Student Teachers' Understanding about the Use of a Computer Software Programme in the Representation and Teaching of Linear Functions

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Recent research on computers as a vehicle to promote mathematical learning has shifted attention to the examination of the role of teacher's knowledge in the use of computer aids. In this paper, I report findings of a study in which the knowledge base of a preservice student teacher in the area of linear functions was analysed in terms of his knowledge about the use of a software that could be used to graph functions. Preliminary results suggest that the student did not integrate knowledge of functions and that of the software well enough to promote deeper levels of understanding about linear functions among his students.

Introduction

Mathematics educators and teachers have invested considerable effort in exploring the potential of computers in helping our students develop better grasp of the subject-matter. That computers could play a significant role in enriching the classroom experiences that teachers provide has received further attention in major curricular documents (Australian Education Council, 1990; National Council of Teachers of Mathematics, 1989). Indeed, it is now generally agreed that the appropriate use of computers in the instructional process is necessary, and that such practice could promote deeper levels of conceptual understanding among students.

Concurrent developments in the area of cognitive psychology and domain expertise have had significant effect on our understanding of deeper levels of processing of mathematical information by the teacher and the student. However, little effort has gone into utilising this knowledge about teachers' conceptual understanding and knowledge construction in the examination of how computers could be used as effective instructional aids. Specifically, there is little data on the question of the relationship between teachers' understanding of mathematical concepts and how this would affect their use of computers as instructional tools.

Background

In recent years researchers interested in improving students' mathematical performance have focused on the quality of knowledge that teachers develop, and how that knowledge is accessed and exploited during planning and delivery of instruction (Clark and Peterson, 1986; Lawson and Chinnappan, 1994; Schoenfeld, 1992). Developments in cognitive psychology particularly in the area of expertise indicate that there are three major components which could be related to the knowledge base of expert teachers which permit them to perform their role more efficiently and effectively: teachers' mathematical content knowledge, the organisation of this knowledge and the blend of knowledge of content and pedagogy. *Mathematical content knowledge* includes information such as mathematical concepts, rules and associated procedures for problem solving. The *organisation of the content knowledge* refers to the links that teachers construct between the various components of content knowledge. The *blend of content and pedagogical knowledge* includes understandings about why some students experience difficulties when learning a particular concept while others find them easy to assimilate, knowledge about useful ways to conceptualise and represent a chosen concept (Feiman-Nemser, 1990), the quality of explanations that teachers generate prior to and during instruction (Leinhardt, 1987) and the characteristics of the learner. This category of knowledge has also been labelled as *pedagogical content knowledge* (Shulman, 1986).

While there may be other components of the teachers' knowledge base which are relevant for teaching mathematics, I regard the above three to be essential if a teacher is to succeed in helping his or her students take an active part in the learning and appreciation of mathematics. The general emphasis on thinking about teachers' mathematical knowledge has been investigated in a number areas of the school mathematics including *functions*. (Even, 1993; Norman, 1993; Wilson, 1994). In the high school curriculum, the understanding of functions, the various forms of functions and their applications is essential for making satisfactory progress in other areas such as calculus and analytical geometry, and higher mathematics that students could choose to pursue in their tertiary studies. Curriculum Standards (National Council of Teachers of Mathematics, 1989) has identified various aspects of functions that are important for a greater depth in students' understanding of functions such as modelling real-world contexts using functions, and classifying and describing families of functions. Included in this category are three further aspects to the understanding of functions: interpretation of functions as represented by graphs, translations among multiple representations of functions and the application of technology to examine general and specific characteristics of functions (Wilson, 1994).

Although some progress has been made in the area of graphical representation of functions and tables, there is less information about the application of technology in the interpretation and teaching of functions. Investigations on teachers' knowledge of functions and the teaching of functions have received little attention particularly in relation to cognitive issues involving the identification and analysis of the nature of knowledge that teachers access when computer aids are used in the teaching/learning process. Specifically, studies of teacher expertise in the past have not examined the nature of teachers' knowledge and understandings that are relevant to using computer softwares in helping students come to terms with definitional aspects of core concepts, classification of types of functions and the use of these ideas in problem solving contexts.

