Developing the Curriculum for Curves Using History and Technology

Enhancing the Power of Historical Ideas Through the Use of Manipulative Representation Tools: Dynamic Geometry Software and LEGO

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For trying to integrate curriculum among geometry, algebra and calculus via technology, the historical roles of technology in mathematics was focused. History shows us the importance of technology for the recursive processes of conceptual changes. David Dennis and Jere Confrey have demonstrated the missing loop for integration by comparing mathematics curriculum with history. The Japanese integrated curriculum lost in World War II tried to use tools but unfortunately it shows the importance of tools because there were no appropriate tools at that time. In this paper, to connect the loop, I show that Dynamic Geometry Software enhances and realizes Descartes dream. I also demonstrate how historical tools and new technology emerge mathematical concepts hidden in tools. Through the discussion, I proposed the new trend of curriculum reform using Dynamic Geometry Software and LEGO for integrating geometry, algebra and calculus via technology.

Inference is Enhanced and also Limited by Technology.

We can not calculate in a notebook if we do not have any paper. It shows that paper enhances our inquiry. In the case of epistemological obstacles, we usually focus on concepts as notions. However we also know that some obstacles arise from tools, technology, and environment such as discussed by Gaston Bachelard (1938). We believe that after we construct a mathematical object on mathematics, the tool for mathematical inquiry itself is reformulate as a mathematical method (see, H. Freundenthal, 1973). Therefore, we usually focused on concepts but the computer enable us to execute these higher mathematical methods by technology. Now, mathematicians usually use computers as tools. Writers usually emphasize inquiry or the laboratory approach in mathematics rather than the structure of Mathematics itself, like the age of Nicolas Bourbaki (J. Horgan, 1993).

In the field of education, Ernest von Glasersfeld has claimed that knowledge is actively built up by the cognitive subject. Realizing the claim in education is not easy if the pupil does not have any appropriate activity tool or any appropriate environment. Davit Tall mentions that computers enable us new sequences of ideas for teaching or learning to avoid cognitive obstacles but they introduce new obstacles depending on the sequences (1989).

Throughout history there are many similar situations. The construction problems are a typical historical example of these situations. In the beginning, the ancients tried to find some kinds of curves to represent the quantity, or get the segment, of geometric mean to enable the construction. We can guess that many of them were familiar with mechanics, linkages, for drawing curves in the process of analysis because they had already constructed well curved Parthenon Shrine, but neither Euclid nor Platonism did not respect to mechanics and requested that only the ruler and compass should be used for describing mathematics synthetically based on the 'Elements'. Pappus taught how to analyze the problems in his Correction VII and discussed mechanics in VIII (Heath 1921). We suspect that these type of books, especially the books of Archimedes who also wrote heuristics and mechanics, were corroded or lost in the age when New-Platonism school selected books to copy from papyrus onto parchment. So readers who read the books that survived, such as Descartes, lamented the loss of heuristics. In the beginning, the ruler and compass and other mechanics enhanced inquiry, but the limitation of the ruler and compass created limitation for futher generations. Descartes broke this limitation and tried to construct Universal Mathematics based on his algebraic approach. This approach provided a new paradigm but it robbed the attractive feature of many ancients' problems through the reformulation of problems. Hundred years later, algebra and calculus are tools for describing and inquiring curves. Of course there are limitations to describe curves using algebra. Ordinary students usually can not interpret algebraic forms easily and lose the opportunity of knowing the beautiful nature of curves through manipulation as Descartes did. We lament the loss of the intuition of Descartes (I will discuss this point later), which has been lost by the generalization of his method.

Missing Loops Between Geometry, Calculus and Algebra.

Figure 1 is a part of a figure by David Tall when he discussd actions and objects in the building of various mathematical knowledge structures. We usually draw similar figures to explain curriculum constitution. But if we compare this figure with the historical flow or schema such as Figure 2, we know that there are missing loops. Curves were originally geometrical objects drawn by tools when the ancients first began to inquire. In the US, David Dennis and Jere Confrey focused on this point. David Dennis stated that, "Analytic geometry could be presented as a feedback loop between the tactile, geometric world of curves and the semiotic world of

