Computer technology in mathematical researches on ecology

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One of indispensable conditions of normal ability to live of the person is comfortable house. Yet from tribes times building carefully chose a place for new houses. The saved experience was successfully applied by boyscout - exits of their tents were oriented according to direction prevailing wind.

During mass urban construction many natural factors remain forget - main accent was done on static durability of a design. Then necessity of an estimation of vibrating durability of buildings in earthquake active regions, but also type and structure of a ground in avoidance "clone" of construction Pizza tower was realised. However till now durability of high-altitude buildings and huge monuments was not considered at influence on them wind load, as was considered, that which strong was not wind, wind loads are very small in comparison with all other loads. There was forget the classical fairy tale about three piglets, in summary - occurrence flatter on monument "Motherland" on Mamaev Kurgan; instant destruction windows of a high-altitude building, carried out in a kind of an aerodynamic structure, at certain wind direction in Boston; occurrence of dangerous fluctuations of exhaust pipes in industrial regions and other.

For city-dweller actuality of a wind conditions is caused first of all not by distribution wind loads, and moving of air weights between houses. In this case at construction of new buildings and it is required to reconstruction existing to determine a space situation of the following zones:

- Discomfort zones for pedestrians, in which wind speed exceeds 7 mps;
- Zones of lowered wind speed (less than 2 mps);
- Zones of possible concentration of harmful substances, speed in which can be more than 2 mps, however mixing of air of these zones and air at their border does not occur;
- Zones of stagnation of air weights, in which speeds of wind less than 1 mps;

The last two zone are characterised by a accumulation and concentration of harmful impurity, present in air.

At solution of similar problems already not to do without by boyscout experience: the rather orientation of buildings is caused by imagination of an architect; district, on which they are located, can have difficult high; the buildings have the various sizes and various floors, and their amount much exceeds amount of tents on small field. All this speaks about necessity of application of modern computer technologies and modern mathematical apparatus.

There are the following methods of the solution of considered above problems:

- Nature experiments in aerodynamic pipes, which are expensive and have limited opportunities of evident representation of the information;
- Nature experiments, based on electrodynamics analogies in aerodynamics, which also have limited opportunities of evident representation of the information, and, besides do not take into account a non-stationary nature of a flow;
- The computer methods, main advantages of which are relative cheapness, opportunity quickly and evidently to present a plenty of variants of a mutual arrangement of buildings, and at presence of the advanced mathematical apparatus, high accuracy of modelling of process of a flow.

Among computer methods of the solution of gas dynamics problems it is in turn possible to allocate following:

- Empirical, when of mathematical dependence are deduced on basis of nature experiments - differ by high speed and low accuracy;
- The method of the solution of Navier-Stokers equation - differs by high accuracy and requirement for huge computing capacities of computer facilities;
- Method of discrete vortices, successfully used by the authors of the of the report yet with times 286-th and giving good results for speeds of wind at which it is possible to not take into account viscosity of air.

In report will be told about complex created by the authors of the programs for solution of aeration problems in ecology, based on development of a method of discrete vortices; comparison of expansion of this complex of the programs to development of computer facilities and software, but also results of real accounts, influencing on the conclusion of ecological examination of Moscow government.

1. PHYSICAL STATEMENT OF THE PROBLEM

We assume that there is a group of buildings on the ground that is taken to be a plane. The form of the buildings may be rather complicated. However, we suppose for simplicity (namely to exclude viscosity) that the form of the buildings is piecewise plane and the wind velocity is much less that the sound velocity. In this case the air is considered to be an ideal incompressible fluid. The vortex trails separating from the buildings can be considered to be fixed in this approach. Therefore, we suppose that the vortex trails separate from the corner lines, and we only corner lines relevant for a given problem (to save the computer time) rather than all of them. Each building is modelled by a vortex layer, and to save computing time the buildings are supplemented by figures symmetric about the ground. The vortex layers on these figures are of the same intensity but of opposite sign of rotation. In this approach the condition of impenetrability is satisfied automatically on the ground (which is flat). This problem can also be solved for the general case of the more complicated topography. To solve the problem is much more difficult and we do not considered it here.