As mentioned earlier, understandings that teachers develop about the interaction between mathematical knowledge of functions, student difficulties with this topic and the role of computers in promoting student's performance levels in this area of functions constitutes what is broadly termed as pedagogical content knowledge about the teaching and learning of functions. An important aspect of this knowledge is for teachers to articulate the strengths and weaknesses of individual students in the classroom, and to use this knowledge to exploit computer softwares in order to help all students. For example, some students experience difficulties when learning about specific ideas while others find this area relatively easy. This situation could be given the following explanation. Mathematical ideas are acquired and stored in the long-term memory in the form of *conceptual schemas* which are organised knowledge structures (Anderson, 1977). A well constructed conceptual schema has many points of contact with which to relate. According to this analysis of understanding, students with a better grasp of a concept, say, gradient, will have built more and better integrated points of contact which could exist in the form of attributes such as steepness, a sketch and ratio of the vertical:horizontal distance. These students will be able make sense of gradients more easily because of the multiple representations they have constructed. This is in contrast to the weaker students who may have fewer or no points of contact with the concept of gradient. A teacher who is aware of these problems about their students is better placed to help both these groups of students.

The question remains as to how could teachers use computers to help students generate the multiple representations of the type I have discussed above. A sound understanding of computer softwares would help teachers utilise the environment to examine ways of building and illustrating a concept that is of interest. Using the example of teaching the concept of the gradient of a straight line, teachers could demonstrate to the students how to draw a straight line on the screen. The next step could involve asking students to draw straight lines with different slopes, and discussing the steepness of these lines in relation to the idea of gradient. In so doing the teacher is supporting the construction of multiple links about gradient with the aid of the computer software. Thus it can be seen that the analysis of how teachers will go about integrating their mathematical knowledge with that of computer environments forms an important issue for researchers.

One way to generate data that is relevant to the above issue of nature of knowledge that classroom teachers use when they utilise computer technology for instructional purposes is to examine the type of mathematical content and pedagogical knowledge that student teachers activate in such situations. Further, by following changes in the quality of that knowledge as the level of expertise of these student teachers increase we are able to inform educators about activities that would help in the training of future teachers in the effective use of computers for mathematics learning.

This study forms the first phase of a larger project about teachers' level of expertise and their use of technology in teaching mathematics. The purpose of the study reported here was to identify the nature of mathematical and related knowledge that preservice student teachers accessed during the teaching of the topic of functions with the aid of a graphing software. The principal research question of this study was: What evidence is there that preservice students have a well-developed knowledge about a) the concept of linear functions, b) the relationships between knowledge of linear function and other areas of mathematics, c) the teaching of linear functions and c) the use of a graphing tool to foster the development of the concept of linear functions and related ideas. In pursuing these questions, I carried out a detailed case study of one student teacher's knowledge base.

Method

Participant

Michael is a third year BEd(Secondary) student currently enrolled in the unit on secondary mathematics methods. Before this study, he has had no formal teaching experience. Michael spent two weeks observing teachers in his second year of the course. During the last two years he has completed mathematics discipline requirements for the BEd(Secondary) which includes calculus, analytic geometry and statistics. Michael has had no prior experience in the use of computer softwares in learning or teaching mathematics, except the application of DERIVE in his first year calculus course.

Material and Procedure

Michael volunteered to participate in this study. The investigator met Michael on two occasions. During the first meeting which lasted about sixty minutes, he was trained in the use of a graphing software, ANUGraph version 2.08 (Smythe and Ward, 1987) for Macintosh LCIII. The investigator introduced the software and showed the various parts of the menu. The purpose of this session was to train Michael in the use of ANUGraph. Michael was given ample time to experiment with this tool and raise questions about the various procedures that can be executed in that environment. Towards the end of this session, Michael was given three focus questions to think about for the next session. These questions addressed the following areas: major concepts about linear functions, the relationships between linear functions and other concepts in mathematics, the teaching of linear functions and associated concepts using ANUGraph. During the second session, Michael was taken through the software again, and asked to respond to the above-mentioned questions. He was encouraged to use the software to illustrate his views. The investigator probed certain responses with questions relevant to the aims of the study.

The interview session was audio taped and transcribed. The transcripts were then analysed for evidence of four groups of knowledge: content knowledge about linear functions, organisation of this content knowledge, pedagogical content knowledge and construction of links between these three knowledge components and knowledge of ANUGraph. A unit of content knowledge refers to any identifiable concept in the area of linear functions. A unit of content knowledge was deemed to be organised when there was an unambiguous indication of correct link(s) made between concepts, procedures, principles or rules.

