Impacts of Using Calculators in Learning Mathematics

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Abstract.

This paper argues that using calculators in learning mathematics may have negative effects if they are used inappropriately: 1. Students may lack conceptual understanding; 2. The procedural perception of a mathematical problem of those students who did not go through a successful structural development in learning algebra could be reinforced; 3. Sometimes the calculator delivers misleading information; 4. Students may develop undesirable problem solving behavior; 5. Differences from conventional notation and notation used with calculators may confuse students. Suggestions aimed at reducing those problems are advanced.

Anyone who presumes to describe the roles of technology in mathematics education faces challenges akin to describing a newly active volcano. (Kaput, 1992, p. 515)

During the debate on allowing children to use calculators in learning mathematics in the 70s and early 80s, the main concern was: If we do not prohibit students from using calculators, will a student possess adequate competence in mathematics once deprived of the device? The debate gradually faded away probably because of the following reasons. First, a bulk of experimental studies seemed to suggest that the students who used calculators frequently were not inferior in arithmetic abilities to those who did not. They even gained in other academic aspects. Second, the rapid development of technology has been penetrating almost all sectors of the society and makes any resistance almost impossible. Third, as a consequence, people realized that it is wiser and more important to innovate mathematics curriculum to adapt to the technology (NCTM, 1989).

Today, the consensus is to enhance the use of technology in learning mathematics. Developing technology-rich activities becomes a fashion. The learning situations portrayed in these activities are almost always rosy. However, as we rush to catch up with the fast pace of the movement, there are questions for which we do not have satisfactory answers. The core of the early concern remains, though at a different level and to a different extent. The whole issue deserves further study because most of the research in 70s and 80s investigated the impact of using simple calculators. Today the problem is more complicated because more advanced calculators, e.g. graphing calculators, are easily available and widely used.

It may be true that technology brings the greatest promise so far for education. It may also be true that the promise comes along with its twin: the risk. There are such lessons. For instance, more than a decade ago, in name of *Computer Aided Instruction*, some students were once left in the front of screens to solve tens even hundreds of similar problems generated by impersonal computers with little help. The result was frustrating. So, the following questions must be answered. Do our students know how to use calculators properly? Is it possible that students will easily develop some misconceptions when they use calculators? Will using the technology foster some particular problem solving behaviors that are undesirable? Are there conflicts between mathematics presented on calculators and mathematics learned from other sources? These questions are universal. Mathematics educators are bound to offer their answers.

This paper raises awareness of the issues mentioned above. Examples introduced were gathered in an on-going study conducted in a four-year college in the United States. During the study, a survey showed that 71% of the mathematics majors in a junior level course either had graphing calculators or had experience with the device. In another two service courses, one third to one half of these non-mathematics majors owned graphing calculators. These data indicate that assessing the impact of graphing calculators is in an urgent need because it is reasonable to anticipate that the number of students who own calculators will continue to grow. The research is also important as more and more textbooks and institutions adopt graphing calculators in their mathematics curricula. In fact, one aspect of the *Calculus Reform* movement is to promote use of calculator for this foundation course.

Conceptual understanding vs. skill

The demand for access to technology by every student rests partly on the argument that algorithms are not the core of the mathematics curriculum, and that there is no need for pencil and paper methods if a calculator is available. There were discussions in 80s about procedural and conceptual knowledge (Hiebert, 1986), instrumental and relational understanding (Skemp, 1987), and so called higher-order thinking (Resnick, 1987). It has been advocated that students should be engaged in more meaningful activities in which they will be able to develop originality and creativity. Calculators, especially graphing calculators, play dual roles in the light of this campaign. It liberates students from rote practice and at the same time provides students with opportunities to *do mathematics*, a phenomenon that emphasizes the similarity between mathematics and other science disciplines in discovering new knowledge. However, when the technology takes over more mathematics, what is left for students? When complicated tasks are accomplished instantly, the embodied concepts are often wrapped up and become opaque to users. The efficiency of getting an answer from a calculator may cover a lack of conceptual understanding of a subject matter.

Here is a thought-provoking example. In answering $16^{-1/2} = ?$, which was on a pre-test for a calculus class, many students responded *0.25*, instead of *1/4*. The decimal answer, as the students admitted, came from using calculators. Although calculating such a power is quite easy for many, the procedure embodies considerable uncertainty for those students who bypassed the mental execution by punching in keys to get the answer. Later, there was evidence showing that some of these students did have difficulties in understanding negative and rational exponents and the rules of exponents.

