Approximate Singularvalue Decomposition of a Matrix with Polynomial Entries

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Abstract

Singularvalues of a matrix play key role in the theory of linear algebra. Therefore, it is well-known and very important fact that every constant matrix M has its singularvalue decomposition, i.e., M can be decomposed into $M = UVW$, where U and W are unitary matrices and V is a diagonal matrix.

When M is a matrix with polynomial entries, it is not so easy to compute such decomposition. However, it is possible to compute polynomial matrices $U^{(k)}, V^{(k)}, W^{(k)}$ which satisfy

$$M \equiv U^{(k)}V^{(k)}W^{(k)} \pmod{(x, y, ..., z)^{k+1}}, \qquad (1)$$

$$\left(U^{(k)}\right)^T U^{(k)} \equiv E \pmod{(x, y, ..., z)^{k+1}}, \qquad (2)$$

$$\left(W^{(k)}\right)^T W^{(k)} \equiv E \pmod{(x, y, ..., z)^{k+1}} \qquad (3)$$

for any integer k (> 0), where $x, y, ..., z$ are variables in M and $V^{(k)}$ is a diagonal matrix. As you can see easily, decomposition in (1) is a singularvalue decomposition of M modulo ideal $(x, y, ..., z)^{k+1}$ and we call the decomposition kth approximate singularvalue decomposition.

In this paper, we use Hensel lifting technique and give a complete algorithm to compute above $U^{(k)}, V^{(k)}, W^{(k)}$. Numerical examples are given to illustrate each step of the algorithm. The algorithm is quite efficient and the Hensel lifting can be performed with only 2×2 constant matrix inversions and basic matrix manipulations.