The Maple Plots of the Rectangular Waveguide Modes

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ABSTRACT

Using CAS maple, the rectangular waveguide modes are shown. With the aid of the plot3d windows. The views of the electromagnetic fields distribution on each cross sections of the waveguide can be extracted from the 3D plots of the fields. The mode index (m, n) are treated as the parameters of the program. By this way, various complicated arrangements of the electromagnetic fields and the face currents of the waveguide can be shown on instant.

1. Rectangular waveguide modes

The eigen modes of electromagnetic waves in the rectangular waveguide are the solutions of the boundary problem of Maxwell′s equations of electromagnetic fields, which can be obtained with the aid of the calculus functions of maple. However, this is not the main interest of this paper.The solutions include two types of independent modes: the TE_{mn} modes and the TM_{mn} modes which are expressed as

TE_{mn}modes:

$$\left[\begin{array}{l} E_x = -i\frac{n\pi}{b}\frac{ck_0}{k_c^2}B_0\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ E_y = i\frac{m\pi}{a}\frac{ck_0}{k_c^2}B_0\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ E_z = 0 \\ B_x = -i\frac{m\pi}{a}\frac{k_z}{k_c^2}B_0\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ B_y = -i\frac{n\pi}{b}\frac{k_z}{k_c^2}B_0\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ B_z = B_0\cos(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \end{array}\right] \tag{1}$$

TM_{mn}modes:

$$\left[\begin{array}{l} E_x = i\frac{m\pi}{a}\frac{k_z}{k_c^2}E_0\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ E_y = i\frac{n\pi}{b}\frac{k_z}{k_c^2}E_0\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ E_z = E_0\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ B_x = -i\frac{n\pi}{b}\frac{k_0}{ck_c^2}E_0\sin(\frac{m\pi x}{a})\cos(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ B_y = i\frac{m\pi}{a}\frac{k_0}{ck_c^2}E_0\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})e^{i(k_z z-\omega t)} \\ B_z = 0 \end{array}\right] \tag{2}$$

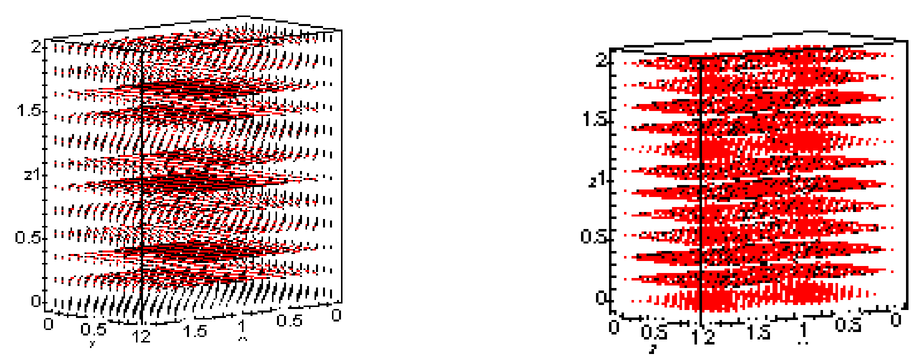
where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, the wave number in the vacuum, and $k_c = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2}$, , the cut-off wave number.

The construction of the electromagnetic fields in the waveguide depends on the mode indexes (m,n), corresponding to various complicated arrangements of the electromagnetic fields. To reveal the distribution of the fields, the traditional idea is to focus on some transverse sections so as to simplify the solutions. However, by this way, only some lopsided views can be got[1].

2. 3D plots of the fields

To show the full views of these modes, the maple function plots[fieldplot3D] is an ideal choice. In order to be able to show the various mode images at will, we treat the mode indexes (m,n) as the parameters of the program. By this way, various complicated arrangements of the electromagnetic fields can be shown on instant.

