

Creating teaching and learning material in Engineering Mathematics using Scientific WorkPlace with External Program Link to Maple

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Abstract

This paper discusses the advantages of using Computer Algebra systems (CAS) to create teaching/learning materials for Engineering Mathematics. Furthermore, it shows that when teacher masters more than one CAS, he is able to draw on the strengths of different systems, in order to create effective materials that will help students in their learning. A sample module, entitled "Numerical Solution to the First Order Differential Equation - Euler's Method ", created using Scientific WorkPlace with External Program Link to Maple is shown. Material created is interactive, in which it allows students to use it for their own exploration. It can also be modified easily to suit the needs of both teachers and students and it can be sent electronically or be posted on the World Wide Web for distance-learning.

Introduction

In the Mathematics and Science Centre, we teach Engineering Mathematics to students of various Engineering Departments. It is important for us, as lecturers, to bear in mind that students from various departments need to master a variety of mathematical concepts and skills so that they are competent in using Mathematics for their own field of study. It is helpful if they can get acquainted with a variety of mathematical tools. In our opinion, Scientific WorkPlace (SWP) is a suitable tool to introduce to the novice in using the Computer Algebra System (CAS). SWP incorporates the facilities for word-processing in CAS. It allows users to perform live mathematics using Maple without actually having to learn the syntax of the latter. When necessary, SWP can be used as a platform to create External Program Link to other CAS, so that more sophisticated task can be accomplished. Like other CAS, SWP is created for some specific purposes, and thus has its own strengths, weaknesses, and limitations. However, if we incorporate other CAS, for example Maple, we would be able to create an even more effective learning environment for our students.

We will demonstrate how this objective can be achieved when a module of teaching material is created using SWP with External Program Link to Maple. This module of the material is used to teach " Solving First Order Differential Equation using Euler's Method ". We will see that SWP provides the facilities for word-processing, for calculating the exact solution or obtaining the series approximation to the solution. Maple, on the other hand, is used to perform Euler's Method to generate the approximated

solution, which is not straightforward if SWP is used. Maple is also used to plot both the exact and approximated solutions, so that comparison can be made.

One of the most important factors for using CAS to prepare teaching materials, is that the materials created is more superior than those prepared traditionally. However, it must be acknowledged that it is a time consuming task for teachers to take up the challenge to write the courseware themselves. Nevertheless, they should be encouraged to do this, as in the long run, the effort spent is worthwhile. This is because the electronic materials, once created, can be used for multiple purposes and can be modified easily. Here, we see that the two systems, SWP and Maple together, provide us with tools to create teaching materials that are interactive with clear exposition. It becomes possible to give more realistic and practical examples to explain concepts, as the laborious calculations have been taken away. Not only does the material give better presentation, it can be modified with very little effort to make different emphasis for different purposes. This is particularly important to most of the teachers, as it provides them with a short cut in which they can pick up written materials, do the minimum modification and be able to use the materials for their own purposes.

As it was said that teaching takes place *in time*, whereas learning takes place *over time*, it is important in a learning process, that students can carry out their own exploration after teaching is over, that they can develop and test their conjectures as well as obtain quick and instructive results. With the use of CAS, we can create an environment for the students to achieve these. Using CAS also makes it possible for us to expose students to a wider scope of the problem solving opportunity.

The following teaching material was written for both students of Building Services Engineering and Civil and Environmental Engineering.

Numerical solution to the first order differential equation - Euler's method

In practice, many first order differential equation (DE) is difficult to, or cannot, be solved by the analytical method. In this case, Euler's method can be used, provided the initial condition is available. We call this the initial value problem.

