The Paradigmatic Roles and Pedagogical Functions of IT in Mathematics Education in East Asia

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Abstract

In East-Asian countries, the paradigmatic roles and pedagogical functions of various components of IT (e.g. internet web-sites, CDROMs, multi-functional computer software and graphing calculators) have not been conceptually discussed and empirically evaluated, despite their increasing importance.

In this paper, section (1) depicts the 4-level paradigmatic roles of IT in mathematics education. Section (2) brings out a 3-dimensional conceptual framework for systemic classification of the pedagogical characteristics of various components of IT in mathematics education. In section (3), based on one in-depth case study of 8 high secondary school students' learning process on complex numbers in Cabri Geometry, a cognitive model for conceptualizing their thoughts is speculated. By pointing out some crucial socio-cultural features, far-reaching implications are drawn in mathematics education, which is beneficial to curriculum development in East-Asian countries in the next century.

1 Four Paradigms Induced by IT in Mathematics Education

By applying Kemmis, Atkin and Wright ' s conceptual model [11, pp.24- 26], paradigmatic changes led by Information Technology (IT) in Mathematics Education are four, in general:

Table 1: 4-level roles of IT in mathematics education

Types of educational paradigm	Characteristics in Teaching / learning Contexts	Extent of the Pedagogical role of IT in mathematics education

Instructional paradigm	Reinforcing programmed Learning through IT by Drilling and practice;	*MORE* IT in control, Topic-based and content-Laden
Revelatory paradigm	Stressing real-life Applications: e.g. Mathematical simulations	In-between
Conjectural paradigm	Promoting deductive / inductive reasoning by testing hypotheses / discovering patterns in problem-solving or open-based investigation or in testing mathematical models	In-between
Emancipatory paradigm	IT used as a labor-saving device in calculating, capturing statistical data and information processing	*MORE* student in control, Learner-centered, content-Free

Unlike scientific paradigms in Kuhn's [6] terminology, IT reformers need to realize that educational paradigms have their distinctive nature in the following table 2. Without thorough understanding of the differences, IT-induced school curricula cannot be effectively implemented.

Table 2: Four-level Comparisons between scientific and educational paradigms

Scientific Paradigms [10, pp.2-6]	**Educational Paradigms**
A new paradigms emerged after *Anomalies / crisis* in the preceding Paradigm;	A new paradigm emerged after some new *educational theories or advancement* in educational resources (e.g. technology);
Absolute in applications: the new Paradigm can be applied everywhere with wider contexts;	*Relativistic* in applications: different educational systems can have different options in choosing the paradigm;
New values in understanding the nature of the living world;	*New values* in understanding educational contexts: cognitive bases of teachers / students, educational improvements / values changes in teaching and learning environments;
One-theory-driven: usually one dominating theory is applied in the new paradigm	*Multi-dimensional*: flexible applications / combinations of various context-based educational theories;

Unlike some Western (e.g. North American and Western / Northern European) countries with *more flexible* school curricula, each mathematics curriculum in some East-Asian countries / places (like Shanghai, Hong Kong, Singapore, South Korea and Taiwan) has *six common characteristics* [7 & 8]:

(i) *summative assessment stressing inter-student academic competitions especially in open examinations*;
(ii) *large student-teacher ratios in classrooms*;
(iii) *repetitive learning through memorization*;
(iv) *teacher-centered pedagogy and / or teacher-led discussion*;
(v) *rigid textbook-based teaching and learning*;
(vi) *uniform school curricula without much diversities at upper secondary level*;

Such similarities imply distinctive paths for initiating educational paradigms in East Asia, completely different from those Western countries. For instance,'instructional paradigm' in table 1 is the most feasible, with sufficient operational, programmable instructions in some PC software. For other paradigms require more open-ended pedagogy (i.e. high degree of students' participation) and diversified school curricula which are *uncommon* in many East-Asian countries currently.

Yet other paradigms are expected to be involved at the beginning of the next century with the forthcoming new syllabuses, stressing flexible school curricula, learner-based pedagogy and individual learning differences with IT integration in some East-Asian countries like Hong Kong [4], Shanghai [15] and South Korea [9].

2 Systemic Classifications of Functions of Various Components of IT

For the evaluation of pedagogical characteristics of various components of IT, a 3-dimensional framework is articulated in the following fig. 1. Such classification is based on three criteria: *their flexibility, sharing of common pedagogical functions and enhancing* student-teacher / student-student *interactions in classroom settings*.