The figures following are the full views of the mode TE_{10} and mode TM_{21} by execute the maple procedures[2] , in which the main maple command is plots[fieldplot3D]. To make a distinction between the electrical and the magnetic fields, different colours are set for them(in maple plot windows on computer, to adjust the visual angle and in a larger scale, some local views can be more appreciable):



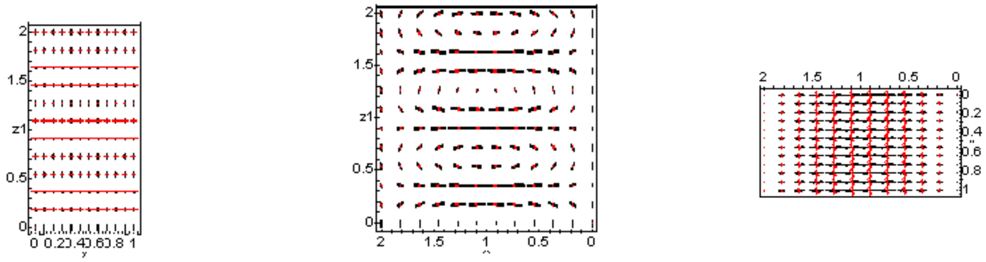
(a) TE_{10} mode (b) TM_{21} mode

Fig.1 Full view of the modes TE_{10} and TM_{21}

To get other high-order modes, it is nothing but to reset the mode indexes (m,n) and execute the program again.

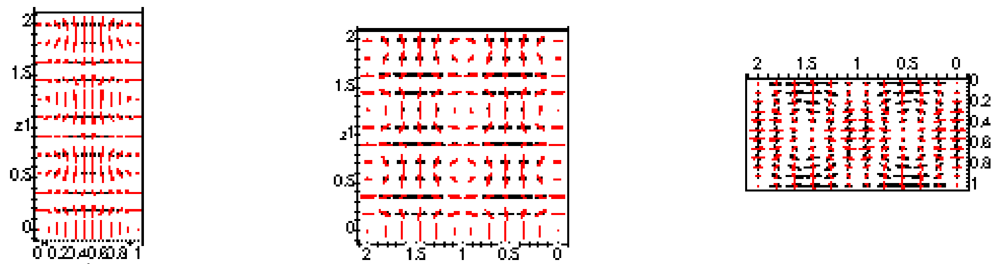
3. The Cross Section Views of the Fields

With the aid of the plot3d windows, the views of the electromagnetic fields distribution on each cross sections of the waveguide can be extracted from the 3D plots of the fields. It is just to adjust the visual angles of the 3D plots by controling the mouse. The following are the views of the wave modes mentioned in last paragraph on each cross sections:



(a) y-z section (b)x-z section (c) x-y section

Fig.2 The views of the cross sections of mode



(a)y-z section (b) x-z section (c) x-y section

Fig.3 The views of the cross sections of mode

4. The Face Currents

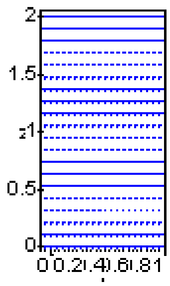
The presence of the guided waves can be observed by picking up some of the electromagnetic energy with a little ”probe”. It is usually convenient to insert the pickup probe via a long thin slot in the guide. Then the probe can be moved back and forth along the guide to sample the fields at various positions. In order to cut up the slot but not to disturb the fields to the full, it is important to know the distribution of the face current .

According to the boundary conditions of magnetic fields, the current density on the faces of the waveguide is

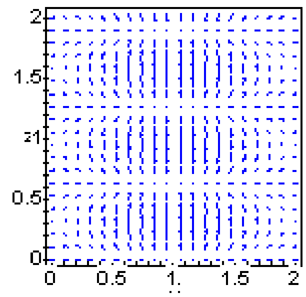
$$\alpha = n \times H \qquad (3)$$

where n is the normal unit vector of the face,$H(= B/\mu_0)$is the strength vector of the magnetic field close upon the face.

Based on the solutionseq.(1) and eq2(2), with the aid of maple command linalg[crossprod], the face current density vector α can be obtained and the command plots[fieldplot] would give the distribution views of the face currents. For example, for the TE_{10} mode, they are



(a)x=0 face



(b)y=0 face

Fig.4 The face currents of the modes

5. Conclusion

Owing to the powerful functions and the flexible language of maple programming, various complicated arrangements of the electromagnetic fields and the face currents of the waveguide modes can be shown instantly and dynamically. It makes possible to understand the boundary problem of the electromagnetic fields in terms of images and provide reliable information for microwave technology.

References

[1] Software for Electromagnetic Fields Inside Waveguides And Cavities (http://home.olemiss.edu/~atef/EM_software/wgc21.html)

[2] ISBN-O-387-94537-7, M.B.Monagan, K.O.Geddes, G.Labahn, S.Vorkoetter, Maple V Programming Guide, NY, USA, Springer,1995.