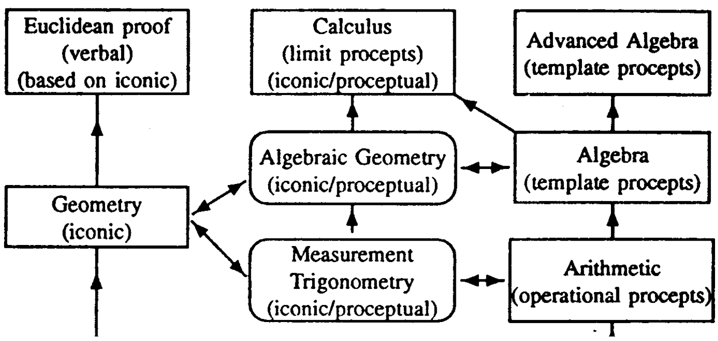


Figure 1 by David Tall (1995)

algebra and numerical data. (An omission of a middle part) How can we construct the missing half of this feedback loop?". They focused on the history of curves in relations to curves drawing tool such as linkages and Dynamic Geometry Software (DGS) in order to develope their curriculum.

In Japan, the curriculum reform movement including this idea but DGS did not existed in the age of World War II. Modernization generations like myself do not know anything about it because we already lost it. Figure 3 was sited in a Japanese book "Current Ideas for Mathematics Education and Curriculum Reform" by Minoru Kuroda. He was also famous and well known as a writer of integrated textbooks in the1920's. The idea of Figure 3 is important for integration between geometry and calculus. In the curriculum reform for secondary schools in 1942, the goal was to learn calculus until the grade of fifteen (or sixteen) years old and the aim was to cultivate mathematical thinking. The method used for integrating different subjects was mathematization. The idea of integration was function and functional thinking (an example of this idea is shown in Figure 3). To teach students such higher concepts, the devices for inquiring tools such as linkages and mechanics (Figure 4) were selected for exploring situations. The textbooks were developed that students would construct mathematics through mathematization which included the inquiry of these situations using hands on tools as shown in Figure 4. But this curriculum did

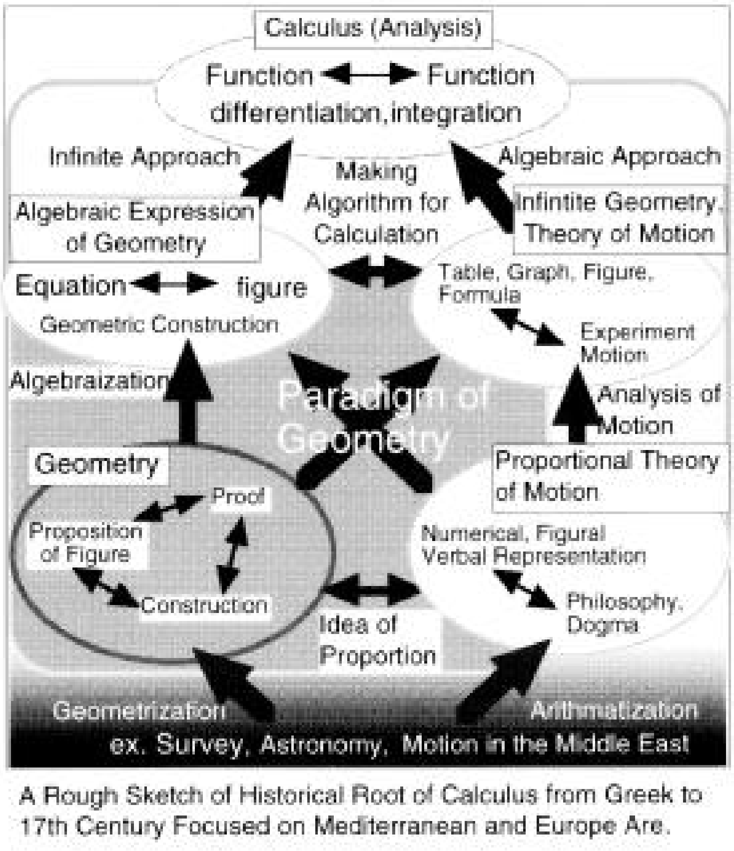
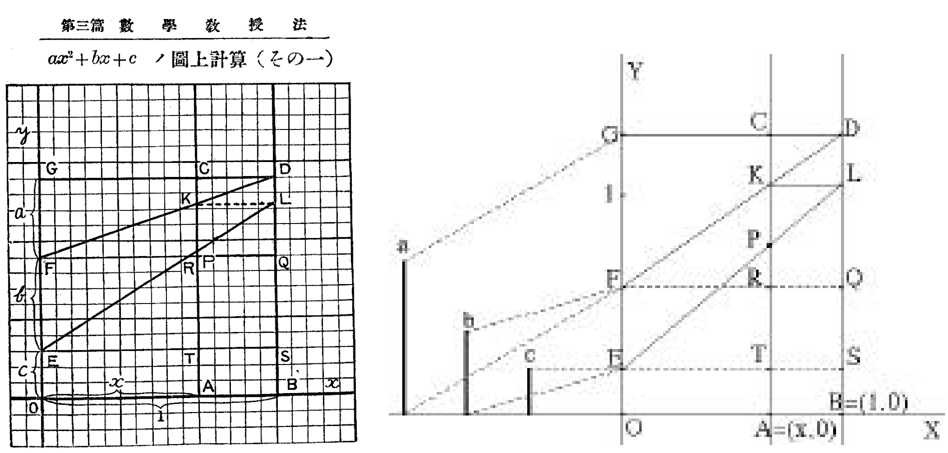
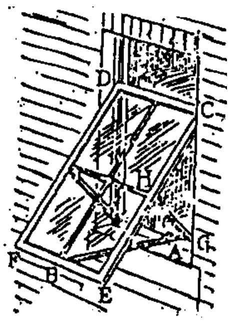