However, this is a rough depiction. For varieties of IT components can occupy locations, depending on *where and how* they are to be used. For instance, Cabri-II 'implanted' into Powerpoint can make the latter very interactive. Another instance is some resourceful 'interactive' internet web- site e.g. http://www.glink.net.hk/~msalee which is established by some enthusiastic in-service teachers in Hong Kong. Users / visitors can freely download 'freeware' (teaching resources) from their workstations. Other web-site like http://www.edp.ust.hk/math provides various channels for exchange of teaching and learning experiences and provision of various branches of mathematical knowledge. On reflection, such web-sites are flexible, non-generic and interactive (*which depends on users*) under the three dimensions in fig.1.

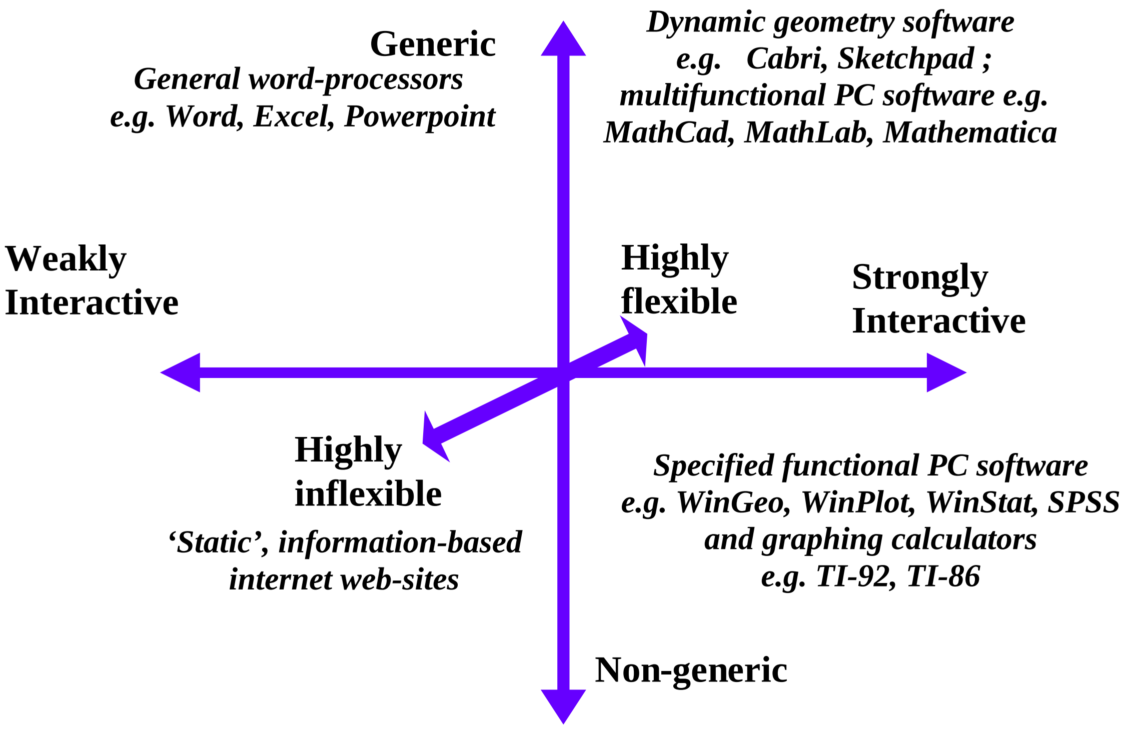

Figure 1: a 3 –dimensional framework for classifying pedagogical characteristics of IT components [Note: the exact 3-dimensional location of each IT component cannot be clearly indicated on the 2-dimensional paper]

Teachers, students and educational researchers need to consider their own teaching, learning and researching contexts by considering the advantages and disadvantages of these IT components in table 3. With *a limited number of IT laboratories* and *less spacious classrooms* in many East-Asian countries [with the above stated characteristics in section (1)], *economical* graphing calculators have their own advantages in day-time lessons. Yet for high-resolution visualization of sketching functional graphs or free geometrical explorations, powerful PC software like Mathematica is *better with high educational PC costs*. So there is always a trade-off !

3 An In-depth Case Study on the Pedagogical Roles of Cabri Geometry

3.1 Research Questions:

(a) What sort of possible *socio-cultural factors* affect students' performance in correlation of algebraic and geometrical solutions in Cabri Geometry?
(b) In what sense(s) can Cabri Geometry be *integrated* into such correlation?
(c) Can we *conceptualize a cognitive model* depicting students' performance and their learning differences?

