

EXPLORE GEOMETRY OVER THE INTERNET USING JAVASKETCHPAD

Zhonghong Jiang
Florida International University

Abstract: This paper introduces a course that I designed for university mathematics education majors to explore geometric concepts and problem solving via technological tools, and illustrates how I organize both in-class and after-class interactive dynamic geometry activities using JavaSketchpad. The use of JavaSketchpad has been found to be effective. It covers the limited exploration of the students through the course homepage. The related web sites of JavaSketchpad encourage the students to acquire further knowledge. The dynamic environment provided by JavaSketchpad puts the students in a position to visually manipulate the geometric objects and hence experience mathematics by seeing it happen or making it happen. This significantly extends the students' visualization and spatial thinking abilities, and facilitates their conceptual understanding.

The Geometer's Sketchpad (Jackiw, 1995) is a dynamic geometry construction and exploration environment available for Macintosh and PC computers from Key Curriculum Press. Geometric objects (from points and lines to rather complicated geometric shapes) can be constructed conveniently through its construction tools and menu options. Once a geometric object is constructed, its shape and/or size can be changed simply by dragging one or more of its component parts around the computer screen, so the object is not a single, static figure, but in essence a prototype for all possible figures of same kind. When changes are made in the figures, the corresponding measurements and related calculations (if any) are changed accordingly. By observing these changes, students can discover what relations and properties are preserved, or what configurations give the best results (e.g., the shortest path). Features such as animation, scripting, and button uses provide students with more capabilities to explore geometric situations and to construct demonstration programs. In the recent years, The Geometer's Sketchpad (GSP hereafter) has been used internationally by many middle and high school mathematics teachers and their students, as well as university faculty of mathematics and mathematics education and their students, in their teaching, learning, and research.

In order for the users to interact with or publish GSP sketches on the Internet, Nicholas Jackiw, the software architect of GSP has released a new software package - JavaSketchpad (1997). With this package, people can share their geometry work with those who may not have desktop GSP, or who use computers that can't access desktop GSP (like most Unix machines). For the teachers and university faculty, they can use the software to distribute interactive dynamic geometry activities and curriculum over the Internet (Jackiw, 1998).

On the part of the students, they can explore geometry dynamically after class or even at home, for their course assignments and for further geometric investigations.

INTERACTIVE DYNAMIC GEOMETRY ACTIVITIES AFTER CLASS

Through my work with mathematics education majors (undergraduate and graduate level preservice mathematics teachers), I found that geometry was a weak spot in these students' previous education. For many of the students, their mathematical reasoning and proof abilities were very limited. Since geometry is an integral part of mathematics (and school mathematics), it is very important to improve the students' geometry background (while improving their overall mathematics background). Based on this consideration, a specific course *Learning Mathematics with Technology* (MAE 3651) has been designed and offered to the preservice mathematics teachers at Florida International University. The main emphasis of this course is to use GSP to help the students explore geometric concepts and problem solving. Examples of the GSP activities in the class, and how the use of GSP facilitates students' learning have been reported in Jiang and McClintock (1997). However, in comparison to the efforts that the students should make to really enhance themselves in geometry, the class exploration time is very limited. JavaSketchpad gives me the opportunity to assign through my home page (<http://www.fiu.edu/~jiangz>) after-class geometric exploration activities (Figure 1) to the students, of whom most do not own the desktop GSP software.



Figure 1. Some dynamic geometry activities assigned through a web page.

The use of JavaSketchpad has been found to be effective. Any student does not have to go to the specific computer lab to work on the assignments. He (or she) can go to any computer lab on campus, which has the Internet access, or even use the dynamic geometry environment at home if he (or she) has a computer with a modem. The dynamic environment on the Internet has played a role very similar to what the desktop environment plays. It puts the students in a position to visually manipulate the geometric objects and hence experience mathematics by

seeing it happen or making it happen. This significantly extends the students' visualization and spatial thinking abilities, facilitates their conceptual understanding, and therefore has benefited almost all of them, especially those visual learners who are not able to make sense of problem situations without visualizing them.

An example is given below to show the benefits of using JavaSketchpad. The students were presented with the following problem: In Triangle ABC, AF and CE are two medians. If angle BAF = angle BCE = 30° , what conjecture(s) can you make? Prove your conjecture(s).

