Building Dynamic Mathematical Models with
Geometry Expert I. Functions and Plane Curves

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Geometry Expert (GEX) is a software for dynamic geometric drawing and automated
geometry theorem proving and discovering. With GEX, we can draw geometric diagrams
dynamically, that is, we can change these diagrams in a continuous way and at the same
time keeping the geometric relations in the figure intact. This feature of GEX provides
a powerful and convenient platform for teaching mathematical concept in an intuitive
way. With GEX, we have proved hundreds of difficult geometry theorems and generated
elegant proofs. Furthermore, we can use GEX to build a database which contains all the
properties of a given diagram that can be deduced using a set of geometric axioms. With
this feature, teachers can easily make exercises and test problems; students can enhance
their ability of solving problems by fully explore the properties of a given diagram.

In this series of papers, we shall show how to build dynamic models for various math-
ematical concepts with three different types of tools:

1. Using ruler and compass with multiple driving points moving on a line or a circle.
   This class includes conics, functions \( y = \frac{\sqrt{f(x)}}{\sqrt{g(x)}} \) where \( f(x) \) and \( g(x) \) are polynomi-
   als and \( n \) and \( m \) are positive integers, trigonometric functions and their inverse
   functions, various curves defined in polar coordinate systems, etc.

2. Using the numerical computation facility provided by C languages, this class in-
   cludes functions of the form: \( y = f(x) \) where \( f(x) \) could be any “elementary
   functions” - \( a^x \), \( x^a \), \( log(x) \), trigonometric functions and their inverse functions –
   and their arithmetic expressions and compositions. This part is quite similar to
   most ”Graphic Calculator”, but is flexible and powerful.

3. Using linkages involving of rotating joints. As proved by Kempe, this class includes
   any algebraic curve \( f(x, y) = 0 \) where \( f(x, y) \) is a polynomial of \( x \) and \( y \).

Dynamic models built with computer software are more powerful, flexible, convenient,
and intuitive than models built of real materials like plastics and wood. Furthermore,
we can show the generating process of these curves by doing animation with GEX. With
these models, GEX provides a “mathematics laboratory” for teaching and learning math-
ematics. In Part I of this paper, we will discuss how to build dynamic models for functions
and plane curves.