doctoral dissertation, Boston College, 1998). Dissertation Abstracts International, 60(1), 56A. *

Dyer, D. M. (1994). The effects of computer and non-computer based instruction on symbolic, graphic, and numerical representations of mathematical functions in applied college algebra. (Doctoral dissertation, University of Maryland, 1994). Dissertation Abstracts International, 55(11), 3438A. *

Estes, K. A. (1990). Graphics technologies as instructional tools in applied calculus: Impact on instructor, students, and conceptual and procedural achievement. (Doctoral dissertation, University of South Florida, 1990). Dissertation Abstracts International, 51, 1971A. *

Fox, L. F. (1998). The effect of a graphing calculator used in an active learning environment on intermediate algebra students' achievement and attitude. (Doctoral dissertation, University of South Florida, 1998). Dissertation Abstracts International, 59(3), 761A. *

Giamati, C. M. (1991). The effect of graphing calculator use on students' understanding of variations on a family of equations and the transformation of their graphs. (Doctoral dissertation, University of Michigan, 1991). Dissertation Abstracts International, 52, 103A. *

Hall M. K. (1992). Impact of the graphing calculator on the instruction of trigonometric functions in precalculus classes. (Doctoral dissertation, Baylor University, 1992). Dissertation Abstracts International, 54, 451A. *

Hembree R. and D. Dessart. (1986). Effects of hand-held calculators in precollege mathematics education: A meta-analysis. Journal for Research in Mathematics Education, 17, 83-99.

Hinerman, S. D. (1997). Graphing calculators in the calculus classroom. (Masters thesis, Salem-Teikyo University, 1997). ERIC document ED416100. *

Hollar, J. C. & K. Norwood (1999). The effect of a graphing-approach intermediate algebra curriculum on students' understanding of function. Journal for Research in Mathematics Education, 30(2), 220-226.*

Hylton-Lindsay, A. A. (1997). The effect of the graphing calculator on metacognitve aspects of student performance in pre-calculus for business. (Doctoral dissertation, Columbia University Teachers College, 1997). Dissertation Abstracts International, 58(9), 3449A. *

King H. J. (1997). Effects of computer-enhanced instruction in college level mathematics as determined by a meta-analysis. (Doctoral dissertation, The University of Tennessee, 1997). Dissertation Abstracts International, 59(1), 114A.

Lesmiester, L. M. (1996). The effect of graphing calculators on secondary mathematics achievement. (Master thesis, University of Houston-Clear Lake, 1996). Masters Abstracts International, 35, 39. *

Mayes, R. L. (1995). An application of a computer algebra system as a tool in college algebra. School Science and Mathematics, 95(2), 61-68. *

Melin-Conejeros, J. (1993). The effect of using a computer algebra system in a mathematics laboratory on the achievement and attitude of calculus students (Doctoral dissertation, University of Iowa, 1992). Dissertation Abstracts International, 53(7), 2283A. *

Merckling, W. J. (1999). Relationship(s) between the perceptual preferences of secondary school students and their achievement in functions using a graphing calculator. (Doctoral dissertation, St. John's University, 1999). Dissertation Abstracts International, 60(2), 371A. *

Monticello, C. A. (1996). Effects of using the graphing calculator as compared to the scientific calculator on achievement and attitude in college algebra. (Doctoral dissertation, Temple University, 1996). Dissertation Abstracts International, 57(3), 128A. *

Mustafa, A. M. (1997). An investigation of the understanding of the numerical experience associated with the global behavior of polynomial functions for students in graphing and non-graphing calculator college algebra courses. (Doctoral dissertation, University of New Orleans, 1997). Dissertation Abstracts International, 58(5), 1629A. *

Norris, C. W. (1994). The impact of using graphic calculators as an aid for the teaching and learning of precalculus in a university setting. (Doctoral dissertation, Michigan State University, 1994). Dissertation Abstracts International, 55(7), 1862A. *

O'Callaghan, B. R. (1998). Computer-intensive algebra and students' conceptual knowledge of functions. Journal for Research in Mathematics Education, 29, 21-40. *

O'Neill, C. F. (1995). Incorporating the graphing calculator into college algebra. (Doctoral dissertation, Kansas State University, 1995). Dissertation Abstracts International, 56(9), 3491A. *

Oster, J. E. (1994). Graphics technology and calculus readiness: The effects of instruction using a programmable scientific graphing calculator on conceptual and procedural understanding in precalculus (Doctoral dissertation, Florida Institute of Technology, 1994). Dissertation Abstracts International, 55(11), 3466. *

Ottinger, T. P. (1993). Conceptual and procedural learning in first year algebra using graphing calculators and computers. (Doctoral dissertation, Georgia State University, 1993). Dissertation Abstract International, 54(8), 2934A. *