If the buildings are high enough and the wind conditions must be studied at the level of entrances only, the problem may be considered as the two-dimensional one. The buildings may be represented as contours of their cross-sections and the incoming flow as a two-dimensional parallel flow.

2. MATHEMATICAL STATEMENT OF THE PROBLEM

The problem of an ideal incompressible fluid flow about bodies in the absence of separating vortex sheets is reduced to the solution of the Neumann exterior problem in which the surfaces (contours) of bodies are it considered to be boundaries. Thus, we are to find the function $U(M)$ satisfying the Laplace equation

$$\Delta U = 0\,, \quad M \in D^{+}\,, \tag{2.1}$$

$$\left.\frac{\partial U}{\partial n_{M_0}}\right|_{S} = -\overline{U}_0 \cdot \overline{n}_{M_0} = f\left(M_0\right), \quad M_0 \in S\,, \tag{2.2}$$

besides U and $gradU \to 0$ as $M \to \infty$, (condition of the damping disturbed velocities at infinity); $\sigma = \bigcup_{k=1}^{m} \sigma_k$ is the totality of piecewise smooth, simple, closed, and mutually nonintersecting surfaces (contours) σ_k, $k = 1,\ldots,m;$ D^+ is the external domain with respect to these surfaces (contours)(Fig.1), $D^- = \bigcup_{k=1}^{m} D_k^-$ is the

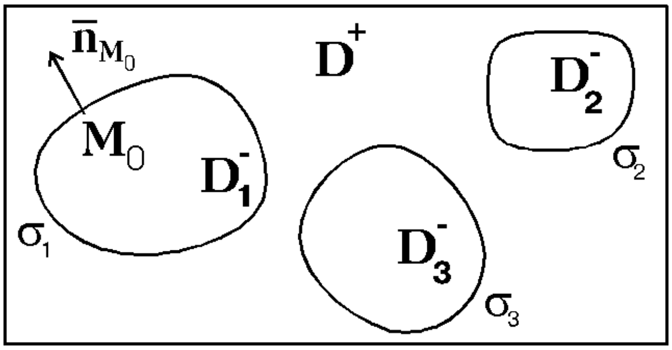

Figure 1

of the surfaces(contours) σ_k; $\bar{n}_{M_0}$ is a unit vector normal to σ at point M_0 directed into the domain D^+; $\overline{U}_0 = gradU_0$, where U_0 is a harmonic function on the whole of the space (plane).

If we designate $f_k(M) = f(M), M \in \sigma_k, k = 1,\ldots,m$ the relations

$$\int_{\sigma_k} f(M)d\sigma_k = 0, k = 1,\ldots,m. \tag{2.3}$$

hold for the functions $f_k(M)$ according to [3].

The solution of the problem (2.1), (2.2) is sought in the form of the potential of a double layer

$$U(M_0) = \int_{\sigma} \frac{\partial}{\partial \bar{n}_M}(\varphi(M, M_0))g(M)d\sigma_M, \tag{2.4}$$

where $\varphi(M, M_0) = (2\pi)^{-1} ln\, r_{MM_0}^{-1}$ for two-dimensional problems, and $\varphi(M, M_0) = (4\pi)^{-1} r_{MM_0}^{-1}$ for three-dimensional problems, and $r_{MM_0} = |\bar{r}_{MM_0}| = |MM_0|$. The function $U(M_o)$ determined by (2.4) is a solution of if the function $g(M)$ is a solution of the integral equation

$$\int_{\sigma} \frac{\partial}{\partial n_{M_0}} \frac{\partial}{\partial n_M}(\varphi(M, M_0))g(M)d\sigma_M = f(M_0), \qquad M_0 \in \sigma, \tag{2.5}$$

where the sign of differentiation with respect to $\overline{n}_{M_0}$ is introduced under the integrand because the normal derivative of the potential of a double layer is continuous when passing through the boundary surface (curve). The integral in (2.5) exists as the Hadamard finite value [5] for functions if $g(M)$ all the partial derivatives belong to $L_2(\sigma)$, i.e. the space of functions which are square integrable on σ, $\sigma \in C^2$ [4].

