On The Leverrier-Faddeev Algorithm

SHUI-HUNG HOU

Department of Applied Mathematics
The Hong Kong Polytechnic University , Hong Kong

Summary

The Leverrier-Faddeev algorithm [1], included in most of the books on linear system theory though often without complete proof, states that the coefficients of the characteristic polynomial

$$p(s) = \det\ (sI - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

of an $n \times n$ matrix A can be obtained by means of the following recursive formulas:

$$\begin{array}{rclcrcl} B_1 &=& I, & & a_1 &=& -\dfrac{1}{1}\,\mathrm{tr}\ (AB_1), \\ B_2 &=& AB_1 + a_1 I, & & a_2 &=& -\dfrac{1}{2}\,\mathrm{tr}\ (AB_2), \\ & \cdots & & & & \cdots & \\ B_n &=& AB_{n-1} + a_{n-1} I, & & a_n &=& -\dfrac{1}{n}\,\mathrm{tr}\ (AB_n). \end{array} \tag{1}$$

Here tr stands for the trace of a matrix.

The matrices $B_1, \cdots, B_n$ are closely related to the adjoint of the matrix $sI - A$ via

$$\mathrm{adj}\ (sI - A) = B_1 s^{n-1} + \cdots + B_{n-1}s + B_n,$$

as one may use the Cayley-Hamilton theorem to verify the relation

$$(sI - A)(B_1 s^{n-1} + \cdots + B_{n-1}s + B_n) = p(s)I.$$

The derivation of the coefficient formula

$$a_k = -\frac{1}{k}\,\mathrm{tr}\ (AB_k), \qquad 1 \le k \le n, \tag{2}$$

on the right hand side in (1) is often omitted in most books [2]–[5] on linear control systems with the exception of a few. The proofs therein are usually rather involved [6, 7].

It is the purpose of this paper to give a novel alternative derivation of (2) by means of Laplace transform. The approach is more readily accessible to students of engineering and applied sciences. Some application of the Leverrier-Faddeev algorithm together with the use of symbolic program *DERIVE* will be considered.

References

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