Since the problem did not specify what kind of triangle ABC was, it was somewhat difficult for the students to construct a triangle which fitted the given situation well without their preimage of the triangle's shape, simply by using paper and pencil type of construction tools. However, with the dynamic features of GSP or JavaSketchpad, it was quite easy to do so. An arbitrary triangle ABC with two medians AF and CE were given in a sketch on a web page. The students could follow the instructions on the page, drag each of the three vertices of the triangle around the screen, and observe the dynamic updates of the angle measurements until both angles became 30° (Figure 2).

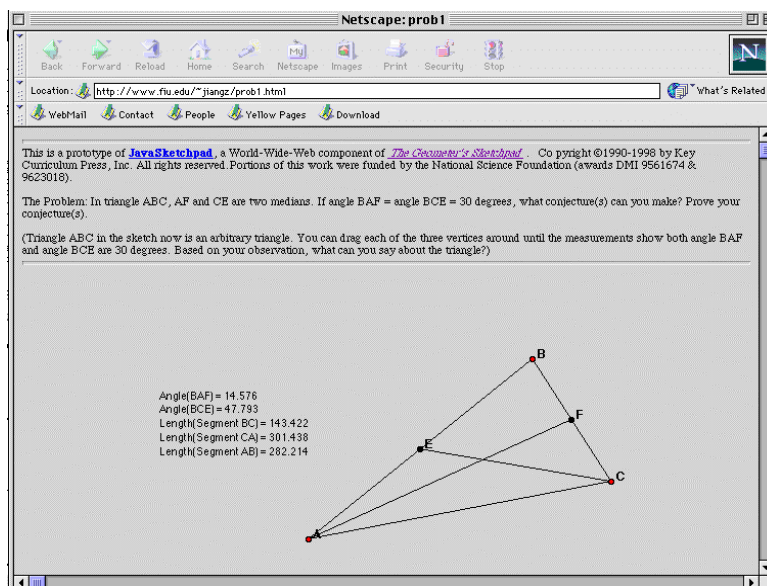


Figure 2. Triangle ABC on the Internet for students to explore.

When both angle BAF and angle BCE became very close to 30° , the students had a situation shown in Figure 3. Noticing that Length(BC) = 239.268 units and Length(AB) = 239.771 units, they came up with a conjecture that Triangle ABC was an isosceles triangle. The small difference between the two lengths did not bother them, since they knew the computer measurements involved some approximations. Some of them continued to drag the vertices to get larger or smaller triangles. However, as long as both angle BAF and angle BCE were kept to be 30° , the measurements showed that sides AB and BC were congruent. All

visual manipulations so far helped the students see that only the shape of this triangle was essential, and Triangle ABC was always an isosceles triangle.

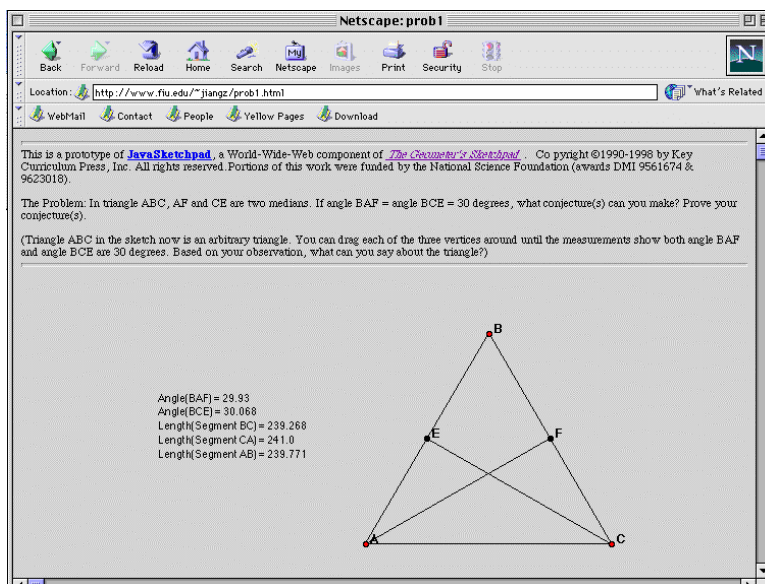


Figure 3. Triangle ABC is shown as an isosceles triangle.

After such an investigation process over the Internet, the students came to the class with their observed finding and ideas to prove it. Through active class discussion, proofs were developed. The following is one of the proofs:

Since angle $BAF = \text{angle } BCE$, and angle B is their common angle, Triangle BAF and Triangle BCE are similar by AA. $BA:BF = BC:BE$ because the corresponding sides of similar triangles are proportional. This implies that $BA \cdot BE = BC \cdot BF$. But $BE = (1/2) BA$, and $BF = (1/2) BC$. Hence, $(1/2) BA^2 = (1/2) BC^2 \Rightarrow BA^2 = BC^2$. Since both BA and BC express lengths here and so are positive, $BA = BC$.