Equation (2.5) has a solution up to *m* arbitrary constants. Indeed, if the function (2.4) is a solution of the Neumann exterior problem for the surface (curve) σ, it is also a solution of the Neumann interior problem for each surface (curve) σ_k, $k = 1,\dots,m$, which has a solution up to a constant. Therefore the unique solution of may be selected from the general solution with the help of the system of equalities

$$g(M_k) = C_k, M_k \in \sigma_k, \quad k = 1,\dots m, \tag{2.6}$$

or the system of equalities

$$\int_{\sigma_k} g(M) d\sigma_k = C_k, \quad k = 1,\dots,m, \tag{2.7}$$

where C_k are arbitrary constants. Since we are usually interested in partial derivatives of the function $g(M)$, the values of constants C_k may be arbitrary, e.g. equal to zero.

According to [4], if the surface (contour) belongs to the class C^2 and the function $g(M)$ to $H^{1,\alpha}$, i.e. the space of functions whose first derivatives belong to the class H^{α}, the operator $A_k(g_k) = A(g)$, M, $M_0 \in \sigma_k$ specified by the left-hand side of (2.5) is bounded from the space $H_k^{1,\alpha}$ on σ_k to the space H_k^{α} on σ_k. We introduce spaces $X_k = H_k^{1,\alpha} \times R_k$ and $Y_k = H_k^{\alpha} \times R_k$ that are the topological product of the spaces $H^{1,\alpha}$ and H^{α}, respectively, by a number axis with the norms $\sqrt{\|g\|^2 + C^2}$ and $\sqrt{\|f\|^2 + C^2}$.

We determine the operator $\tilde{A}_k$ from the space X_k to the space Y_k as follows. The element $\left(g_k, \gamma_{0k}\right)$ from the space X_k is placed in correspondence with the element $\left(f_k, C_k\right)$ from the space Y_k according to the rule:

$$\gamma_{0k} + \int_{\sigma_k} \frac{\partial}{\partial \overline{n}_{M_0}} \frac{\partial}{\partial \overline{n}_M} \left(\varphi_k\left(M, M_0\right)\right) g_k\left(M\right) d\sigma_{kM} = f_k\left(M_0\right), \qquad M_0 \in \sigma_k, \tag{2.8}$$

$$\int_{\sigma_k} g_k\left(M\right) d\sigma_k = C_k. \tag{2.9}$$

The operator $\tilde{A}_k$ is specified on the whole space X_k. It is a bounded and one-to-one operator, i.e. it is continuously invertible in the pair of the spaces $\left(X_k, Y_k\right)$.

We now denote by $\left(X_k, Y_k\right)$ the direct sum of the spaces $X_k \left(Y_k\right), \quad k = 1, \ldots, m$, and specify the operator $\tilde{A}$ from X to Y by the rule:

$$\sum_{k=1}^{m} \gamma_{0k} \delta\left(k, p\right) + \sum_{k=1}^{n} \int_{\sigma_k} \frac{\partial}{\partial \overline{n}_{M_0}} \frac{\partial}{\partial n_M} \left(\varphi\left(M, M_0\right)\right) g_k\left(M\right) d\sigma_{kM} = f_p\left(M_0\right), \tag{2.10}$$

$$M_0 \in \sigma_p, \quad p = 1, \ldots, m,$$

$$\int_{\sigma_p} g_p\left(M\right) d\sigma_p = C_p, \quad p = 1, \ldots, m, \tag{2.11}$$

where $\delta\left(k, p\right)$ is the Kronecker-Capelli delta equal to unity for $k = p$ and to zero for $k \neq p$.