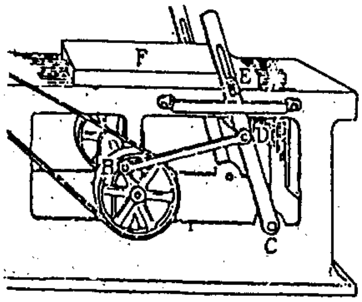
Figure 2 by Masami Isoda (1996)



If A= (x, 0), B= (1, 0),
FG= a, EF= b, OE= c,
Then
RK : a = x : 1,
RK = a x,
SL = b + a x,
SL : PT = 1 : x,
PT = $a x^2 + b x$.
Thus AP= $a x^2 + b x + c$.

Figure 3. by Minoru Kuroda (1927) (Redrawned by Isoda. 1998)





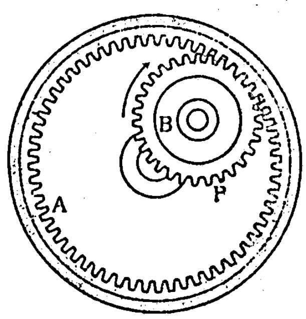
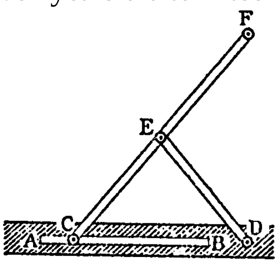
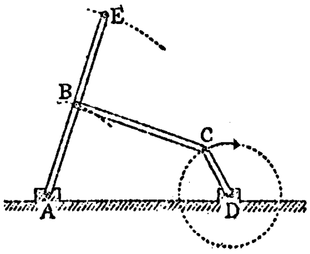
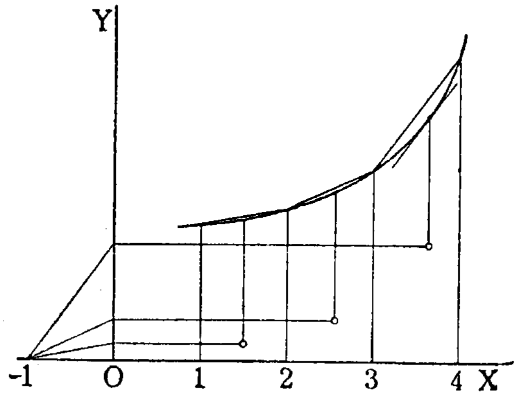


Figure 4. All of figures are sited from mathematics textbooks written for secondary school students from eleven years old to fifteen years old in 1943 and 1944.







not put into well practice because in 1944 almost all these tools were burned or hidden in shelters and students went to factories in shelters and used the tools for work and not for learning mathematics. After the end of the war, in 1945, these positive ideas could not keep at the difficult situation that having only a few tools in the factories and the occupied nation have to change the ideas by the claim of the General Headquarters of the Allied Powers (US).

