MATHEMATICA FOR THE BEGINNER

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Abstract

In this tutorial we will explain some elementary aspects of the computer algebra system *Mathematica* 3.0. The tutorial is mainly divided into five sections : Arithmetic and Numerical Calculations, Algebra, Calculus, Graphing in Two and Three Dimensions, and Animations. In the first section Arithmetic and Numerical Calculations, we will cover the topics such as exponentiation, square roots, logarithms, absolute values, factoring integers, and approximating numbers to arbitrary precision. This section at the same time provides the user a feel for the syntax of the *Mathematica* language. In the second section covering Algebra, we will show how to factor, expand and simplify algebraic expressions. One highlight in this section is how to use *Mathematica* to mimic the functional notation. In the same section, we will show how to solve single equations and systems of equations. In the third section on Calculus, we discuss differentiation, indefinite and definite integration, and limits of functions. The fourth section covers one of the most important aspects of the tutorial, Two and Three Dimensional Graphing. Starting with graphing a simple function, we will discuss in detail very many aspects of graphing. Among them include various plot options in *Mathematica*, such as, "PlotStyle", "AspectRatio", "Axes", "AxesLabel", "Frame", "PlotLabel", etc. We will also explain parametric and implicit plotting as well. The final section is devoted to showing how to use *Mathematica* as an animation tool.

■ 1. Arithmetic and Numerical Calculations

1. Fraction calculations

```
(3 / 2 + 4 / 5) / (-2 / 3 + (5 / 6) ^2)
```

```
414/5
```

2. Approximations

```
N[Sqrt[2], 100]
```

```
1.41421356237309504880168872420969807856967187537694807317667973799073247\
8462107038850387534327641573
```

3. Approximating other radicals and exponents

```
N[3 (5)^(1 / 3) + 23^4, 13]
```

```
279846.1299278
```

4. Calculations involving pi and e

```
N[Pi, 200]
```

```
3.14159265358979323846264338327950288419716939937510582097494459230781640\
6286208998628034825342117067982148086513282306647093844609550582231725359\
4081284811174502841027019385211055596446229489549303820
```

```
N[Exp[1], 100]
```

```
2.71828182845904523536028747135266249775724709369995957496696762772407663\
0353547594571382178525166427
```

5. Absolute values

```
Abs[E - Pi]
```

```
-E + Pi
```

```
N[%]
```

```
0.423311
```

6. Logarithms

```
N[Log[100], 12]
```

```
4.60517018599
```

```
N[Log[10, E], 12]
```

```
0.434294481903
```

```
Log[2, 8]
```

```
3
```

7. Trigonometric functions

```
N[Sin[30]]
```

```
-0.988032
```

```
Sin[30 Degree]
```

$$\frac{1}{2}$$

```
Cos[Pi / 4]
```

$$\frac{1}{\text{Sqrt}[2]}$$

8. Complex numbers

```
(2 + 3 I) (4 - 2 I)
```

```
14 + 8 I
```

```
Abs[14 + 8 I]
```

```
2 Sqrt[65]
```

■ 2. Algebra

1. Factoring polynomials

```
Factor[x^8 - 1]
```

$$(-1 + x)\ (1 + x)\ (1 + x^2)\ (1 + x^4)$$

```
Factor[x^12 - y^12]
```

$$(x - y)\ (x + y)\ (x^2 + y^2)\ (x^2 - x\,y + y^2)\ (x^2 + x\,y + y^2)\ (x^4 - x^2\,y^2 + y^4)$$

2. Expanding polynomials.

```
Expand[(2 x - 3 y)^10]
```

$$1024\,x^{10} - 15360\,x^9\,y + 103680\,x^8\,y^2 - 414720\,x^7\,y^3 + 1088640\,x^6\,y^4 - 1959552\,x^5\,y^5 + 2449440\,x^4\,y^6 - 2099520\,x^3\,y^7 + 1180980\,x^2\,y^8 - 393660\,x\,y^9 + 59049\,y^{10}$$

```
Factor[%]
```

$$(2\,x - 3\,y)^{10}$$

3. Simplifying algebraic expressions

```
Simplify[x (1 + x) + (2 x - 3)^2]
```

$$9 - 11\,x + 5\,x^2$$

```
Simplify[1 / (1 + x) + 1 / (1 - x)]
```

$$\frac{-2}{-1 + x^2}$$

3. Evaluating expressions at a given value

```
2 x^3 - 3 x^2 + 4 x + 5 /. x -> 3
```

```
44
```

```
(2 x * y) / Sqrt[x^2 + y^2] /. {x -> 2.3, y -> 4.3}
```

```
4.05621
```