In another class, when ten students (non-math majors) who owned TI-82 graphing calculators were asked if they knew how to graph $y = \sin x$, all of them answered yes. Further, when asked who knew the properties of the function, none of them could come up with an answer. Two students uttered just a little about the shape of the function but nothing analytic.

The examples tell us that conceptual understanding may be missing even though students can use the technology skilfully. Therefore, when we use calculator to create learning activities, we should not neglect the underlying concepts and solely rely on the visual effects. For example, students can be guided to find mathematical relationships such as

$$\sin^2 x + \cos^2 x = 1, \quad \text{or} \quad \sin(90^\circ - x) = \cos x,$$

through using a graphing calculator. But the “discovery” may seem to be artificial. The understanding of the relationships will be reinforced if we can back it up with an analytic approach. Otherwise the understanding is not very desirable from the cognitive perspective.

Procedural vs. structural perception

According to Kieran (1992), students often experience enormous difficulties in their transition from learning arithmetic to learning algebra. She pointed out that this happens when a student does not have a structural view of the subject and holds a procedural view instead. She argued that algebra should be conceived of as a branch of mathematics that deals with general numerical relationships and mathematical structures together with the operations functioning on those structures. As an indication of students’ difficulties, they are uneasy in handling algebraic entities. When asked what is *AB* in Figure 1 where *AC = 8* and *CB = 2x*, young students might offer *16x* or *10x* as answers. Some may even come up with a single number. Another often observed phenomenon is that the input-output procedural notion of function is much readily accepted by students than other notions. These difficulties may be so persistent that they could follow some students through their college years. Actually, a big complaint of the college instructors is the poor algebraic preparation of their students.

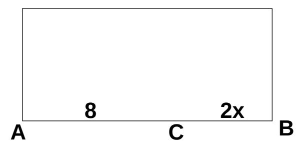

Figure 1

Although the calculator creates new teachability and learnability, the way that it attacks a problem is basically numerical in nature. For example, on a TI-80's series calculator, no matter if it is finding a zero, searching for extremes of a function, or getting the determinant of a matrix, it always proceeds in numerical forms. Consequently, for those students who are very number-oriented, calculators provide them with a feeling of safety and an "outlet". In fact, to many teachers, it is not a surprise to get the following response to a coming examination: "Can we use calculators?" If the permission is given, these students will feel relief because at least they can come up with "something" in the test. Using calculator will also reinforce the procedural perception of a mathematical problem for those students who did not go through a successful structural development in learning algebra. Clement and his colleagues (1981) showed that students may solve a problem either procedurally or structurally depending on different environments. They found that even when a problem involved literal terms, students still tried to interpret the problem as a procedural one if a computer was presented.

It must be pointed out that the discussion above does not mean that the structural view is superior to procedural one. Different perspectives are needed in different situations. As Kieran explained, mathematicians continuously experience transition from one perspective to the other. The progression results in the development of mathematics. Likewise, to be mathematically competent, students must go through similar transitions. Among these transitions, the one that goes from arithmetic to algebra is particularly crucial.

Which is correct ?

Any graph of a function is a human invention that visually represents a relationship between variables. Accordingly, our judgement about a graph's correctness is only based on some underlying agreements. One of the great advantages of technology is its remarkable ability to display the visual representation efficiently. When we cheer it, we are also aware that it fulfils the goal with certain limitations. For example, often a zigzag shape of a graph on a computer screen is in fact a smooth curve. It is even harder to show an accurate graph on a calculator due to the small size of its screen. Together with other technical problems, it is no wonder that distortions or even erroneous representations may occur. While teachers are usually capable of detecting such inaccuracies and mistakes, it is questionable if their students can do the same. Since today's students are so overwhelmed by high tech products and are so much used to WYSIWYG (*What you see is what you get*), they may readily accept whatever a calculator produces with little reservation. The following examples show that in using technology to explore the mathematical world, something unforeseen may happen. Students, who have little at their disposal to critique the technology, are not immune from misunderstanding.

Example*: Find the number of solutions of sin 100x = 0.5x*

The cognitive requirements for solving this problem with and without calculator are completely different. When the technology is involved, one main challenge is to pick up appropriate *window variables*. It is possible, indeed likely, to be trapped by using preset values that are inappropriate. In this case, Figure 2a shows an incorrect graph of *y = sin 100x - 0.5x* on a TI-82 graphing calculator on the interval (-3, 3). Entirely different graphs will be displayed when different windows are used (Figure 2b and Figure 2c show the graphs of the same function on (-0.5, 5) and (-0.5, 0.5) respectively).