While affirming that the students did a good job, I pointed out that the 30° angle condition had not really been addressed in the proof mentioned above. The proof only used the congruence of angle BAF and angle BCE, indicating that the two angles did not have to be 30° angles. What did this imply? Was the 30° angle condition significant or not? The students were encouraged to continue their investigations after class. Through further manipulating the Java sketch shown in Figure 3, the students found that isosceles triangle ABC could be dragged to be an equilateral triangle while both angle BAF and angle BCE remained 30° . The students sent me e-mail messages one after another, reporting their new finding. With some hint from me, the students finally developed the proof for the new conjecture.

EXPLORING JAVA SKETCHES ON MANY IMPORTANT WEB SITES

The students have been benefited from JavaSketchpad not only in completing their related course work, but also in getting access to many more GSP exploration environments, including those on many important and interesting web sites such as the home page of Key Curriculum Press (<http://www.keypress.com/>) and the links made available by that page. Those net-based examples of JavaSketchpad in action have opened a much wider geometry learning world to the students, and effectively helped them construct geometric concepts, strengthen problem solving abilities, and apply geometric ideas to the real world situations. As an example, let us see how our students have achieved a better understanding of the concept and applications of ellipse (as well as other conic sections) through visiting the web site *Geometry in Motion* (<http://members.xoom.com/dpscher/>) by Daniel Scher. In a traditional mathematics class, students usually learn the concept of ellipse by the definition (“the set of all points, the sum of whose distances from two fixed points is a constant”), and/or from a cross section point of view (“formed by a plane that cuts all the elements of a right circular cone”). Because of the limited information offered by the textbook, the students only have a superficial understanding of ellipse. In contrast, by exploring *Geometry in Motion*, our students experienced the dynamic forming process of an ellipse from multiple points of view such as intersecting circles, the sliding triangle, the folded circle, congruent triangles, and concentric circles. In the Falling Ladder and the Bent Straw situations (Figures 4a and 4b), by changing the position of the designated point and dragging it, the students were able to generate various ellipses with different eccentricities. These activities revealed that ellipses vividly exist in the real world around us. (Through visiting other web sites or further studies, students would realize that ellipse has wide applications in physical science, astronomy, and engineering.)

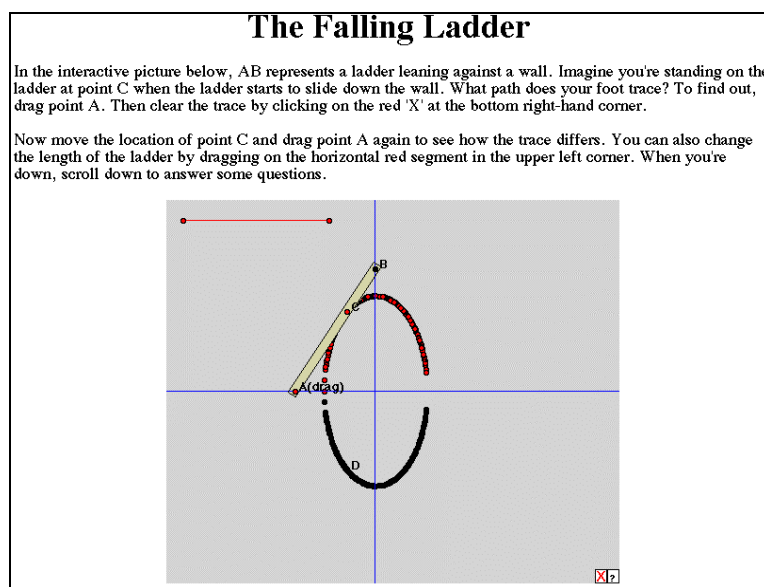


Figure 4a. The Falling Ladder situation.