Results

Table 1 shows some examples of the three knowledge components that were considered to be important for teaching linear functions. All three examples in the content knowledge category were relevant to the teaching of linear functions. In the organised content knowledge category, the examples presented in the table shows that Michael has a good command of the various mathematical concepts (scales, x- and y-coordinates, ratios), and that he had constructed important connections between these concepts. The example involving pedagogical content knowledge category suggests that Michael expects his students to be motivated by the use of ANUGraph and that student exploration constitutes a useful way to encourage students to develop further understandings about the concept via this software.

Table 1 : Selected examples of Michael's knowledge base

Knowledge component	Selected examples
Content knowledge	equivalent equations substitution steepness of a slope
Organised content knowledge	linear function involves graphing scaling is shown on x and y axis y = 2x means the ratio of y:x is 2:1, for every point on the line you go one unit across and two units up.
Pedagogical content knowledge	let the kids play with the software and they will pick up things by discovery

In order to generate further information about the extent of Michael's knowledge in the area of teaching linear functions, I searched the transcripts again and determined the frequency of occurrence of items of knowledge that could be placed in any one of the above three categories. These three categories appear under the Knowledge Component (KC) column of Table 2. As the focus of the study was on the *interaction* between knowledge about the use of the computer software and knowledge about teaching linear function, the data was subjected to a second analysis in which I looked for evidence of links between an item of knowledge from the above categories and ANUGraph. For example, if Michael mentioned the x-coordinate of a point, and showed how the software helped his students identify this coordinate, then his content knowledge about the x-coordinate was considered to be linked to the software. We determined the number of times this occurred for each of the three categories of Knowledge Components. This data, referred to as Knowledge Components Related to Software (KCRS) appears in the second column of Table 2.

Table 2 shows that Michael had built up a reasonable amount of knowledge about linear functions in all the four areas hypothesised in this study. He was able to access 36 items of content knowledge, 12 units of organised units of content knowledge and 12 units of knowledge that showed he was aware of the learning and teaching of the concepts mentioned during the interview. I subsequently determined the percentage of these knowledge items that were related to the software in question. This analysis was required to examine the issue of how well Michael's mathematical and pedagogical knowledge was integrated with the use of ANUGraph.

Table 2: Frequency of knowledge components

Knowledge Component (KC)		Knowledge Components that were related to software (KCRS)	KCRS as a percentage of KC
Content Knowledge	36	13	36
Organised Content knowledge	12	2	17
Pedagogical Content knowledge	12	2	17

Table 2 shows that despite the training given in the use of the software, Michael did not make sufficient use of the many of the options available within ANUGraph. This is evidenced by the fact that only 36% of content knowledge was related to use of the software. This situation for example, is illustrated more clearly in the content area of plotting a linear function. Michael discussed two ways of plotting a linear function. Firstly, he outlined a method of determining two points on the graph when the y-intercept and gradient are given, and then joining these two points. In the second method, Michael suggested that students could plot a linear function by using a set of values for x and y coordinates. >From a teaching viewpoint, both these approaches are sound and they do provide an insight into aspects of understanding the geometric and algebraic representations of linear functions. However, the construction of these forms of the linear function could be significantly enhanced by the *ANUdata* option provided in the software. This option not only provides an efficient method to plot a linear function from a table of x and y values, it also encourages students to determine the equation of the straight line.

The data presented in Table 2 also indicates that there are gaps in Michael's knowledge in the way the software could have been used to build on or highlight the links between various components of knowledge associated with linear functions. One such area of involves the solution of two linear equations which could have been achieved with dramatic effect by plotting the two equations and determining their coordinates of the point of intersection using the *show-coordinates* and *zoom* options available in the software.

The investigator expected Michael to make a few comments about the teaching and learning aspects of linear functions, and how the implementation of his chosen approaches would enhance or hinder student participation and learning outcomes. As shows in Table 2, Michael made 12 remarks that is related to the pedagogical content area. In almost all his explanations, Michael seems to be preoccupied with how *he* would learn linear functions with little consideration to the expectations, abilities, attitudes and other characteristics of his students. What is equally interesting is the minimal connections made between the pedagogical content knowledge and the software itself.

In order to provide a better picture about how Michael could have exploited the software let's look at Figure 1. Figure 1 shows the two linear function that were constructed by Michael in his attempt to show their steepness. While this effort clearly constitutes an important way to use the software, Michael did not use this situation to illustrate other concepts which were located in his repertoire of content knowledge, such as points on a given graph should satisfy the algebraic relationships represented by the equations. For example, by using the *show-coordinates* option he could have moved points P and Q along the respective straight line graphs and investigate how this affects the relationships between the coordinates. Furthermore, in relation to the function y=x, Michael made the following observation:

Graph of y=x, line going through straight through and we would expect that it will be equal to 45 degrees...