3.2 Subjects

Eight (6 male and 2 female) grade 10-11 students of above average mathematics ability [based on their open and school exam. results] were grouped into 4 pairs. They were the private tutees of the author in some after-school tutorial lessons. Subjects in each pair were studying in one of four prestigious secondary schools in 1998-1999.

3.3 Pre-conditions before the study

These eight subjects were chosen, based on their familiarity of the topic and friendships. The problems were not seen or solved by them before the study. They had a four-hour training program in gaining operational knowledge and sufficient practical time on using Cabri Geometry installed in the graphing calculators modeled TI-92 with a good mastery of discussion strategies.

3.4 Instructions during the study

Each pair was given the following problem (#) and spent 30-45 minutes solving it through Cabri Geometry II. Their solution and thinking process needed to be recorded in detail by hands or stored in soft copies. The pair discussion was not interrupted by the author who only recorded the process by field notes.

3.5 Methodology

Basically, it is a qualitative research. Before the study, the subjects were interviewed informally about their school environment (e.g. their day-time mathematics teachers' pedagogical style, teaching beliefs and the subjects' cognitive skills in the investigating topic of complex numbers). Their learning habits and high-order thinking (e.g. problem-solving) were examined through some after-school private tutoring lessons conducted by the author.

3.6 ***Problem (#)***

For any complex numbers $\mathbf{Z_1}$, $\mathbf{Z_2}$,

(#) **(i)** Prove that:

$$|\mathbf{Z_1 - Z_2}| = \sqrt{2} \quad \textit{IF AND ONLY IF} \quad |\mathbf{Z_1 + Z_2}| = \sqrt{2}$$

With $|\mathbf{Z_1}| = |\mathbf{Z_2}| = \mathbf{1}$, where $|Z|$ denotes the modulus of a complex number Z.
[Hint: make geometrical-algebraic ***CORRELATION*** through Cabri II]

(#) **(ii)** Prove that:

$$|\mathbf{Z_1 - Z_2}| = |\mathbf{Z_1 + Z_2}| \quad \textit{IF AND ONLY IF} \quad \mathbf{arg}\,(\mathbf{Z_1 / Z_2}) = \boldsymbol{\pi / 2}$$

where arg Z denotes the argument of Z .
[Hint: make geometrical-algebraic ***CORRELATION*** through Cabri II]

(#) **(iii)** Can you ***VISUALIZE*** any relationship(s) between (i) and (ii)? Can you ***CORRELATE*** with those algebraic proofs to (i) and (ii) "

3.7 Important steps in visualizing the geometric-algebraic gaps

For **(#) (i)**, visualization of the diagonals of a square (formed by its adjacent sides $\mathbf{Z_1}$ and $\mathbf{Z_2}$ of unity modulus) are of length $\sqrt{2}$ by dynamic dragging;

For **(#) (ii)**, visualization of the angle between the adjacent sides $\mathbf{Z_1}$ and $\mathbf{Z_2}$ of a parallelogram being equal to $\pi / 2$ i.e. becoming a rectangle when the length of their diagonals $Z_1 + Z_2$ and $Z_1 - Z_2$ are equal to each other by dynamic dragging in Cabri Geometry;

For **(#) (iii)**, **(#) (i)** is a *particular* case of **(#) (ii)**, by understanding that the rectangle in **(#) (ii)** is *not necessarily* a square in **(#) (i)**;

Algebraic solution to **(#) (ii)** was commonly done by some subject pairs:

$$
\begin{array}{lrcl}
 & |Z_1 + Z_2|^2 & = & |Z_1 - Z_2|^2 \\
\textbf{iff} & (Z_1 + Z_2)(Z_1^* + Z_2^*) & = & (Z_1 - Z_2)(Z_1^* - Z_2^*) \\
\textbf{iff} & Z_1^* Z_2 + Z_1 Z_2^* & = & 0 \\
\textbf{iff} & \mathrm{Re}(Z_1 Z_2^*) & = & 0 \text{ ------- Step (xx)}
\end{array}
$$

[Note. **Re (Z), Z *** denote the real component of **Z** and the conjugate of **Z** respectively whereas **iff** stands for "if and only if"]

In fact, other pairs substituted in the form **Z** = a + b**i** [**i** = $\sqrt{-1}$] with similar results.