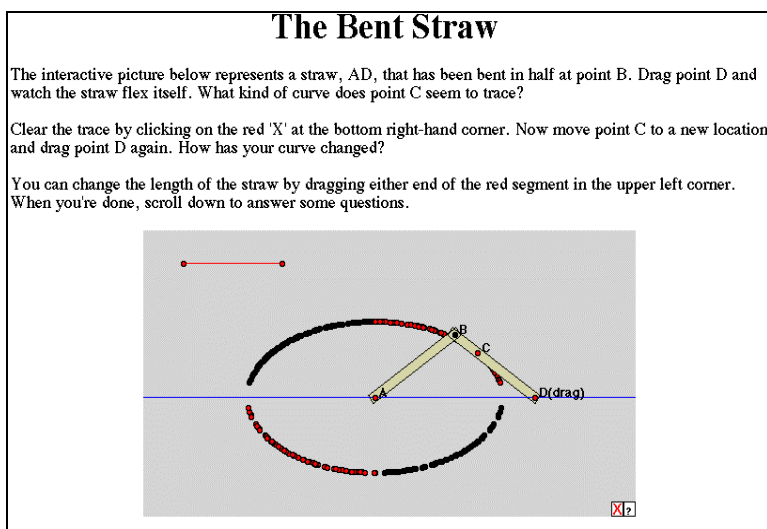


Figure 4b. The Bent Straw situation.

Further more, the page Van Schooten's Ellipse introduced the students to a mechanical linkage created by Fran van Schooten, a Dutch mathematician in the 17th century. The students tried actively to figure out why the linkage draws ellipses and used this activity as a test to see if they understood ellipse. Needless to mention more, we have already seen that in the web-based environment, our students learned ellipse through dynamically exploring the related materials with much wider richness. Therefore, they reached a more comprehensive and profound understanding of the concept and applications of ellipse than they can in the traditional class.

SHARING INNOVATIVE GEOMETRIC IDEAS WITH OUTSIDE INTERNET USERS

In comparison to exploring Java sketches on other people's web pages, it seems more challenging and exciting for our students to post their innovative geometric ideas on our own web pages to share them with outside Internet users who are interested in geometry. Inspired by so many wonderful features of GSP and JavaSketchpad, our students are not satisfied with only learning from others. They have started their own investigation and research activities. Dixon found in her research (1997) that the GSP instructional environment was more effective than the traditional instructional environment at improving students' two-dimensional visualization, but was not more effective at improving students' three-dimensional visualization. However, she offered an interpretation of the results that the students' spatial visualization did improve in both two and three dimensions, but this improvement was measured inadequately. To examine the effectiveness of the GSP instructional environment in the three-dimensional case, some of our students have created many interesting, thought-provoking GSP sketches and used them with their fellow students and some middle and high school students. Based on this effort, they have provided strong evidence that the use of GSP can effectively enhance students' three-dimensional visualization and hence the students' understanding of three-dimensional

geometric concepts (see Rodgers, 1997 and Zhang, 1999). Through JavaSketchpad, they have posted some of these sketches over the Internet (see Research Examples at <http://www.fiu.edu/~jiangz>). One of the Java sketches called *Rotation of an Icosahedron* is shown in Figure 5 (thin lines expressing the edges which cannot be seen from the front view). When one drags the free point on the small circle or clicks the Rotation button, he (or she) can see the icosahedron from different perspectives.

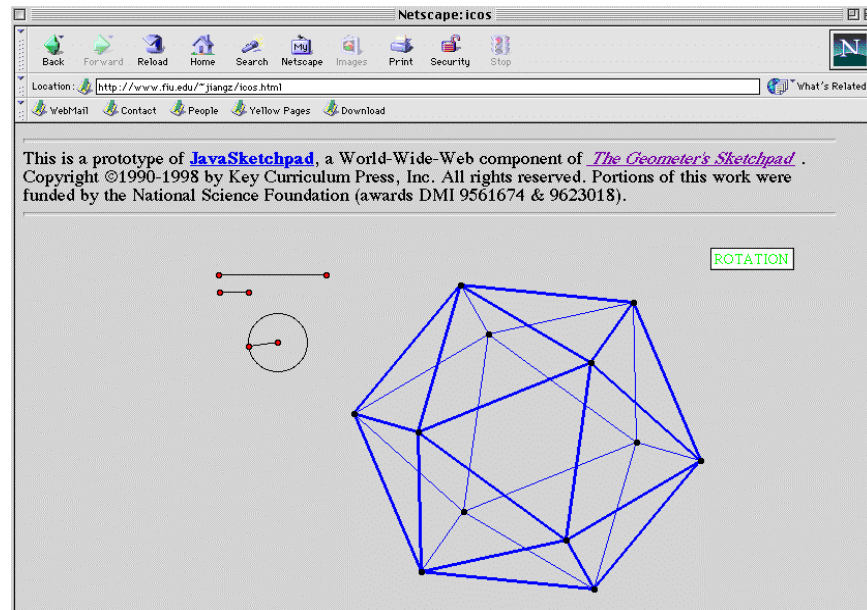


Figure 5. *Rotation of an Icosahedron.*

Figure 6 shows the same icosahedron, but with the thin edges hidden. A task was assigned to the students: When given part of the edges of the icosahedron (the part one can see from the front view) as shown in Figure 6, please draw the rest of the edges based on your spatial visualization.