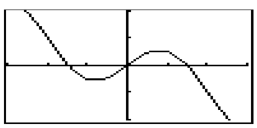

Figure 2a

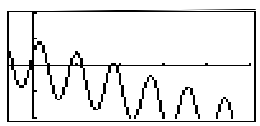

Figure 2b

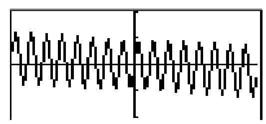

Figure 2c

Two issues deserve discussion. First, the peculiar situation and the inconsistency can really confuse students. For them, the problem may not be as simple as choosing one among the three. Having run into such a rare circumstance, they may develop a feeling of crisis: "May similar situations happen without my knowledge?" Or, "How will I know if a graph is correct from now on?" The way of knowing suddenly becomes open to question. A trusted medium and a source of knowledge can no longer be relied upon with total assurance on an adequate foundation. Second, one may argue the benefit of leading students to discern the correct from the incorrect in this problem. But an articulation of choosing appropriate window variables actually down plays the value of using the technology in the first place. If one understands the analytic feature of the problem, there is little need to display the graph. So we have a dilemma and the value of using technology becomes arguable for this problem.

In addition, limitations of the technology is also a factor that leads to inaccurate information. For example, $y = (x^2 - 5x + 6)/(x - 2)$ is discontinuous at *x = 2*. But most graphing calculators are incapable of displaying the discontinuity adequately. What is plotted is identical to that of $y = x - 3$, which is the very error that an instructor desperately wants his students to avoid.

The next example illustrates that when the limitations are not understood by students, unwanted learning outcomes can be produced. Consider the problem: "Is any member in the set {11, 111, . . 11...1, . . . } a perfect square?" posed in class. Surprisingly, one student immediately answered: "Yes." When asked which number was a perfect square and how the conclusion was reached, she pointed to her calculator that was showing:

$$\sqrt{1111111111111111111} = 1054092553\,.$$

Had she thought the problem a little carefully, she would not have made such a claim. However, she embraced the answer without any doubt.

While the previous examples suggest that the technology always strives to give us something no matter what, sometimes it failed to do so. For example, the graph of the function $y = x^{2/3}$ is not wholly displayed on TI-82. Only the part in the first quadrant is displayed (Figure 3a). To get a more complete picture, one should either rewrite the function as $y = (x^2)^{1/3}$ or $y = (x^{1/3})^2$ (Figure 3b).

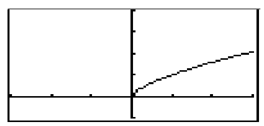

Figure 3a

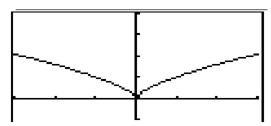

Figure 3b

All these examples show when technology is used, the user's discretion is very much needed. The responsibility can be broad and the rules are not explicit. The mnemonic burden of remembering the rules may overwhelm students. In addition to that, to apply the rules properly may require knowledge of other subjects. We have no grounds to assume that students always possess it. When we encourage students to use technology, we leave them to face multiple demands on their knowledge simultaneously rather lead them to learn in a systematic approach. This is not an easy decision and we need to be very selective.

Problem solving behavior: overusing calculator

The square root problem in the last section also brings up a phenomenon, which I call *overuse*. Clearly, it was anticipated that the problem should be solved by applying some basic knowledge of number theory. Since the answer to the perfect square problem is actually *No*, there is no way that it can be solved through testing the elements in the infinite set one by one on a calculator. That student's problem solving behavior, as many teachers have observed, is very typical. It is not unusual to find that some students were desperately searching for answers with their calculators during a test that actually has little to do with the device. In doing so, they misinterpret a problem and give up efforts to look up for other solution paths. They quickly fail in the *understanding a problem* phase (Polya, 1957). Actually, their repertoire of problem solving strategies is impoverished. *Trial and error*, through a calculator, becomes their main tactic. And a calculator, due to its nature, serves this type of student as a crutch. Inevitably, the situation leads to the overuse of the technology.