Consider the initial value problem $\frac{dy}{dx} = f(x, y)$ where $y(x_0) = y_0$. Assume the true solution $y = y(x)$ exists in the domain $[a, b]$. Given a point x in $[a, b]$ and for a small value of h , the gradient at any point on $y(x)$ in the domain $[a, b]$ is given by $y'(x)$, which can be approximated by $y'(x) \approx \frac{y(x+h) - y(x)}{(x+h) - x}$. This gives the Euler's method approximation of $y(x+h)$ where $y(x+h) \approx y(x) + hy'(x)$. Therefore, $y(x+h) \approx y(x) + hf(x, y(x))$ and it is normally written as $y_{n+1} = y_n + hf(x_n, y_n)$. Here, y_{n+1} gives the approximated value of $y(x_{n+1})$, where $x_n = x_0 + nh$, within the domain $[a, b]$. It can be seen that the solution of the differential equation is given by a set of approximated values in the interval with the step size h . In general, the approximation to the solution can be improved by reducing h .

The following are a number of classical examples that we would use in the classroom. We will see that with the SWP and Maple, they become more comprehensive and interesting. We use SWP to obtain the exact solution or the series approximation to the solution. The External Program Link is created in each of the example, so that at a click of the mouse, Maple would be activated and that Euler's method approximation to the solution can be obtained. Furthermore, students are able to change the parameters of the problems, within the Maple Worksheet, to obtain solutions to their own problems. This document

can be experimented live with Scientific WorkPlace or Scientific NoteBook. For an msg file of this document, click [here](#), or see atcm97.tex at Appendix section.

Example 1: Given a differential equation $\frac{dy}{dx} = 2y$ where $y(0) = 1$.

Method 1. Direct Computation from SWP: Define the DE in the matrix form: $\frac{dy}{dx} = 2y$. Choose **Solve**
 $y(0) = 1$.

ODE + Exact. We get "Exact solution is : $y(x) = e^{2x}$ ".

Method 2. Approximate the solution for $0 \leq x \leq 2$, using Euler's method with various step size h .

Compare the approximations with the true solution $y=e^{2x}$.

At this point, we can use the SWP command, **External Program Call**, under **Insert + Field**, to create a link to Maple, click [here](#) for a maple file or see maple1.mws at the Appendix section. To activate the link we choose **Tools + Action**.

The command activates the External Program Link to the Maple worksheet, thereafter we are able to execute the Maple worksheet to obtain the Euler's approximation to the solution of the differential equation, as well as obtain the plot of the true solution and the approximated solution as shown in **figure 1**.

Figure 1 shows the graph generated by Maple. It shows the true solution to the DE and the Euler's method approximation to the solution with step size $h=0.5$.

If we reduce the step size $h=0.1$, Maple can calculate another set of better approximations, and obtain the plot as shown in **figure 2**, in which comparison can be made between approximations obtained using different step sizes, as well as with the true solution.

The plot in **figure 2** shows that the approximation with step size $h=0.1$, drawn in circles, moves closer to the true solution, the curve, compared to the approximation with step size $h=0.5$, drawn in diamonds. Note that all the Maple Worksheets are appended below.

We can see in this example that Maple program can be used quite easily to perform Euler's method, which SWP is not able to achieve. The graphs give excellent illustration that the approximated solution improves as the step size, h , reduces. One of the most important feature is that the document is *live*. With simple instructions given, students can experiment it themselves; they can use "copy and paste" and solve the same problem using various step size h , to see how the approximation approaches the true solution obtained by SWP. Teachers, on the other hand, can copy the source code, make very little necessary alteration, and use it to solve other similar problems.

When Euler's method is used to approximate the solution of the first order differential equation, we often believe that the approximation is 'better' around the initial condition and that the approximation 'gets worse' as we move far away from the initial condition. The next example will show us if this is always true.

Example 2: Given $\frac{dy}{dx} = 2x - y$, where $y(0) = 1$, for $0 \leq x \leq 5$.

Method 1: Direct Computation from SWP: Define the DE in matrix form: $\frac{dy}{dx} = 2x - y$, $y(0) = 1$. Choose **Solve**

ODE + Exact, we get "Exact solution is : $y(x) = 2x - 2 + 3e^{-x}$ ".

Method 2: Approximate the solution of the differential equation using Euler's method with the various chosen step size h .

Compare the approximation with the true solution $y=3e^{-x}+2x-2$.

