Fundamental Theorem of Calculus

with

Spreadsheet

BY

NOBUHIRO TAKAHASHI

(Kanagawa Prefectures IKUTA High-School in Japan)

A Computer spreadsheet (for example: Excel, Lotus 123,etc.) is very popular software for general use, and it is a powerful tool for mathematics as well. In this paper, we study the possibility of mathematical education with a spreadsheet. We compute tables, draw curved lines and so on, for Fundamental Theorem of Calculus. The Theorem is a very important theorem in high school mathematics. But it is difficult to understand the Theorem for students. So, we try to explain this theorem using the spreadsheet. Many Japanese high school mathematical texts (like mathematics in the educational guidelines of The Ministry of Education) represent this (introduction, the theorem and it's proof) as follows:

Suppose you want the area of the region S bounded by $y = f(x)$, $f(x) \geq 0$ for all x in $[a,b]$, lines $x = a, x = b$, and the x axis. $f(x)$ must be continuous in $[a,b]$. For the purpose of calculating this area, consider the area of region bounded by $y = f(x)$, $f(x) \geq 0$ for all x in $[a,b]$, $x = a$, $x = b$ $(a < t < b)$, and the x axis. This area is a function of t. Let $S(t)$ be the area function, then $S(a) = 0,\ S(b) = S$. And then, to get derived function $S'(t)$ evaluate increment $\Delta S(t)$ by using figure. By taking a limit, we gain $S(t)' = f(t)$. Therefore, $S(t)$ is indefinite integral of $f(t)$, and the area S is definite integral over $[a,b]$.

We experiment the above process of the proof by using the method of approximation on a worksheet: At first, we make a table giving values to functions $f(t)$ at the right-hand endpoints, which are divide into n intervals, each of length $\Delta t = \dfrac{b-a}{n}$. The next, to get a table of area function $S(t)$, the method used is to approximate the area $S(t)$ we wish to find with the sum of the areas of rectangles. The last, by calculating the rate of change $\dfrac{S(t+\Delta t)-S(t)}{\Delta t}$ in a table, we show that $S'(t) = f(t)$ at these points. Using these tables of $f(t)$, $S(t)$, and $S'(t)$, we can construct these graphs on the worksheet. Furthermore, we show that the degree of approximation is improved by increasing n. In this way, we give the students a hint of proof of the theorem.

Consequently, the reason that a spreadsheet is very useful for mathematical education follow: 1) Using the table, 2) Using the graph, 3) Using the macro, etc.