AN OPEN LEARNING MATHEMATICS FOUNDATION COURSE WITH MATHCAD

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Abstract

We present some details on an Open Learning mathematics foundation course which uses Mathcad. For 1998, a thousand plus students enrolled on the course in Singapore which represents 42 tutorial groups conducted by part-time tutors. The course is taken by adult students with various backgrounds who plan to graduate with a Mathematics, Computer science, Technology or Psychology degree together with possible minor components. In particular, we give examples on the use of Mathcad in developing students mathematical skills and its use in assessment via compulsory assignments. The Course is presented in partnership with UK Open University.

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1 Introduction

Singapore Institute of Management (SIM) was appointed by the Singapore Ministry of Education in 1992 to run Open University Degree Programme (OUDP) in partnership with UK Open University (UKOU). OUDP follows very closely the approach UKOU uses towards teaching. We adopt the multi-media instructional system, the main components being: self-learning study units, audio and video tapes, computer technologies, tutorials and lectures.

An important component of open learning is Tutor Marked Assignments (TMAs). The TMAs are compulsory and contribute towards a student's degree but, more importantly, are very much part of the learning process itself. The TMAs typically contain a collection of questions which guide students through mathematical ideas in terms of applications, notation and theory. The TMAs contain a number of questions which involve investigations and applications with Mathcad.

Mathcad is used extensively throughout the mathematics foundation course. There are many pointers to using Mathcad in the study units as well as in the TMAs. Techniques of Mathcad are continuously introduced through out the course as and when needed along side the development of mathematical ideas. Mathcad[2] version 5 is used but almost any user-friendly computer algebra system would do just as well.

2 The Math's Degree

The general structure of the mathematics degree (with no minor component) is as follows: There are two basic types of courses, 10 credit and 20 credit courses at levels 1 to 4. To obtain an ordinary degree a student must obtain 120 credits of core and elective courses. A student must complete a minimum core of 80 credit units with the remaining 40 credits chosen from electives.

A student can take no more than 40 credits in a given year and only takes the 20 credit foundation course in their first year. For honours degree an extra 40 credits is required which must be of level 3 and 4 type courses. Students are assessed in two ways:

i) Tutor Marked Assignments (TMAs). There are about 4 assignments for a 10 credit course and 8 assignments for a 20 credit course (plus some so called Computer Marked Assignments for some courses (multiple choice

[2] One of the main reasons Mathcad was chosen is because of its word processing capabilities.

questions)). The continuous assessment represents about 50% of a student's final grade.

ii) An examination at the end of the academic year which represents the further 50% of a student's assessment. Students must pass both the continuous assessment and the examination separately.

3 The Foundation Course

The mathematics foundation course consists of two 10 credit courses: **MSZS121 - Using Mathematics** and **MSZS221 - Exploring Mathematics** [3] . The two course are designed to be taken separately or together but a student should not attempt MSZS221 without doing MSZS121 first since many of the concepts in MSZS221 build on those in MSZS121 which includes most of the Mathcad preparation materials.

The prerequisites are a good recent pass at GCSE or O level for MSZS121[4]. To take both courses together, which is the case at OUDP, students will need to be very fluent in their use of algebra and trigonometry, and be reasonably familiar with linear, quadratic, trigonometric and exponential functions, if they are going to cope with these two courses without previous exposure to some of the topics covered in MZSZ121. A - level mathematics, or its equivalent, will provide this.

3.1 Brief description of MSZS121 Using Mathematics

The 10 credit course MSZS121 consists of four blocks A, B, C and D with each block containing between 3 and 5 chapters, each chapter representing a study week. Before these are studied, students read **Chapter A0 - Getting to know Mathcad**. This introduces students to the basic operation and editing instructions of Mathcad. Brief details of the four blocks are:

3.1.1 Block A

Chapters A1 (Modelling physical process) and A2 (Modelling growth) both deal with mathematical modelling; in these chapters, the mathematical ideas of sequences arise directly out of real-life contexts and are applied directly to understanding them.