From history we can learn how to connect the missing half of the loop in our curriculum that we should promote the appropriate activity using the appropriate tools. For this connection, David Dennis and Jere Confrey used linkages and DGS. Colette Laborde (1996) discussed the significance of DGS to represent linkage mechanisms and Maria G. Bartolini Bussi has been engaging in innovative projects using linkages and DGS since the early 1980's. At ATCM'97 held in Penang, Malaysia, we saw similar research contexts. Jen-chung Chuan also discussed Horner's Method such as in figure 3 and showed an idea of linkages which I also presented at Penang. Barry McCrae presented Fermat's Point Problem which also have the view of history. In the following papers, I will discuss some of historical ideas which demonstrate how to make connections with the missing half of the loop using linkages and DGS.

A Problem Posing by Descartes's Using Linkage

In Geometry (1637), Descartes began to show the constructions such as the multiplication (figure 5), the quotient, and the extraction. If we use his idea for using DGS, we can easily draw (construct) any algebraic equations like a construction of Figure 3. Thus, if he had used DGS, he may able to spread his Universal

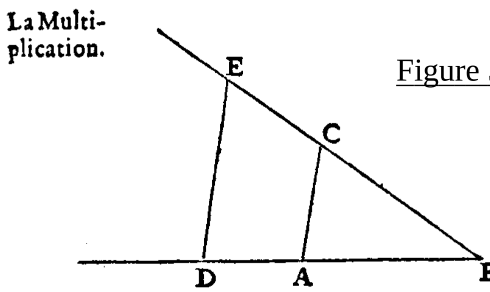


Figure 5

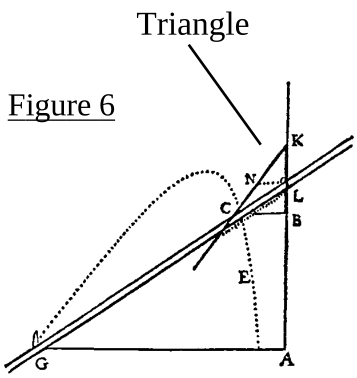


Figure 6

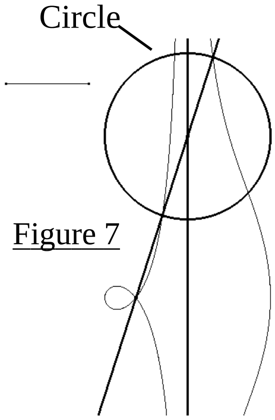


Figure 7

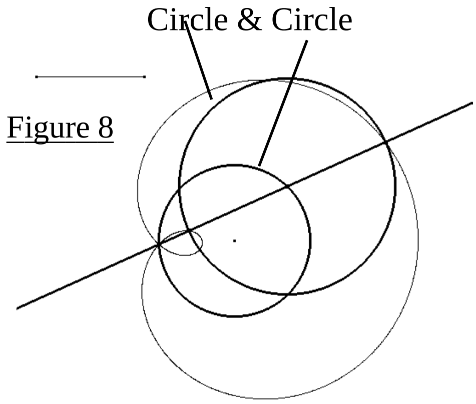


Figure 8

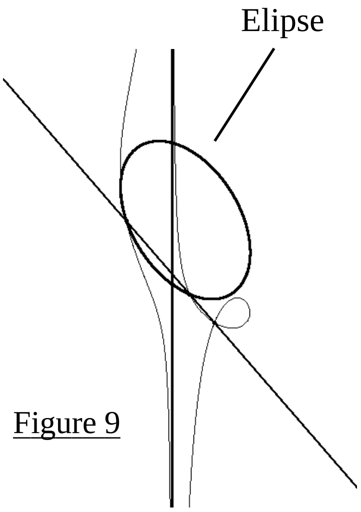


Figure 9

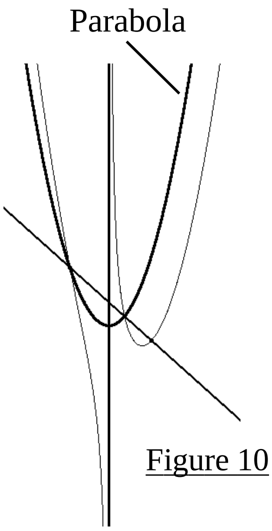


Figure 10

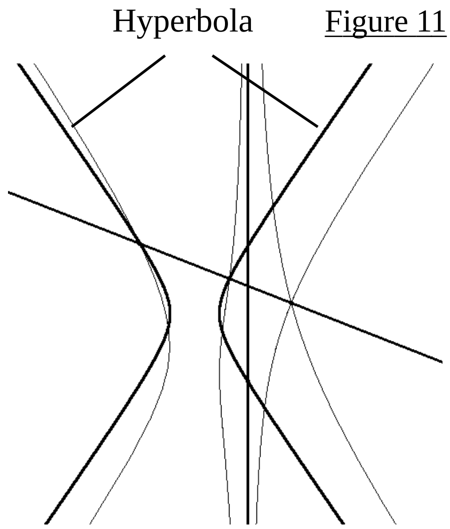


Figure 11