The above statement again shows that he expected students to work out that the angle between the graph and the x-axis has a magnitude of 45 degree. This point could have been more clearly and more spectacularly by using the cursor to find out the coordinates of, say point Q, and asking students to use the right-angled triangle created by dropping the vertical and horizontal segments. This activity could then be followed up by applying the tangent ratio to the angle in question. Such an approach has the potential to encourage students appreciate not only the power of the computer environment, but more importantly, help student make connections between knowledge about trigonometry acquired in a non-Cartesian system.

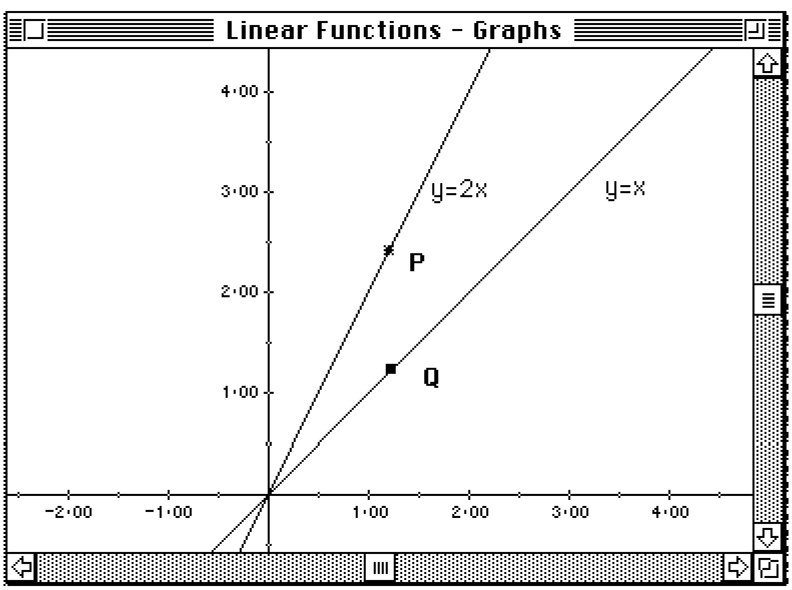

Figure 1

Discussion

The purpose of this pilot study was to generate data about the nature of student teachers' knowledge in four areas, namely, knowledge about linear functions, how this knowledge is connected to other mathematical knowledge, knowledge about the teaching and learning of linear functions, and the use of a software to enhance the understanding of linear and related functions.

In order to address these questions, this study used a framework for teacher knowledge representation which included three major knowledge components: mathematical content knowledge, organisation of mathematical content knowledge and pedagogical content knowledge. Analysis of one student teacher's knowledge base suggests three patterns. Firstly, Michael has a reasonably well developed knowledge in the content area of linear functions. He was able to provide at least three ways to represent linear functions and several examples to illustrate this point.

In the second area of interest concerning the structure of knowledge of functions, one could detect a number of gaps in the knowledge base of the participant. For instance, Michael did not make any connection between the concept of linear functions and the solution of equations via graphical or any other means. This gap in the relational knowledge is particularly significant given that the software could have easily been utilised for this purpose. Table 1 provides further evidence for the lack of connections between items of knowledge involving gradient of a line. Michael attempted to explain gradient by referring to the idea of steepness of the slope. While this is a reasonable, he could have drawn on his knowledge about tangent of the angle between the line and the x-axis, and related this to gradient of the line. Alternatively, he could have used the idea of 'going across and going up' (Table 1) to illustrate gradient which he did not do. Prawat (1989) argued that a well organised knowledge structure aids in the accessing and use of that knowledge during teaching. The poorly connected state of Michael's knowledge suggests that he could experience problems in a) constructing alternative representations of the mathematical concepts, b) recreating these representations within a computer environment and c) activating available representations flexibly during the teaching process.

Interview data also suggests that Michael is less concerned about the learner in the teaching about linear functions. There were few instances where he made reference to the learner in the form of learner's prior knowledge, their attitudes to and beliefs about the topic, and the value of using ANUGraph. Knowledge about the learner and how the learner would process the given content knowledge (Peterson, 1988) constitutes a critical factor in the acquisition and further development of the content knowledge. In this regard, on the basis of what Michael has said during the interview, one is led to the conclusion that he is not aware of the importance of understanding the learner in the learning/teaching situation.

