Computing of Focal Quantities for E_3^1 Systems Using Wu elimination

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In this paper, we consider the following cubic polynomial differential system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + Ax^2 + 3Bxy + Cy^2 + Kx^3 + 3Lx^2y + Mxy^2 + Ny^3 \end{cases} \tag{1}$$

The problem was motivated from the famous Hilbert 16th question. A lot of work on quadratic systems have been done. Now, many mathematicians have turn their interest to cubic systems. For the purpose of analysing the number of limit cycles, it is necessary to calculate the focal values. However, it is more complicated and difficult, so that much attemp to perform the calculations by hand is bound to fail. Therefore, mathematicians indicate that a special symbolic computing technology must be developed.

Using Wu elimination and Poincaré method, we design a special algorithm to calculate the focal value for system (**??**). This algorithm is acheived by programming a MATHEMATICA program, and we obtained some satisfactory results.

By our algorithm, we obtained expressions of the first 7 focal values for system (**??**). And hence, it is follows that a series of conclusion on the degree of the origin $O(0,0)$ as a fine focus. For example, we got the following theorem.

Theorem 1 *Let $a_0 = A + C$, $a_5 = A - C \neq 0$. Then the origin is a 6-th fine focus if and only if the following conditions are satisfied.*

1. $L + N = 0$;
2. $M = -4a_0^2 - a_0a_5 - 3K$;
3. $N^2 = (123a_0^4 - 26a_0^3a_5 - 25a_0^2a_5^2 - 2a_0a_5^3 - 24a_0^2K - 48a_0a_5K - 4a_5^2K - 12K^2)/12$;
4. $10234a_0^6 - 2283a_0^5a_5 - 2008a_0^4a_5^2 + 31a_0^3a_5^3 + 54a_0^2a_5^4 + 2a_0a_5^5 - 3354a_0^3a_5K - 20a_0^2a_5^2K + 140a_0a_5^3K + 4a_5^4K + 80a_5^2K^2 = 0$;
5. $a_0 = t_0a_5(K = K_1)$; $\quad a_0 = t_ia_5(K = K_2)$, $\quad i = 3, \cdots, 8$;

Moreover, if the first $i-1$ conditions are satisfied and the i-th condition is not satisfied, then the origin is an i-th fine focus.