[3] The two courses are called MST121 and MS221 respectively in the UK.

[4] UKOU offer a lower level course: MU120 "Open Mathematics" which would be a suitable prerequisite for MSZS121 but is not offered in Singapore.

Chapter A3 (Representing circles) introduces a quite different aspect of mathematics, namely the study of mathematical concepts and structures *in their own right*, as things which are fascinating and worth thinking about independently of any immediate application. Cartesian and parametric coordinates are used to present curves.

Chapter A4 (Modelling with functions) returns to the theme of using mathematics to model processes in the world we live in, using mathematical objects known as functions.

3.1.2 Block B

Chapter B1 (Functions and calculations) looks at certain general aspects of communicating with computers, and expressing how calculations are to be performed.

Chapters B2 (Modelling with sequences) and B3 (Modelling with matrices) develop the theme of modelling introduced in Block A. Chapter B2 considers two models using sequences. These models do not involve any new mathematics, but are explored in some depth. Chapter B3 goes on to consider models involving *linked* sequences. To handle such models, we introduce a new mathematical idea: *matrices*.

3.1.3 Block C

Chapter C1 (Differentiation and modelling) concerns the rate at which variables change, whether the change is an increase or decrease and whether the change stops. *Differentiation* is introduced to help in analyzing these matters.

Chapter C2 (Integration and Modelling) takes a different view towards change. *Integration* is introduced, as the inverse process of differentiation.

The third and final chapters of Block C (C3: Choosing a function for a model) discusses how appropriate functions can be chosen during the mathematical modelling process. Ideas introduced both in the previous chapters and in earlier blocks are reviewed and brought together in this discussion.

3.1.4 Block D

Chapter D1 (Chance) begins by considering a concept which is fundamental to all models for chance events and which underpins statistical thinking: *probability*. In chapters D1 and D2 (Modelling variation), two models for the variation observed in a variable are discussed, one model for a discrete variable and one for a continuous variable.

Chapters D3 (Estimating), D4 (Comparing) and D5 (Looking for relationships) are all concerned with drawing inferences about populations from samples of data; each chapter looks at one type of statistical investigation. Chapter D3 look at estimating an unknown quantity; Chapter D4 investigates differences between populations by comparing samples of data; and Chapter D5 is about looking for relationships between variables.

Block D of MSZS121 mainly uses specialized statistical and simulation packages in place of Mathcad. As such Block D of MSZS121 will not be discussed in this paper.

3.2 Brief description of MSZS221 Exploring Mathematics

The 10 credit course MSZS221 consists of four study blocks A, B, C and D with each block containing four chapters. Brief details are:

3.2.1 Chapter A

Chapter A1 (Exploring numbers and formulas) introduces second order recurrences. It looks at various ways of solving equations and how these have led to number systems and their representations including integers, rational numbers and non-rational real numbers.

Chapter A2 (Exploring conics) introduces the family of curves known as conics, both geometrically and algebraically, and the properties of parabolas, ellipses and hyperbolas are examined.

Chapter A3 (Exploring transformations) introduces transformations in the plane which preserve lengths and angles, that is, translations, reflections and rotations, and shows how these transformations can be symbolized, combined into composites, and decomposed into a sequence of reflections.

Chapter A4 (Functions from geometry) deals with the addition and scalar multiplication of vectors. It introduces functions of two variables and how the surfaces which are produced by such functions can be investigated through their contours and cross-sections.

3.2.2 Block B

Chapter B1 (Exploring functions) takes a systematic look at real functions examining when a function has an inverse and how two functions can be combined into a composite function.

Chapter B2 (Algorithms and recursion) of this course develops ideas about algorithms. It also devises and uses the Binomial Theorem.

Chapter B3 (Iterating functions) explains the various forms of long-term behavior exhibited by sequences generated by the logistic recurrence.

Chapter B4 (Iterating matrices) also looks at why certain patterns of long-term behavior occur and introduces some important concepts relating to matrices: *eigenvalues and eigenvectors*.

