

Building Dynamic Mathematical Models with Geometry Expert I. Functions and Plane Curves

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Geometry Expert (GEX) is a software for dynamic geometric drawing and automated geometry theorem proving and discovering. With GEX, we can draw geometric diagrams dynamically, that is, we can change these diagrams in a continuous way and at the same time keeping the geometric relations in the figure intact. This feature of GEX provides a powerful and convenient platform for teaching mathematical concept in an intuitive way. With GEX, we have proved hundreds of difficult geometry theorems and generated elegant proofs. Furthermore, we can use GEX to build a database which contains all the properties of a given diagram that can be deduced using a set of geometric axioms. With this feature, teachers can easily make exercises and test problems; students can enhance their ability of solving problems by fully explore the properties of a given diagram.

In this series of papers, we shall show how to build dynamic models for various mathematical concepts with three different types of tools:

1. Using ruler and compass with multiple driving points moving on a line or a circle. This class includes conics, functions $y = \frac{\sqrt[n]{f(x)}}{\sqrt[m]{g(x)}}$ where $f(x)$ and $g(x)$ are polynomials and n and m are positive integers, trigonometric functions and their inverse functions, various curves defined in polar coordinate systems, etc.
2. Using the numerical computation facility provided by C languages, this class includes functions of the form: $y = f(x)$ where $f(x)$ could be any “elementary functions” – a^x , x^a , $\log(x)$, trigonometric functions and their inverse functions – and their arithmetic expressions and compositions. This part is quite similar to most “Graphic Calculator”, but is flexible and powerful.
3. Using linkages involving of rotating joints. As proved by Kempe, this class includes any algebraic curve $f(x, y) = 0$ where $f(x, y)$ is a polynomial of x and y .

Dynamic models built with computer software are more powerful, flexible, convenient, and intuitive than models built of real materials like plastics and wood. Furthermore, we can show the generating process of these curves by doing animation with GEX. With these models, GEX provides a “mathematics laboratory” for teaching and learning mathematics. In Part I of this paper, we will discuss how to build dynamic models for functions and plane curves.