Mathematics more and more. But in the age of Descartes, curves were only figures defined by geometry and drawn using devices such as ruler and compass, linkages, and mechanics, etc. In Geometry, after he demonstrated the ancients' geometric construction problem for getting a segment which could be solved by his algebraic method, he discussed the power of his methods using curves which ancients like Pappus did not discuss well. In these discussion, we find his methods of problem formulating using devices as follows. First, he applied his methods to inquire curves made by the linkage using a triangle (Figure 6, 1637). After he discussed that this curve is hyperbola, he change the condition as follows: "If CNK be a circle having it center at L, we shall describe the first conchoid of the ancients (figure 7, but not the first one), while if we use a parabola having KB as axis we shall describe the curve which is the first and simplest of the curves required in the problem of Pappus (Figure 10)" By changing conditions, he discussed other curves using linkages and generalized the ancients' idea of concoide. We can do the same process easily using DGS like figures 7 to 11 (by Cabri Geometry II). We should know that these constructions are done using the same idea, the only difference is that the parts of mechanics (figures) shown in figures 6 to 11.

Now, teachers and students draw an algebraic graph and observe it via changing parameters using Graphing Software. But students usually find it hard to understand the meaning of algebraic equations such as conchoid (Figure 7) and rimason (Figure 8) because they can not find the reason how to make such an equation. When students are engaged in Descartes type activities, they can