4. Functional notation and evaluating functions

```
f[x_] := 2 x^2 - 3 x + 5
```

```
f[3]
```

```
14
```

```
g[x_, y_] := (x - 2 y) / Sqrt[x^2 + y^2]
```

```
g[3, 4]
```

```
-1
```

5. Making a table of values for a given function

```
Table[2 x^2 - 3, {x, -5, 5, 0.5}]
```

```
{47, 37.5, 29., 21.5, 15., 9.5, 5., 1.5, -1., -2.5, -3., -2.5, -1., 1.5, 5.
  21.5, 29., 37.5, 47.}
```

6. Summations and products

```
Sum[i^3 - 2 i^2 + 5, {i, 1, 12}]
```

```
4844
```

```
Product[2 i + 3, {i, 1, 12}]
```

```
71152682225625
```

7. Solving equations

```
Solve[3 x^2 - 2 x - 1 == 0, x]
```

```
{{x -> -(1/3)}, {x -> 1}}
```

```
x /. %
```

```
{-(1/3), 1}
```

```
Solve[x^4 + x^3 + 6 x^2 + 8 x - 16 == 0, x]
```

```
{{x -> -2}, {x -> 1}, {x -> -2 I Sqrt[2]}, {x -> 2 I Sqrt[2]}}
```

```
NSolve[(2 x + 1) ^2 - 1 == x^3 + 2 x, x]
```

```
{{x -> -0.44949}, {x -> 0.}, {x -> 4.44949}}
```

```
FindRoot[(2 x + 1) ^2 - 1 == x^3 + 2 x, {x, 6}]
```

```
{x -> 4.44949}
```

8. Solving systems of equations

```
Solve[{2 x + 3 y == 4, -3 x + 4 y == 1}, {x, y}]
```

```
{{x -> 13/17, y -> 14/17}}
```

■ 3. Calculus

1. Derivatives of functions

```
D[x^4, x]
```

$$4\ x^3$$

```
D[x^4, {x, 2}]
```

$$12\ x^2$$

Here is another method of finding and evaluating derivatives of a function.

```
f[x_] := x^4
```

```
f'[x]
```

$$4\ x^3$$

```
f''[x]
```

$$12\ x^2$$

```
f'[5]
```

```
500
```

2. Indefinite and definite integrals of functions

```
Integrate[x^2, x]
```

$$\frac{x^3}{3}$$

```
Integrate[x^2, {x, 1, 2}]
```

$$\frac{7}{3}$$

```
NIntegrate[Sin[x^2], {x, 0, 1}]
```

```
0.310268
```

3. Limits of functions

```
Limit[(x^2 - 4) / (x - 2), x -> 2]
```

```
4
```

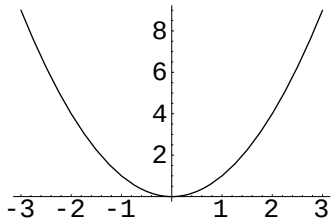
```
Limit[1 / x, x -> Infinity]
```

```
0
```

■ 4. Graphing Functions

1. Single variable functions - Basic plot command

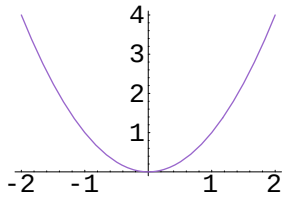
```
Plot[x^2, {x, -3, 3}]
```



-Graphics-

2. Single variable functions - Plots with styling

```
Plot[x^2, {x, -2, 2}, PlotStyle -> {RGBColor[0.6, 0.4, 0.8]}]
```



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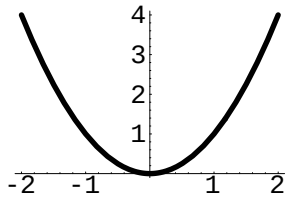
```
Plot[x^2, {x, -2, 2}, PlotStyle -> {Thickness[1 / 50]}]
```