3.8 Measurement criteria

1. degree of interaction: questioning or leading in-depth discussion:
   0 – 2 times --- weak; 3-4 times --- medium; more than 4 times ---- strong;
2. open-ended day-time pedagogy: whether day-time teachers ever taught the subjects in broader perspectives: geometrical interpretation of some algebraic statements concerning complex numbers by drilling exercises and discussing the problems during lessons;
3. bridging up the *geometric- algebraic gaps*: visualization between:
   (a) $\arg(Z_1 / Z_2) = \pi / 2$ [*geometrical representation*] and
   (b) $\mathrm{Re}(Z_1 Z_2^*) = 0$ in the above step **(xx)** [*algebraic representation*]
   formulated in their last step in the algebraic solution to **(#) (ii)** through the dynamic dragging of geometric figures in Cabri Geometry.

3.9 Research Results

Table 3: a summary of students' solution profile in the study before probing

Pair 1: Tom & Jimmy (Grade 10)	**Pair 2: Henry & Sam (Grade 11)**	**Pair 3: Cat. & Peggy (Grade 10)**	**Pair 4: Peter & Gary (Grade 11)**
Member interaction: Strong	*Member interaction:* Weak	*Member interaction:* Moderate	*Member interaction:* Weak
Time used: 40- 45 minutes	*Time used*: 30-32 minutes	*Time used*: 40-45 minutes	*Time used*: not more than 30 minutes
Day-time pedagogy: Open-ended	*Day-time pedagogy:* Open-ended	*Day-time pedagogy:* Closed-ended	*Day-time pedagogy:* Closed –ended
Problem-solving profile: Algebraic proof in **(#)(i)**,	Problem-solving profile: Algebraic proof in **(#)(i)**,	Problem-solving profile: Algebraic proof in **(#)(i)**;	Problem-solving profile: Algebraic proof in **(#)(i)**;

NO correlation in **(#)(i)**; Realizing geometrical meanings through Cabri and being able to construct algebraic proof in **(#) (ii)**, *WITH* bridging up geometrical-algebraic gaps through discussion using Cabri in **(#) (ii)**; Visualizing **(#) (i)** is particular case **of (#) (ii)** *WITHOUT* correlation .	*NO* correlation in **(#)(i)**; Realizing geometrical meanings *NOT* through Cabri but through hand-drawings and being able to construct algebraic proof in **(#) (ii)**, and *NO* correlation in **(#)(ii)**; *WITHOUT* bridging up geometrical-algebraic gaps in **(#)(ii)**; Stated *WITHOUT* proof to the case **(#) (iii).**	*NO* correlation in **(#)(i)**; *NOT* realizing geometrical meanings by any means but being able to construct algebraic proof in **(#) (ii)**; *WITHOUT* bridging up geometrical-algebraic gaps by making unsuccessful attempts in **(#) (ii)**; *Reassess the algebraic proofs and see* (#) *(i) is a particular case of* **(#)** ***(ii)***.	*NO* correlation in **(#)(i)**; *NOT* realizing geometrical meanings by any means but being able to construct algebraic proof in **(#) (ii)**; *WITHOUT* bridging up geometrical-algebraic gaps *WITH NO* attempts in **(#) (ii)**; *NO* solution to **(#) (iii).**

Follow-up Probing
Afterwards, the author asked the subject pair no.1 to redo **(#)(i)** using Cabri II and checked whether the pair partner really understood the correlation by asking them:

(#) (iv) What are the values of $|\mathbf{Z}_1 - 2\mathbf{Z}_2|$ and $|2\mathbf{Z}_1 - \mathbf{Z}_2|$ (with $|\mathbf{Z}_1| = |\mathbf{Z}_2| = \mathbf{1}$) using algebraic and geometric methods through Cabri Geometry and find their correlations?

For pair no.2, 3 and 4, the author asked them to rethink **(#)(ii)** by visualizing the geometric meanings of $\mathbf{Z}_1 - \mathbf{Z}_2$ and $\mathbf{Z}_1 + \mathbf{Z}_2$ and then similarly redo **(#) (i)**in Cabri geometry.
For pair no.3, the author asked them to redo **(#) (iii)** after finishing **(#) (i)** and **(#) (ii)**.