It follows from the properties of the operator A that the operator $\tilde{A}$ is continuously invertible in the pair of the spaces $\left(X, Y\right)$. In the following it is the system of equations that we shall numerically solve.

3. CONSIDERATION OF THE SYMMETRY OF SURFACES (CONTOURS)

Let us introduce the Cartesian coordinates *OXYZ* in the space and the coordinates *OXY* on the plane. Let the surfaces (contours) σ_k be symmetric about the plane *OXY* (*OX* axis). The concept of symmetry of the surface is defined below. The incoming flow $\overline{U}_0$ is symmetric about the plane *OXY* (*OX*-axis) if the vectors $\overline{U}_{01}$ and $\overline{U}_{02}$

at the symmetric points M_{01} and M_{02} are symmetric about the plane OXY (OX-axis). Analogously we define the symmetry of the surface. The surface (contour) σ is said to be symmetric about the plane OXY (OX axes) if for any $M_0(x,y,z)\left(M_0(x,y)\right)\in\sigma$ there exists $\tilde{M}_0(x,y,-z)\left(\tilde{M}_0(x,-y)\right)\in\sigma$ and the unit vectors of the external normal $\bar{n}_{M_0}$ and $\bar{n}_{\tilde{M}_0}$ are symmetric about the plane OXY (OX-axis). Then the function $f_p(M_0)=-\left(\overline{U_0},\bar{n}_{M_0}\right)$ will be even with respect to the plane OXY,(OX-axes), i.e. $f_p(M_0)=f_p(\tilde{M}_0)$.

We show that the operator A in this case has the property of transforming even functions $g(M)$ to even functions $f(M_0)$ and odd functions to odd ones. Indeed, equation (2.5) for the two-dimensional case is

$$\frac{1}{2\pi}\int_{\sigma}\frac{\left(\bar{n}_M,\bar{n}_{M_0}\right)r^2_{MM_0}-2\left(\bar{r}_{MM_0},\bar{n}_M\right)\left(\bar{r}_{MM_0},\bar{n}_{M_0}\right)}{r^4_{MM_0}}g(M)\,d\sigma_M=f(M_0),\quad M_0\in\sigma \tag{3.1}$$

and for the three-dimensional case

$$\frac{1}{4\pi}\int_{\sigma}\frac{\left(\bar{n}_M,\bar{n}_{M_0}\right)r^2_{MM_0}-3\left(\bar{r}_{MM_0},\bar{n}_M\right)\left(\bar{r}_{MM_0},\bar{n}_{M_0}\right)}{r^5_{MM_0}}g(M)\,d\sigma_M=f(M_0),\quad M_0\in\sigma \tag{3.2}$$

Because of the symmetry of the incoming flow $\overline{U}_0$ and the surface (contours) σ we find that $f(M_0)$ is an even function with respect to the plane OXY (OX axis).

Further we consider only the two-dimensional case.

The left-hand side of (3.1) is written as

$$\frac{1}{2\pi}\left[\int_{-\pi}^{0}A\left(M(s),M_0(s_0)\right)g(M)\,ds+\int_{0}^{\pi}A\left(M(s),M_0(s_0)\right)g(M)\,ds\right]=f(M_0).$$

(we suppose that σ is described by the parameter of arc length that varies on the segment $[-\pi,\pi]$, and a part of the contour that lies in the upper half-plane is described on the segment $s\in[0,\pi]$).