Palmiter, J. R. (1991). Effects of computer algebra systems on concept and skill acquisition in calculus. Journal for Research in Mathematics Education, 22(2), 151-156. *

Pankow, C. K. (1994). The effects of college student use of graphics calculators on the learning of algebraic concepts (Doctoral dissertation, The University of North Dakota, 1994). Dissertation Abstracts International, 55(12), 3374A. *

Paschal, S. G. (1994). Effects of a visualization-enhanced course in college algebra using graphing calculators and videotapes. (Doctoral dissertation, Georgia State University, 1994). Dissertation Abstracts International, 55(9), 2754A. *

Quesada, A. R. and Maxwell M. E. (1994). "The effects of using graphing calculators to enhance college students' performance in precalculus." Educational Studies in Mathematics, 27(2), 205-215. *

Rich, S. B. (1991). The effect of the use of graphing calculators on the learning of function concepts in precalculus mathematics. (Doctoral dissertation, University of Iowa, 1990). Dissertation Abstracts International, 52, 835A. *

Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic forms. Educational Studies in Mathematics, 21, 431-450. *

Schrock, C. S. (1990). Calculus and computing: An exploratory study to examine the effectiveness of using a computer algebra system to develop conceptual understanding in a first-semester calculus course. (Doctoral diss., Kansas State University, 1989). Dissertation Abstracts International, 50, 1926A. *

Scott, B. A. (1995). The effect of graphing calculators in Algebra II classrooms: A study comparing achievement, attitude, and confidence. (Doctoral dissertation, University of North Texas, 1994). Dissertation Abstracts International, 55(9), 2755A. *

Shoaf-Grubbs, M. M. (1992). The effect of the graphics calculator on female students' cognitive levels and visual thinking. (Doctoral dissertation, Columbia University, 1992). Dissertation Abstracts International, 54(1), 119A. *

Smith, B. A. (1996). A meta-analysis of outcomes from the use of calculators in mathematics education. (Doctoral dissertation, Texas A & M University, 1995). Dissertation Abstracts International, 57, 787A. (University Microfilms No. DA9724626).

Smith, K. B. (1991). An investigation of the computer algebra system Derive into instruction of linear programming (Doctoral dissertation, University of South Carolina, 1991). Dissertation Abstracts International, 52, 3550A. *

Stick, M. E. (1997). Calculus reform and graphing calculators: A university view. Mathematics Teacher, 90(5), 356-60, 63. *

Suydam, M. N. (1980). The use of calculator in precollege education: A state of the art review. Columbus, OH: Calculator Information Center.

Suydam, M. N. (1976). Electronic hand calculators: The implication for precollege education. Final Report. Washington, D. C: National Science Foundation.

Taylor, L. J. C. (1991). Assessing the graphing levels of understanding and quadratics knowledge of C^2PC students. In F. Demana & B. K. Waits (Eds.), Proceedings of the 2nd annual conference on technology in collegiate mathematics. Reading, MA: Addison-Wesley Publishing Company. *

Thomasson, S. J. (1992). The effects of the graphing calculator on the achievement and attitude of college students enrolled in elementary algebra (Doctoral dissertation, University of Tennessee, 1992). Dissertation Abstracts International, 53(11), 3835A. *

Tolias, G. (1993). The effects of using graphing technology in college precalculus. (Doctoral dissertation, The University of Texas at Austin, 1993). Dissertation Abstracts International, 54(4), 1274A. *

Trout, C. R. (1993). The effect of a computer algebra system in intermediate college algebra (Doctoral dissertation, University of Texas at Austin, 1993). Dissertation Abstracts International, 54, 127A. *

Upshaw, J. T. (1993). The effect of the calculator-based, graph-exploration method of instruction on advanced placement calculus achievement. (Doctoral dissertation, University of South Carolina, 1993). Dissertation Abstracts International, 54, 4023A. *

Utter, F. W. (1996). Relationships among AP calculus teachers' pedagogical content beliefs, classroom practice, and their student's achievement. (Doctoral dissertation, Oregon State University, 1996). Dissertation Abstracts International, 57(12), 5091A. *

Vasques, J. L. (1991). The effect of the calculator on student achievement in graphing linear functions. (Doctoral dissertation, University of Florida, 1990). Dissertation Abstracts International, 51, 3660A. *

Weber, T. E. (1998). Graphing Technology and its effect on solving inequalities. (Doctoral dissertation, Wayne State University, 1998). Dissertation Abstracts International, 60(1), 88A. *

Wilkins, C. W. (1995). The effect of the graphing calculator on student achievement in factoring quadratic equations. (Doctoral dissertation, Mississippi State University, 1995). Dissertation Abstracts International, 56(6), 2159A. *

***	Study Included in the Review**