3.2.3 Block C

Chapters C1 (Exploring differentiation) and C2 (Exploring integration) explore further the topics of differentiation and integration, respectively, and introduces rules for handling products and composites.

Chapter C3 (Taylor polynomials) introduces the topic of Taylor approximations and Taylor series. This provides one answer to the question of how a computer or calculator, whose arithmetic operations are basically restricted to addition and multiplication, can evaluate functions such as exp, cos and arctan.

Chapter C4 (Modelling with differential equations) differs markedly from most chapters of MSZS221 in that it reverts to consideration of applications of the Calculus. The amount of new mathematics introduced here is therefore limited to the solution of certain first-order differential equations.

3.2.4 Block D

Complex numbers, the topic in Chapter D1 (Complex numbers), are a "new" type of number. Within the real numbers, certain algebraic equations, such as $x^2 + 1 = 0$, have no solutions, but if we supplement $\mathcal{R}$ then we obtain a system within which all polynomial equations have solutions.

In Chapter D2 (Number Theory), we consider numbers of the simplest type, integers. Over the years, these have provided many intriguing conjectures, the study of which has contributed to the development of mathematical ideas of much wider relevance.

Addition and multiplication of real numbers are examples of operations. A set with an operation satisfying particular properties is called a *group*, the topic of Chapter D3 (Groups).

Chapter D4 (Proof and reasoning) concerns "effective argument", a topic that has been a theme throughout MSZS221. Proving that results hold is an important pillar of pure mathematics.

3.3 How the course is studied

The two courses MSZS121 and MSZS221 are taken concurrently by OUDP students in their foundation (first) year. Typically, Block A of MSZS121 is studied followed by Block A of MSZS221 etc. Each Block of both courses has an associated Computer Book. These contain many Mathcad based exercises and investigations which are refereed to by the chapters in that Block. Many of the exercises and investigations in turn refer to live Mathcad files designed for the specific exercises or investigations.

4 Examples using Mathcad

Extensive use of Mathcad is exploited throughout the course. As mentioned earlier, each Block has an associated computer book which contains many Mathcad based exercises and investigations associated with the various chapters of that Block. The computer books also have live Mathcad files associated with them with many of the exercises and investigations set up for immediate use. Quite typically, the exercises and investigations elaborates, extends, improves and generalizes ideas in the course chapters.

The compulsory Tutor Marked Assignments also contain Mathcad based exercises and investigations, some of which have associated live and ready to use Mathcad files. These exercises and investigations are expected to be solved by the students after reading appropriate chapters, the chapters being indicated at the beginning of each exercise or investigation.

The very nature and style of the various assignments are geared towards using, describing, explaining and conjecting mathematics in both "stand alone mathematics" and solving real-life problems. Lets now look at some examples from both the computer book and the compulsory Tutor Marked Assignment exercises and investigations.

4.1 Example 1

Chapter A1 (Modelling physical processes) of MSZS121 considers two mathematical models. The first is "The New Moon Model" which we consider here and is the first piece of mathematics encountered by students. The problem is to predict the dates of the new moon for next year based on this year's dates. This problem nicely motivates the use of simple recursive relations (an arithmetic sequence in this case) as well as expose students to typical thought process when creating a mathematical model. After appropriately labeling the days of the new moon and using a yearly average for the number of days between each new moon, a mathematical model is suggested for the new moon cycles:

$$m_1 = 10.965, \;\; m_{new} = m_{previous} + 29.545,$$

where 29.545 represents the average time between new moons and $m_1 = 10.965$ is the decimal time of the first new moon (first new moon in 1994 occurred on the 11 January at 11:10pm with midnight on the start of the new year being defined as zero time).

The mathematical nature of this model is investigated and formalized with the help of Mathcad files and predictions with criticism are developed. Students are finally encouraged to generate a similar model for the full moon cycle.