Let us assume that we have $g(M(s))=g(M(-s))$ for the symmetric points, then

$$\frac{1}{2p}\left[-\int_{+p}^{0} A\left(M(-s),M_0(s_0)\right)g\left(M(s)\right)ds+\int_{0}^{p} A\left(M(s),M_0(s_0)\right)g\left(M(s)\right)ds\right]= f\left(M_0\right)$$

or $$\frac{1}{2p}\int_{0}^{p} g(M)\left[A\left(M(-s),M_0(s_0)\right)+A\left(M(s),M_0(s_0)\right)\right]ds= f\left(M_0\right), \quad M_0\in[-p,p]$$

Note that the function $A\left(M(-s), M(s_0)\right)+A\left(M(s), M\right)(s_0)$ is even relatively s_0. Using this fact we prove the above statement. It fact, using the symmetry of the body, we obtain:

(1) $\bar{n}_{M(-s)}\cdot\bar{n}_{M(-s_0)}=\bar{n}_{M(s)}\cdot\bar{n}_{M(s_0)}$;

(2) $\left(\bar{r}_{M(s)M(s_0)},\bar{n}_{M(s)}\right)\left(\bar{r}_{M(s)M(s_0)},\bar{n}_{M(s_0)}\right)=\left(\bar{r}_{M(-s)M(-s_0)},\bar{n}_{M(-s)}\right)\left(\bar{r}_{M(-s)M(-s_0)},\bar{n}_{M(-s_0)}\right)$.

The equalities follow from

$$\bar{r}_{M_1M_2}=\left(x_2-x_1,y_2-y_1,z_2-z_1\right),$$
$$\bar{r}_{\tilde{M}_1\tilde{M}_2}=\left(x_2-x_1,y_2-y_1,-\left(z_2-z_1\right)\right)$$

$$\bar{n}_M=\left(n_1,n_2,n_3\right),$$
$$\bar{n}_{\tilde{M}}=\left(n_1,n_2,-n_3\right).$$

Trom the equalities we get

$$A\left(M(-s),M(-s_0)\right)=A\left(M(s),M(s_0)\right),$$

$$A\left(M(s),M(-s_0)\right)=A\left(M(-s),M(s_0)\right).$$

Let the equation for the contour (surface)s be $Y=f(x)$ or $\bar{n}_{M_0}\perp OY$ (we have analogous equations for the surface s) in the neighbourhood of the point M_0, then

$$\int_{-p}^{p} g(s)ds=\int_{-p}^{0} g(s)ds+\int_{0}^{p} g(s)ds=-\int_{p}^{0} g(s)ds+\int_{0}^{p} g(s)ds=2\int_{0}^{p} g(s)ds$$

If the function $g(M)$ is even (symmetric) with respect to the plane *OXY* (*OX*-axis), we fix the value of $g\left(M^*\right)$ by fixing the value of $g\left(\tilde{M}^*\right)$, or we fix the value of

$$\int\limits_{\sigma_{1/2}} g(M)\,d\sigma_{1/2} = \frac{1}{2}\int\limits_{\sigma} g(M)\,ds$$

by fixing

$$\int\limits_{\sigma} g(M)\,d\sigma\ ,$$

where $\sigma_{1/2}$ denotes a part of the surface (contour) that lies about the plane OXY (OX axis). The same is true for

$$\int\limits_{\sigma} f(M)\,d\sigma$$

Thus, for the whuch are even (symmetric) with respect to the plane OXY (OX axis) functions $g(M)$ and the surfaces (contours) σ, equation (2.5), conditions (2.3) and (2.7) are equivalent to the equation

$$\int\limits_{\sigma_{1/2}}\left[A\left(\tilde{M},M_0\right)+A\left(M,M_0\right)\right]g(M)\,d\sigma_{1/2} = f(M_0),\quad M_0\in\sigma_{1/2} \tag{3.3}$$

and to the conditions

$$\int\limits_{\sigma_{1/2}} g(M)\,d\sigma_{1/2} = C\ , \tag{3.4}$$

$$\int\limits_{\sigma_{1/2}} f(M)\,d\sigma_{1/2} = 0\ . \tag{3.5}$$

If we again determine the operators $\tilde{A}_k$ from the space $H^{1,\alpha}(\sigma_{1/2})\times R$ to $H^{\alpha}(\sigma_{1/2})\times R$, we obtain the equation (for a single surface σ_k)