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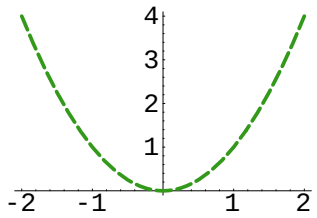
```
Plot[x^2, {x, -2, 2}, PlotStyle ->
 {{RGBColor[0.2, 0.6, 0.1],
    Thickness[1 / 80], Dashing[{0.05, 0.02}]}}]
```



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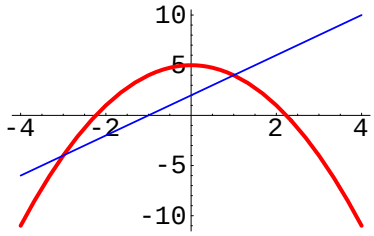
3 . Graphing more than one function

```
Plot[{5 - x^2, 2 x + 2}, {x, -4, 4}, PlotStyle ->
{{RGBColor[1, 0, 0], Thickness[1 / 90]},
{RGBColor[0, 0, 1], Thickness[1 / 200]}}]
```



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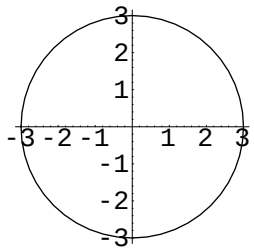
4. Implicit plots

```
<< Graphics`ImplicitPlot`
```

```
ImplicitPlot[x^2 + y^2 == 9, {x, -4, 4}]
```



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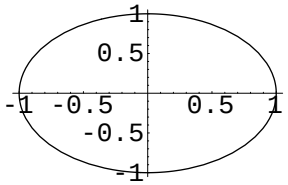
5. Parametric plots

```
ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi}]
```




```
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```
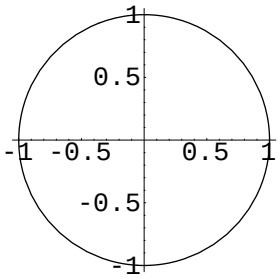

```
Show[%, AspectRatio -> Automatic]
```




```
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```
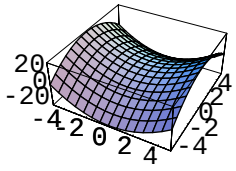

6. Graphing two variable functions

```
Plot3D[x^2 - y^2, {x, -5, 5}, {y, -5, 5}]
```




```
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```
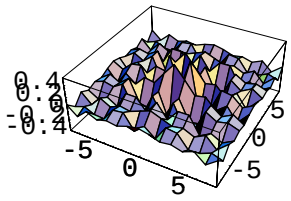

```
Plot3D[Sin[x^2 + y^2] / Sqrt[x^2 + y^2], {x, -8, 8}, {y, -8, 8}]
```

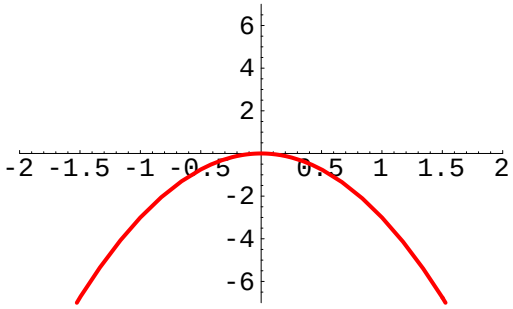



```
-SurfaceGraphics-
```

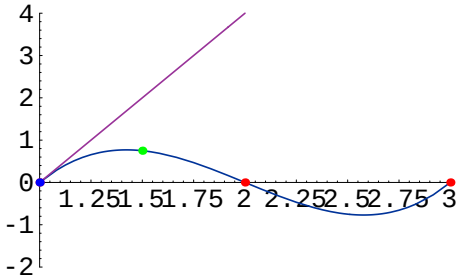
■ 5. Animations

1. Animating a parabola

```
Do[Plot[k * x^2, {x, -2, 2}, PlotRange -> {{-2, 2}, {-7, 7}},
PlotStyle -> {{Thickness[1 / 120], RGBColor[1, 0, 0]}}],
 {k, -3, 3, 0.5}]
```



2. Cubic polynomials, roots and tangents



3. Animation of a 3-D surface

```
Do[Plot3D[k*Sin[x^2 + y^2]/Sqrt[x^2 + y^2], {x, -8, 8},
{y, -8,8}, PlotRange -> {-7, 7}], {k, -7, 7, 0.5}]
```