Here again, an External Program Link to Maple is created. (For the maple file, click here or see maple2.mws at the Appendix section.) With this, we choose **Tools + Action** and the External Program Link activates the Maple Worksheet. We execute the worksheet and the Euler's approximation to the solution of differential equation is obtained and the graph plotted.

The graph in **Figure 3** shows that the critical values lie around the turning point of the curve, where the approximation is inaccurate. This is a special case where the true solution approaches a straight line, $y=2x-2$, as x increases; this means Euler's method approximation approaches the true solution as x increases, regardless of the step size h .

The special characteristics of this problem tend to be over-looked, if not because the true solution are obtained easily for making comparison with the approximated solution. So, the effectiveness of these computational tools is self-evident.

It will be interesting to discuss with students the reasons why the Euler's method does not give good approximation when x is close to 1, compared to the approximation when x is large. Through the discussion, we would be able to tease out from the students their past knowledge on the derivatives, etc. The illustration also prompt them to be aware of the special case in applying Euler's method and not to draw general conclusion on the method. All these are made possible because the laborious calculation has been taken care of by the use of CAS.

The next example shows a DE which has no closed solution. SWP can, however, provide us with the series approximation to the solution. We also use Maple to generate the Euler's method of approximation. It is helpful, in this case, that we are able to reduce the step size, h , to a fairly small value and watch how the approximated solution converges.

Example 3: Given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 0$.

Method 1. Direct computation using SWP: Define the DE in matrix form as before: $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$.

When we use **Solve ODE + Exact** or **Solve ODE + Numeric**, we cannot get a plot. However, when we use **Solve ODE + Series**, we get the Series solution is : $y(x) = \frac{1}{3}x^3 + O(x^6)$.

Method 2: Apply Euler's method with various step size h to approximate the solution of the initial value problem: $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$, on the interval $0 \leq x \leq 2$. Note here that the analytical solution can not be obtained.

With the **External Porgam Link** created and choose **Tools + Action**. Maple is activated as before. (For the corresponding Maple file, click here or see maple3.mws at the Appendix section.)

In the worksheet, Maple first obtains the approximation using step size $h=0.1$ and then we set $h=0.01$ and Maple goes through 200 steps to generate the array containing the approximated solution. These values are used to plot the graph shown in **figure 4**. In this Worksheet, a very small step size, h , was chosen to generate the approximated curve. The intensive calculation, which is extremely difficult to perform using any ordinary method, is made possible using Maple.

The graph shows that if the step size h is not small enough, the approximation becomes unreliable very quickly as x increases.

As there is no analytical solution to this problem, we can only compare the Euler's approximation with the series approximation obtained by the SWP.

After being shown the examples and supervised hands-on sessions, students should be competent in solving their own problems. Problem solving would reinforce their understanding of the concepts and skills. Students can follow the steps laid out in the examples to solve their problems using SWP, as well as modify the Maple Worksheet to suit their needs. Finally, we can assign them more challenging tasks with proper assignment statements. The task can be of more calculation intensive which requires multiple skills, and integrated knowledge. It can be investigative activities or other relevant tasks. The tasks would further improve their problem solving skills. Students can be asked to attend to the assignment and report their findings using SWP. They can be asked to submit their assignment electronically.

Conclusion

When Computer Algebra System is incorporated in the teaching of mathematics, it will create a much better teaching/learning environment. Teaching materials which are effective will make learning a happy experience. It provides new opportunity for exploration, investigation to test conjecture and draw conclusion. It also enables students to develop more mathematical and computational skills which are closer to that which they will require in the future working environment. It will motivate both the teachers and the students.

Teachers need time to develop teaching modules. The effort, however, is worthwhile as these materials, once created, can be modified quite easily for future use. For those who do not wish to develop their own material, there are available courseware obtainable from various sources that can be modified to suit their needs. It will motivate both the teachers and the students when CAS is used.

High speed computers are needed to run the CAS effectively for the development of these materials, as well as conducting on-line teaching.