$$\gamma_{0k} + \int\limits_{\sigma_{1/2}}\left[A\left(\tilde{M},M_0\right)+A\left(M,M_0\right)\right]g(M)\,d\sigma_{k,1/2,M} = f(M_0),\quad M_0\in\sigma_{k,1/2}\,, \tag{3.6}$$

$$\int\limits_{\sigma_{1/2}} g(M)\,d\sigma_{1/2} = C\ , \tag{3.7}$$

which is uniquely solving for any $f(M)$ and C.

For the system of surfaces we obtain

$$\sum_{k=1}^{m} \gamma_{0k}\,\delta(k,p) + \sum_{k=1}^{m} \int\limits_{\sigma_{k,1/2}} \left[A\left(\tilde{M}, M_0\right) + A\left(M, M_0\right) \right] g_k(M)\, d\sigma_{k,1/2,M} = f_p(M_0),$$

$$M_0 \in \sigma_{p,1/2}, \quad p = 1,\dots,m, \qquad \textbf{(3.8)}$$

$$\int\limits_{\sigma_{p,1/2}} g_p(M)\, d\sigma_{p,1/2} = C_p, \quad p = 1,\dots,m, \qquad \textbf{(3.9)}$$

where are γ_{0k} regulating variables [1].

4. NUMERICAL METHOD AND EXAMPLES OF CALCULATIONS

We solve the system (3.8),(3.9) numerically by the method of discrete closed vortex frames (discrete vortex pairs). In [2.7] the authors show how to construct vortex trails and give a scheme of the numerical solution of (3.6), (3.7) for a single surface. However, the symmetry is not taken into account and operator $\tilde{A}$ is not constructed which, in our opinion, useful in the justification of the above numerical method for the three-dimensional case while the two-dimensional case can be justified using.

We consider, as an example, the calculation of a nonlinear nonstationary flow around of building. We use the following method for building surface.

(a) The surface of each object is represented as a set of independent modules (walls, the pitches of a roof, a roof, columns, etc).

(b) The form and curvature of the surface of each module is determined (according to the Coons method by boundary curves and mixing functions. These functions are determined for two-and three-dimensional modules whereas the boundary curves are given analytically or by points. Besides, additional support curves may be given to define the form of a complicated surface more exactly.

(c) The modules are a constructive mapping of a square such that a rectilinear grid on the square is transformed into a curvilinear grid on the module.

When calculating the flow, the curvilinear grids allow us to approximate the surface by a system of closed quadrangular (triangular) vortex frames.

In order to check whether the geometric of a surface is adequate we have developed a program of visualization of three-dimensional objects which allow us to observe them at any angle. We have used an algorithm for removing the invisible lines which is similar to the algorithm of Z-buffer but is superior as to speed and memory, because only the vortex frames are used for determining the shading rather than each pixel of the screen. The frames may only be employed when a special auxiliary grid is placed on the projection plane. Moreover, in order to better visualize objects of complex forms the cells can be toned, taking into account illumination.