In the analysis which focused on how and to what extent the student teacher related knowledge about the software to knowledge of linear functions, there is some evidence that Michael was able to perform most of the routine functions such as constructing the equation for a function and graphing it. He showed sufficient understanding of how the visual impact of this software could be utilised for the purposes of illustrating gradients of a family of linear functions. However, he did not exhibit a similar ability at using the software to solve or pose novel problems involving the construction of linear functions. For example, as mentioned earlier, the software has the facility to generate a linear equation for a given table of values by using the *ANUdata* option. Despite being alerted to the availability of this option, Michael made no use of this information. The use of this option would challenge students having to make connections between two representations of linear functions: the tabular and algebraic. The ability to move flexibly between these two representations is considered to be indicative of deeper understandings in this area (Moschkovich, Schoenfeld and Arcavi, 1993). This facility was not exploited by the student teacher. Furthermore, ANUGraph has the potential to be used as a tool for testing conjectures about linear functions. Michael did not show any attempt at utilising this option either.

It must be pointed out data provided here is preliminary in nature. In generating the frequency data shown in Table 2, I was mainly interested in determining instances of the various categories of knowledge components. Table 2 does not show links that might have been established between items identified in the three categories. It is proposed that future analyses of data should examine these connections which could help us gain insights into why certain links were or were not established.

Implications

The use of computers is increasingly been accepted as a viable alternative to the traditional paper and blackboard approaches to teaching mathematical concepts. While this view has its merits, it is based on the assumption that teachers will be working from a sufficiently developed and organised knowledge base about the content area of linear functions and the power of the software in order to construct and demonstrate multiple representations of mathematical concepts. Tentative results from this pilot study suggest that while preservice teachers may show acceptable levels of content knowledge, they may not have this knowledge integrated adequately with their knowledge about computer softwares. Preservice students could also be expected to pay little or no attention to the learning needs of individual students in the class. While it is too early to generalise on the basis of this study, the results do seem to suggest that teacher education programs need to analyse the mathematical content and software interface carefully. Any effort along these lines has the potential to inform teachers about how we could harness the power of computer technology more effectively and completely during mathematical instruction.

References

Anderson, R. C. (1977). The notion of schemata and the educational enterprise: General discussion of the conference. In R. C. Anderson, R. J. Spiro & W. E. Montague (Eds.), *Schooling and the acquisition of knowledge*, Hillsdale, NJ: Lawrence Erlbaum.

Australian Education Council (1990). *National Statement on Mathematics for Australian Schools* . Carlton, Vic: Curriculum Corporation.

Chi, M. T. H., Feltovich, P. J. & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, *5*, 121-152.

Clark, C., & Peterson, P.L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 255-297). New York: Macmillan.

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*(2), 94-116.

Feiman-Nemser, S. (1990). Teacher preparation: Structural and conceptual alternatives. In W.R. Houston, M. Haberman & J. Sikula (Eds.), *Handbook of Research on Teacher Education* (pp. 212-233). New York: Macmillan.

Lawson, M.J., & Chinnappan, M.(1994). Generative activity during geometry problem solving: Comparison of the performance of high-achieving and low-achieving students. *Cognition and Instruction*, 12 (1), 61-93.

Leinhardt, G. (1987). The development of an expert explanation: An analysis of a sequence of subtraction lessons. *Cognition and Instruction, 4*(4), 225-282.

Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema & T .P. Carpenter (Eds.), *Integrating Research on the Graphical Representation of Functions* (pp. 69-100). Hillsdale, NJ: Lawrence Erlbaum.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council.

Norman, F. A. (1993). Integrating research on teachers' knowledge of functions and their graphs. In T. A. Romberg, E. Fennema & T .P. Carpenter (Eds.), *Integrating Research on the Graphical Representation of Functions* (pp. 159-187). Hillsdale, NJ: Lawrence Erlbaum.

Peterson, P. (1988). Teachers' and students' cognitional knowledge for classroom teaching and learning. *Educational Researcher*, *17*, 5-14.

Prawat, R. (1989). Promoting access to knowledge, strategy and disposition in students. *Review of Educational Research*, *59*, 1-42.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In Douglas A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, (pp. 334-370). New York: Macmillan.

Shulman, L.S. (1986). Paradigms and research programs in the study of teaching: A contemporary perspective. In M. C. Wittrock (Ed.) *Handbook on Research in Teaching*. New York: MacMillan.

Smythe, N., & Ward, M. (1987). *ANUGraph*. Canberra: The Australian National University.

Wilson, M. R. (1994). One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education*, *25*(4), 346-370.