Most of the students had difficulty in drawing the missing edges in the first place because they lacked the necessary three-dimensional spatial visualization. However, After they dragged the free point on the circle, or clicked the Rotation button, the views of the icosahedron from all possible perspectives stimulated their visualization needed for completing the task. They realized that there were pairs of pentagons (such as pentagon ABCDE and its not-shown “partner”) involved. The two pentagons in each pair were parallel to each other (in terms of the planes containing them), and their corresponding sides were also parallel to each other. With this idea, the students first tried to locate the missing pentagons and then successfully got all missing edges drawn. Through a series of activities like this one, the students’ three-dimensional spatial visualization has been significantly improved.

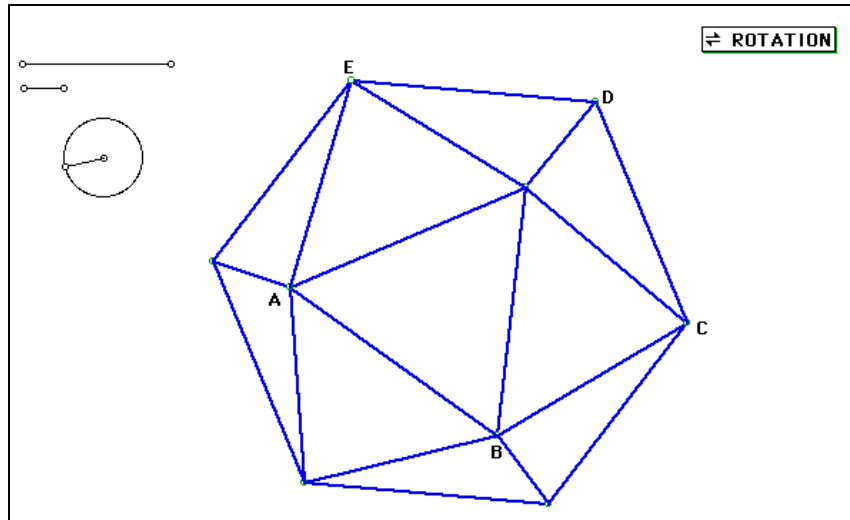


Figure 6. Can you draw the rest of the edges of this icosahedron?

CONCLUSION

JavaSketchpad is a work in progress at the present time, but its current version is already an excellent tool for students, teachers, educators, and researchers to explore geometry and share geometric ideas over the Internet. I have used JavaSketchpad to help reach my curriculum goals by extending in-class GSP exploration activities to times other than the class time and places outside the classroom including students' homes. My students, especially those who do not own desktop GSP software, have enjoyed considerable convenience in exploring dynamic geometry without coming to a specific lab at specific times. In addition, they have been provided with many more opportunities to learn creative geometric ideas from the educators, researchers, and outstanding mathematics students inside or mostly, outside the campus. Some of our more advanced students have been creating innovative Java sketches and sharing them with outside people through our own home pages. Seeing the endless usefulness of GSP, we hope that the completed version of JavaSketchpad will be released soon, and its stabilized features will contribute much more to mathematics teaching, learning, and research.

REFERENCES

- Dixon, J. (1997). Computer Use and Visualization in Students' Construction of Reflection and Rotation Concepts. *School Science and Mathematics*. Vol. 97, No. 7, pp.352-358.
- Jackiw, Nicholas. (1995). *The Geometer's Sketchpad*. Berkeley, CA: Key Curriculum Press. Software.
- Jackiw, Nicholas. (1997). *JavaSketchpad*. Berkeley, CA: Key Curriculum Press. Software.

Jackiw, Nicholas. (1998). *JavaSketchpad: Dynamic Geometry for the Internet*.
<http://www.keypress.com/sketchpad/index.html>.

Jiang, Z. & McClintock, E. (1997). Using the Geometer's Sketchpad with Preservice Teachers. In Geometry Turned On. Washington, DC: The Mathematical Association of America.

Rodgers, J. (1997). A Constructivist Approach to Geometry with GSP as Measured by van Hiele: A Qualitative Study. Master thesis at Florida International University.

Zhang, J. (1999). Icosahedron, the Geometer's Sketchpad and Golden Ratio: Dynamic Geometry Environment in Progress. Master thesis at Florida International University.