Seeing the rapid development of technology and its immense impact on various disciplines, Steen (1988) defined mathematics as "a science of patterns". According to this view, the way to study mathematics as a discipline is essentially the same as with science, i.e. “systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems” (Schoenfeld, 1992). The perspective is reflected in the call (NCTM, 1989, 1992) that we should guide students to *do mathematics*. The call argues that real learning takes place when students feel the need to question and probe problems like most mathematicians do: to explore, communicate, hypothesize, verify, revise, etc. In the process, hopefully, they will be able to identify patterns and gain insight into the problems. While Steen’s assertion and NCTM’s stand are quite general, the nature of their idea is referring to the methodology used to study the discipline. Unfortunately, there is a tendency to interpret the point in a narrow sense and overstate the function of pattern finding in mathematics education. We all know that we cannot teach students to learn all of mathematics using only a discovery approach. Employing *trial and error* and *looking for a pattern* as the primary way to learn mathematics and problem solving cannot take students very far. Part of the beauty of mathematics lies on the fact that it accommodates so many different intelligent ideas to solve various problems. Being able to solve a problem in multiple ways and to approach totally different scenarios with a same strategy are both valued. We should tell students that using calculator to look for a pattern is appropriate only for certain types of problems. And they should understand that it does not guarantee a success and it is rudimentary in many cases.

We observed that the degree of the students’ belief and dependence on the calculator is often out of bounds. It turns out to be to their disadvantage in learning mathematics. The cognitive challenge residing in the perfect square problem is well designed. But the student misunderstood the challenge to be simply a matter of testing members of the set {1, 11, 111, . . . 11...1, . . .} with the square root key on a calculator until an integer appeared. As a result, the value of the original problem was entirely lost without her knowledge. In another introductory mathematics course, the immediate reaction from some students to the question “How many consecutive zeroes are there in 100! starting from its ones place?” was resorting to their calculators. They did not see the real challenge. To their disappointment, only an error message was displayed because of the huge magnitude of the number. As the following example shows, learning to apply calculus knowledge becomes another matter when only looking for a pattern on calculator is considered. It happened in a service class.

Problem*: A walnut grower estimates from past records that if 20 trees are planted per acre, each tree will average 60 pounds of nuts per year. If for each additional tree planted per acre the average yield per tree drops 2 pounds, how many trees should be planted to maximize the yield per acre? What is the maximum yield?*

Several students solved it by comparing different yields generated from calculators. Although, as one student put it later, she knew “there should be another way” to solve the problem, when the calculator was presented, she said that she couldn’t help but use it. A simple pattern and a numerical answer is what she thought that she should pursue. In that situation, the calculator just assisted her to realize her goal quickly, but unfortunately superficially and inappropriately.

The notation systems

A salient feature of mathematics is its elaborate notation system. A massive number of symbols and terms have been adopted by mathematicians through out history. The meaning of the notations together with the rules and agreements that govern their uses are shared by the people in this particular community. The systems have become an indispensable part of mathematical language semantically and syntactically. Mastering mathematical notations is never an easy task for many students because the multiple functions of some signs and their subtle differences are not always explained explicitly and clearly to them by teachers and textbooks. This maturity takes time. It is expected that through a long journey, students will be able to acquire the meanings gradually and their misconceptions can be corrected.

When computer or calculator is used, however, learning the notations may become more difficult for two reasons. First, the intermediate process of mathematical activity of a calculator is hidden from an observer. Therefore, it is difficult to debug students' mistakes resulting from their misunderstanding of those notations. Second, the technology may furnish very different meanings and functions to the notations used in conventional mathematics. This one in particular could cause much confusion of students and produce dreadful results. It is not difficult to find many such instances.

The "=" sign is one of the most frequently used notations and has many functions. It can suggest a relation such as an identity, an equation, or an expression. It can also imply a definition and a correspondence. Generally, it conveys a logic message . Nevertheless, the "=" key used in many calculators (or it's equivalents: "*Enter*" key or "*Return*" key) basically refers to an instruction that often means to "Accept this assignment", "Execute this expression", "Switch to this mode," etc. It is more a command key. With such a significant difference, it is not surprising to see that using the symbol to mean "Now I am going to do this:" is a frequently occurring mistake. It is exactly how the sign is used by a calculator. Here we have reason to speculate on the negative impact of using technology. This misuse may also make the transition from studying arithmetic to algebra, which is discussed in the section of "Procedural vs. structural", more difficult. Because in arithmetic, the sign is more action oriented.

Things may be worse when the sign is allowed to be used in a way that is deemed as incorrect or unthinkable in other situations. Say one wants to get *P* as a product of *x*, *y*, and then *5P*. With a calculator, after one gets *P*, one can press "*", "*5*", and "=" in a sequence to mean $P * 5$. But notice how easily it can be interpreted as:

$$x * y = P * 5 = z,$$

and how often teachers are annoyed by similar mistakes made by students!