4.2 Example 2

This TMA problem is based on MSZS121 chapter B2 (Modelling with sequences). A Mathcad file is given which is ready to be used to investigate recurrence system. In particular, the following is investigated:

$$x_1 = 1, \;\; x_{i+1} = \frac{1}{3}(2x_i + \frac{3}{x_i}),$$

for $i = 1, 2, ...$ Students are invited to use this recurrence system to evaluate $\sqrt{3}$ to four decimal places and asked to illustrate (using graph and table) how rapidly the iteration converges. Finally, students are asked to investigate the convergence of the above sequence for different values of x_1 and hence state the values of x_1 for which x_i converges to $\sqrt{3}$, and the values of x_1 for which x_i converges to $-\sqrt{3}$.

4.3 Example 3

For chapter A4 (Functions from geometry) of MSZS221, there is a TMA question associated with this chapter which again requires a specific Mathcad file to be used. Students are asked to use Mathcad to investigate functions of the form:

$$t : \mathcal{R}^2 \to \mathcal{R}$$
$$(x, y) \mapsto x^2 + ky^2$$

for different values of k. Students are asked to use the Mathcad file to define a grid of points P for values of x from -5 to 5 and values of y from -5 to 5, with 21 points in each range. Students are invited to use this grid to create a surface plot and a contour plot for the function $t(x, y) = x^2 + 3y^3$. The students are then asked to identify the shapes of the curves shown in the contour plot which was produced. Students are then asked to use Mathcad and investigate what happens to the contours when the value of k decreases but remains positive. In particular $k = 1$ and $k = 1/3$ is suggested. The student is asked to **Describe** and **Explain** the changes and illustrate their answers with appropriate printout. Students are then invited to predict the effect on the contours on setting $k = 0$ and $k = -1$ giving brief reasons. Finally, the students are asked to use Mathcad to test their predictions and describe what happens to the contours of the surfaces for $k \leq 0$.

4.4 Example 4

A TMA question associated with MSZS221 B3 (Iterating functions) considers fixed points of a function $f : \mathcal{R} \to \mathcal{R}$:

$$f(x) = \frac{40x^3}{(1 + 2x^2)^3}.$$

Students are initially invited to investigate the fixed points of f using Mathcad (or otherwise) followed by the use of the Mathcad "root" facility, to obtain each of the fixed points of f to 3 decimal places. The student is then asked to use Mathcad to find the gradient of f at each of the fixed points of f and hence classify the fixed points as attracting, repelling or indifferent.

The student is then asked to repeat the exercise for the function $f^2 = f \circ f$. Finally, for various starting points x_0, students are asked to describe the long-term behavior of the iteration sequence $f^n(x_0)$ and to give a brief explanation of why they expect this behavior.

4.5 Example 5

In this final example, we look at a simple investigation from the computer book of Block C of MSZS121 associated with chapter C3 (Choosing a function for a model). In particular we describe the way oscillating functions are developed, again being mainly motivated from real life applications which is typical of MSZS121. A nice activity from the computer book invites students to investigate the function:

$$y = a + b \sin(kx + c),$$

where a, b, k and c are the parameters of the function. In particular, students are asked to observe: altering a changes the mean value about which the functions oscillates; altering b (without reversing its sign) changes the amplitude of the oscillation; altering k changes the frequency which the function oscillates, or equivalently, changes the wavelength or period of the oscillations; altering c shifts the graph to the left or right; reversing the sign of the second term has the effect of reflecting the graph about the mean line of oscillations; replacing sin by cos, or vice versa, shifts the graph sideways a quarter of a wavelength (to the left from sin to cos).

The main examples illustrated in this paper are all taken from earlier chapters of the courses so as to be of relevance to a maximum number of readers.

All the above examples are illustrated with live Mathcad files at ATCM'98 together with some comments from students. See the Mathcad file oudp2.mcd for a summary of the Mathcad files presented at ATCM'98.

For further details of the mathematics foundation course see the UKOU web site at:

http://www.open.ac.uk/OU/Studying/mathcomp.html.