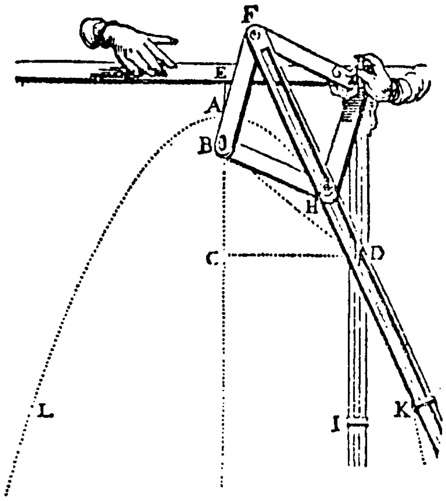


Figure 12

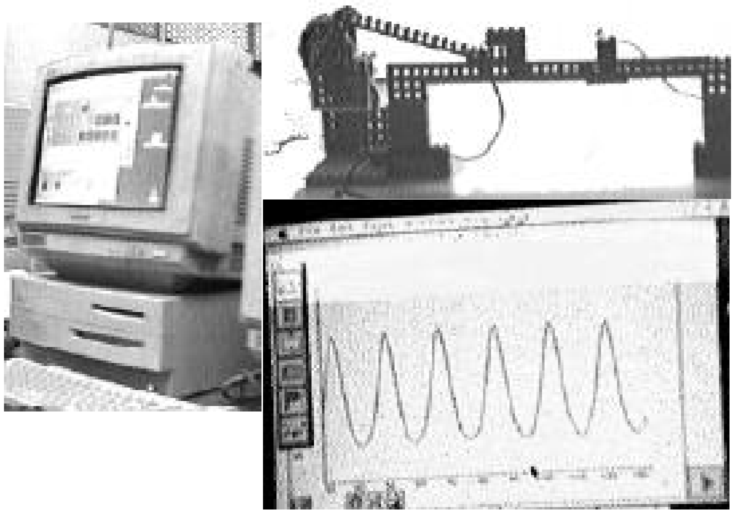


Figure 14. Motion of Crank Mechanism

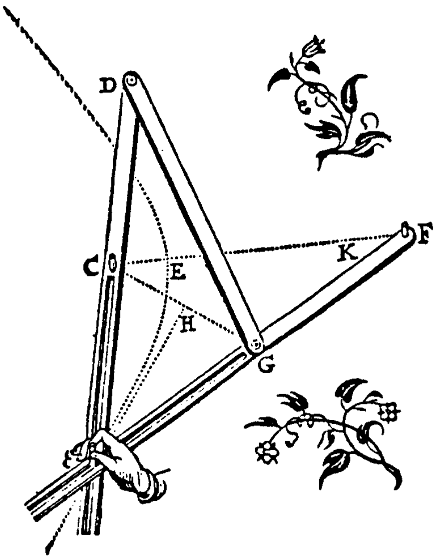


Figure 13

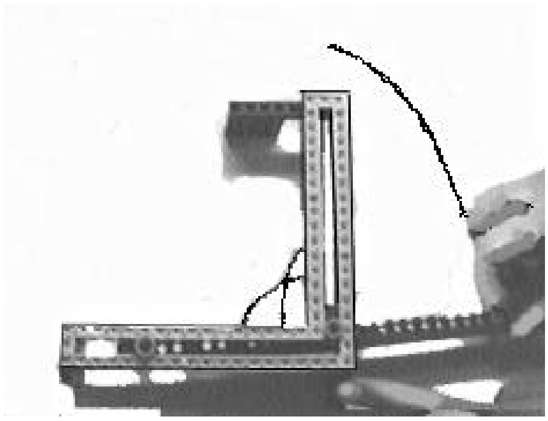


Figure 15

recognize curves and can represent them algebraically through their geometrical observation and reasoning. This type of activity is one of the activities for the feedback loop.

Higher Mathematical Concepts Hidden in Tools.

We can see several linkages and curve drawing mechanics on the internet site (http://155.185.1.61:80/labmat/usa1.htm) offered by Maria G. Bartolini Bussi (my group translated the Japanese version with her permission: http://130.158.186.11/mathedu/mathedu.html). Each tool represents a mathematical concept or a definition itself. But more important point is that we can find the hidden concept through manipulating of the tool.

Figures 12 and 13 are van Schooten’s linkages (1657, re-sited from David Dennis 1995 and Jan A. van Maanen 1995). In figure 12, we can find a tangent of a parabola as a part of a linkage. We do not have to teach this part as a tangent, but students should know the reason why we can draw a parabola using the linkage of figure 12. Through the drawing, students can easily find it as the tangent and can inquire how to draw it as such as discussed by Apollonius of Perga and Galileo

Galilei. If we change the directrix to a circle, like the changing of parts done by Descartes, we can find other conic sections. Figure 13 demonstrate another method of drawing. The linkage of figure 13 is a typical mechanism named a four-linked mechanic as shown in figure 4. In this mechanism, the operating-handed point in figure 13 is the center in a moment of which Izak Newton discussed in Principia Mathematica (1687). The extra case of figure 13, a locus of the midpoint on DE is the lemniscate of Jakob Bernoulli. We do not have to teach these concepts when students inquire the curve using the devices of figure 13, but students should reflect on this experience when students can discuss them more mathematically. The visual and manipulative feature of these devices helps student to reflect on their own experiences.

Today, we have mechatronics which enable us to discuss these hidden concepts as the subject matter. The Jere Confrey group and my group used LEGO for such a mechatronics. LEGO enable us numerical observations of the workings of linkages (Figure 14) as well as geometric curve drawings and observation such as the linkage of Figure 15.

Final Remarks

To connect the missing half of the loop among curriculum is one of most fruitful area to use history for understanding how mathematics is connected and how the historical idea was enhanced via tools of earlier times. One of meaningful uses of technology should be supported by studying such a historical view including tools.

Ten years ago, NCTM standards (1989) enhanced exploring algebra using graphing tools. In the reform process of it, there are several strong objections from the perspectives of algebraic reasoning and proofs which has been weakened by enhancement of the graphing tool. On the other hand, I have discussed here that there are several movements which enhance curve drawing using DGS have try to connect the missing loop. This discussion noted as important in the new movement of the world which includes students' algebraic and geometric reasoning.

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