The potential destruction of students' understanding of the sign can also result from a "convenient" rule supported by some calculators. For example, by pressing "=" repeatedly, an accumulated effect of certain action will be produced. So, $5\times3\times3\times3$ is obtained by pressing "=, =, =" in a row after $5*3$ are input. It simplifies the physical actions at the price of altering the meaning of an essential element in a universal language.

An analysis of the multiple interpretations of the sign reminds us, first, how important it is to clarify the difference between the functions of notations in mathematics and those in a technology environment, and second, that interpreting a symbol in an additional but vague way will require students' extra effort in understanding a language that is already a roadblock to many of them.

To mathematics teachers, there is nothing more frustrating than finding that what they try hard to teach students to avoid can get the green light on a calculator. For example, "*All 'open' parenthetical elements are closed automatically at the end of an expression.*" (TI-82 Graphics Calculator Guidebook, 1-21) A nice feature for a calculator though, this is just another conflict that will leave students puzzled. When the parentheses are arbitrary, misinterpretation can result.

Conclusion and Suggestions

The discussion above together with the examples show us that concern for the negative impact of using calculators, especially graphing calculators, is very real. Because calculators are generally numerical in nature, students may not acquire solid conceptual understanding. Their view of mathematics will probably be more procedural and accordingly their problem solving skills may be limited. The development of their structural view about mathematics could also be hindered. Moreover, because of it design, a calculator may deliver misleading information and create confusion in learning notation.

There is no cure but adaptation. Simply prohibiting the use of calculator has been proved to be a failure. The focus ought to be when, where, and how to use the device. Students should be informed very clearly that calculator is only one of many tools used to learn mathematics and it is a miracle only to a certain extent. The teacher is responsible to show students both positive (quickness , multiple representations, etc.) and negative (possible distortion even deception, self-contained rules, etc.) aspects. It will be especially beneficial to demonstrate how the technology is capable and fascinating in one case and incapable and helpless in another. It is also useful to engage students in solving problems in different environments, technologically rich or not, and demonstrate how one problem can be interpreted in very different cognitive challenges.

The industry always thinks of developing more powerful products and educators have accepted whatever is marketed. Yesterday the industry gave us four operation calculators. Today, there are computer algebra systems for both computers and calculators. Tomorrow, sure enough, more powerful ones will come out. We try hard to cope with the reality and have been comforting ourselves by saying that those features of calculators are only basic skills and students should pursue higher-order thinking. But where should a line be drawn to say that we want to keep those pieces of mathematics for human beings? It is very critical that mathematics educators share their concerns with industry and campaign for pedagogically more attractive and plausible calculators. The power of a calculator does not just mean the ability to accomplish more mathematical tasks. It should also lead to developing powerful mathematical minds. It should be an important criterion. When Shank and Edelson (1989/1990) advocated the innovation of instructional techniques in the following, they meant that the old technology should be replaced. I would rather use the quote to alert the teachers to the penetration of new technology in learning mathematics:

> *Many of the instructional techniques currently in use are there because technology makes them easy to employ, not because they are educationally sound. (p. 20)*

References

Clement, J., Lochhead, J., & Monk, G. (1981). Translation difficulties in learning mathematics. *American Mathematical Monthly, 88,* 286-290.

Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

Kaput, J. J. (1992). Technology and mathematics education. *Handbook of research on mathematics teaching and learning (D. A. Grouws, Ed.),* 515-556, New York: Macmillan Publishing.

Kieran, C. (1992). The learning and teaching of school algebra. *Handbook of research on mathematics teaching and learning (D. A. Grouws, Ed.),* 390-419, New York: Macmillan Publishing.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, Virginia: The National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics.

Polya, G. (1957). How to solve it: A new aspect of mathematical method (2nd ed.). Garden City, NY: Doubleday.

Resnick, L. B. (1987). *Education and learning to think*. Washington, DC: National Academy Press.

Shank, R., & Edelson, D. (1989/90). A role for AI in education: Using technology to reshape education. *Journal of Artificial Intelligence in Education, 1(2),* 3-20.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook of research on mathematics teaching and learning (D. A. Grouws, Ed.),* 334-370, New York: Macmillan Publishing.

Skemp, R. R. (1987). *The psychology of learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

Steen, L. (1988). The science of patterns. *Science, 240,* 611-616.

Texas Instruments Inc. (1993). *TI-82 Graphics Calculator Guidebook.* Texas Instruments Inc.