Based on Biggs' SOLO taxonomy in [2], the answer to the above research question(c) in **3.1** is as follows**:**

Table 4: a cognitive model depicting students' performance / learning differences

SOLO Level	Description of the level	Evaluations of pair performance
Pre-structural	Unable to give any algebraic-geometrical correlation	*Before probing,* pair nos. 3 and 4 on **(#) (i)** **and (#) (ii)**, pair nos. 2 on **(#) (ii)** *After probing,* Still no result for pair nos. 3 and 4 on **(#) (i)** and **(#) (ii),**
Uni-structural	Able to give one particular algebraic-geometrical correlation in either **(#) (i)** or **(ii)**	*Before probing,* pair no.1 on **(#) (ii)** *After probing,* pair no.1 on **(#) (i);** pair no. 2 on **(#) (ii)**
Multi-structural	Being able to bridge up the geometrical-algebraic gap by questioning or dragging in Cabri Geometry in both **(#) (i)** and **(ii)**	*After probing,* pair no. 2 on **(#) (i)** and **(#) (ii)**
Relational	Being able to relate the cases in (i) and (ii) either algebraically or geometrically in **(#)(iii)**	*After probing,* pair no.1 on **(#) (iii)**
Extended abstract	Being able to relate the cases algebraically and geometrically in other similar cases	*After probing,* Pair no.1 can do **(#) (iv)**

3.10 Data Interpretation

1. Strong pair interactions and suitable author's (or tutor's) probing were necessary to develop subjects' high-order thinking (i.e. correlation of the geometric and algebraic representations) which involved ***strategic know-hows in making geometrical-algebraic correlation by suitable dragging and dialectical questioning in Cabri Geometry*** [answer to research question (a) in **3.1**].
2. Pedagogy and assessment by day-time school teachers seemed to strongly determine subjects' certain ways of thinking e.g. no positive result for the pair nos. 3 and 4 before and after probing. When explaining the geometrical meanings after the study, the subject pair no.3 complained*:* [answer to research question (a) in **3.1**] *"This is not the usual way we learnt from our day-time teachers before*. Examinations only require algebraic proof, don't they ?"
3. To sum up, subject knowledge in broader perspectives (geometrical and algebraic) can be mastered by the subjects when gaining suitable IT strategic (know-how) knowledge of bridging up the geometric-algebraic gaps in Cabri Geometry. [answer to research question (b) in **3.1**].
4. Subjects' study records revealed Cabri Geometry's *three-fold* didactic roles: ***complemented*** what their traditional paper-and-pencil works cannot do; ***repeated*** *and* ***reinforced*** fruitfully what they learnt from traditional paper-and-pencil works.

3.11 Limitations:

(i) a smaller number of subjects; (ii) only one topic being covered; (iii) the models being inapplicable to normal classroom settings especially with passive learning styles of Asians (c.f. Biggs' Asian learners' model in [1]); (iv) no investigations into subjects' individual learning disparities in Cabri Geometry [c.f. 13]; (v) lack of in-depth cognitive studies on how the subjects' gain subject knowledge by applying IT strategic knowledge in Cabri geometry; (vi) assuming no possible 'hidden' socio-cultural factors in the above cross-case comparisons in table 3.

3.12 Implications of the model in table 4

1. Learning mathematical concepts in IT media (e.g. Cabri geometry) require more high-level strategic knowledge for teachers / learners and more cognitive 'leaps' (in SOLO levels) in group/ pair settings. To some extent, this explains some unexpected learning outcomes or complicated learning behavior encountered by other researchers [e.g.14] in similar teaching / learning contexts.
2. In using IT, both *multi-level* formative and summative assessments involving the SOLO levels are required in future open examinations, which currently dominate upper secondary mathematics curricula in East-Asian countries.
   Heated discussion [e.g. 3 & 5] recently centers on the disparities between examination setters' expectations and student candidates' performance with an ineffective / inefficient use of graphing calculators. But *lack of training in teachers' and students' mastery of IT strategic knowledge* in day-time lessons may be the causes for such disparities.
3. Important *socio-cultural factors* such as teachers' value-beliefs in IT media or philosophy of mathematics education in [12], students' individual differences and the interactive process of students' constructing knowledge in pair discussion further complicates the IT teaching and learning process. So there comes an urgent need to find out on *how and whether IT alone* really enhance teaching and learning (topic-based / IT component-based) or *to what extent* IT need to be *integrated fruitfully* into traditional pedagogy in East-Asian countries.
4. In most 'congested' classrooms in most East-Asian countries, the above multi-level learning / teaching context is not easily achieved. The subjects in the above study pinpointed the *impracticality* of their time-consuming (albeit in-depth) learning in day-time schools. Another contextual factor is the influence of school / classroom culture over students' mode of cognitive thinking or their interactions when visualizing geometric-algebraic correlations in the IT media. So more forthcoming researches should be focused on *how* to bridge up the geometric-algebraic gaps in the IT media in various groups / classes of students with mixed abilities.

END

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