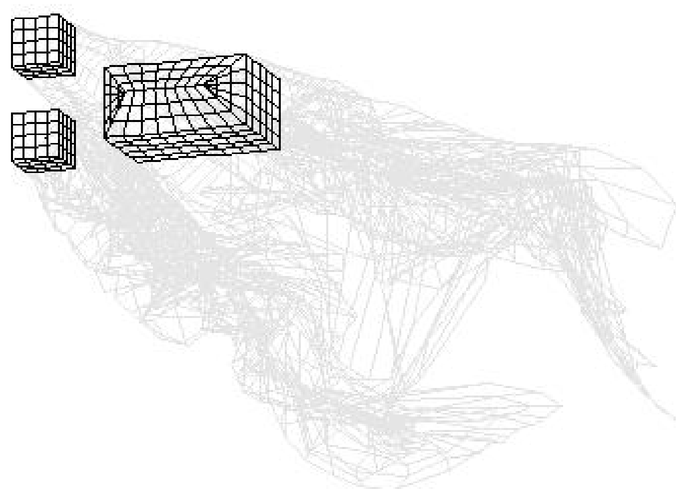

Figure 2

When calculating the aerodynamic flow about bodies with corners we must fix the lines of the separating vortex sheets with the aid of the above visualization program in the dialog mode. The intensities of the separating vortex frames are determined by the intensities of the frames whose sides belong to the line of the separating vortex sheet. The frames are determined automatically in the process of calculations.

The calculations of the aerodynamic flow about a system of buildings show how the vortex location varies with time. This can be observed with the aid of the visualization program (Fig.2). Besides, we have obtained the intensities of the free vortex frames and time variations in the vortex frame intensities approximating the surface of the buildings. The vortex frame intensities and the vortex location are necessary for calculating the velocity fields and the required aerodynamic characteristics.

The velocity fields are calculated on fixed planes arbitrarily oriented in space. To run the program we must determine the sections of the buildings and a rectangular domain on the plane, as well as a grid at the nodes of which the flow velocities are calculated. The domain sizes are determined by the location of the vortex sheet as the main source of disturbances. Moreover, the flow velocities are calculated only at the points lying outside the buildings and above the ground.

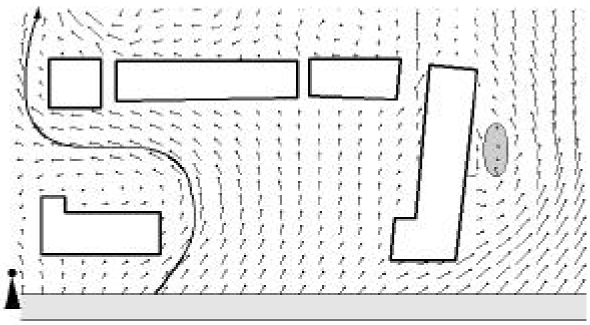

Figure 3(a)

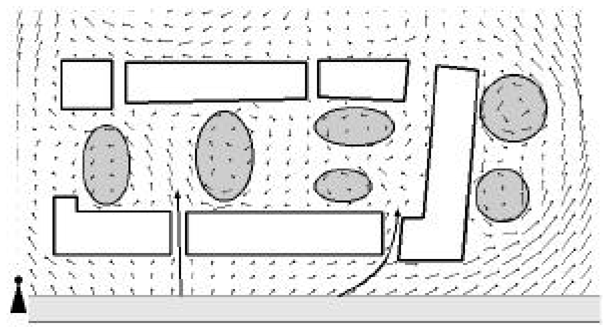

Figure 3(b)

On the basis of the above design procedure, the appropriate computer program, as well as the study of a degree of discretization and the realiability of the procedure we obtain the practical results while carrying out an examination of the version of building an area in the centre of Moscow. The calculations were carried out for different wind directions, using the seasonal wind rose (here the calculations are given for winter time); for the existing and projected.

The problem of aeration was also important as there are the traffic lights on the highway in the immediate vicinity of the buildings and the drivers waiting for the green light do not shut off the engines. In the existing version the bulk of the air containing a large amount of the exhaust gases would move along an S-shaped trajectory (Fig. 3a) from the highway but a small vorticity zone C of low intensity would also form. In the presence of the projected building two vorticity zones instead of one would form, and their intensity would be approximately 1.5 as much. What really matters is that zones of stagnation that can exist rather long would form in the inner yard. The air in the zones would be saturated with the exhaust gases coming from the highway. This, it is owing to the aerodynamic examination that the project has not been realized.

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