Teaching and Learning Calculus Using Computer

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ABSTRACT: *Learning mathematics is a major focus of educational institutions at all levels and technology has long been an important teaching tool in the field of mathematics. The new technological tools such as computers and computer software have provided educators and students with more opportunities to teach and to learn mathematics in new ways. The used of multimedia interactive courseware with its ability to visualize concepts will help instructors to explain calculus concepts, which previously, cannot be illustrated through the conventional methods of teaching. This paper will discuss the effectiveness of the Teaching and Mastering Calculus Computer Courseware (TEMACCC) on the achievement of diploma students in the subject Calculus. Specifically, this paper will discuss the effectiveness of TEMACCC versus traditional teaching and tutorial type on the achievement of diploma students in the subject of Calculus. This research is a quasi-experimental research involving students from three groups assigned to a control group (28 students), TECCC group (24 students) and TEMACCC group (23 students). TECCC and TEMACCC group are defined as two treatment groups. The statistical analysis revealed that there was a statistically significant difference between students' achievement in the control and TEMACCC groups. Findings indicate that students in TEMACCC group have better performance compared to those in the other two groups. This research implies that TEMACCC could be use as a tool in teaching and learning calculus.*

1.   Introduction

Computer technology is becoming an important entity in the lives of student today especially in tertiary education. At this level, technology use can transform a classroom from a boring place to an exciting learning environment for mathematics. Besides that, the methods of teaching and learning mathematics have changed as a result of the evaluation of new education technologies. As [10] had clearly recognized that technology should play a crucial role in the mathematics classroom, it is recommended that all students should be provided with the necessary tools to enhance the learning of mathematics. Computers should also be made available for demonstration purposes, individual work and group work.

It is important for educators to find new ways using technology to enhance teaching methods and thereby to improve learning. There are various types of mathematics software that can be found in the market today. Most of the software programmes were developed based upon the needs of the learners. Software programmes such as tutorial software is to access students' knowledge and create a plan that will help strengthen the students' weakness. This type of software will provide individualized instructional plans based on the objectives that the student has not mastered yet. There is also mathematical software for the instructional purposes. Instead of teaching

mathematics on black board or whiteboard, computers have provided mathematics teachers with a brand new channel of teaching difficult and abstract concepts using the multimedia technologies. Learning mathematics on the computer screens can be regarded as a mode of visual entertainment for students. Educators can present materials during the class sessions and help students visualize abstract concept that can be hardly explained using the traditional method. By using technology during the classroom, the educator will play a different role, instead acting as presenter of the class materials to provide of methods to meet the needs of the individual learner. The students are no longer competing against themselves in order to achieve their individual objectives.

Calculus is one of the most important and fundamental courses for students in tertiary education especially students pursuing in the engineering and science field. During the first year, instructors are supposed to teach students the basic concepts and theories of calculus by helping them to understand the profound mathematical ideas and improve their abilities to think logically, deeply and creatively. However, these students are not given opportunities to construct what they have learned in most traditional calculus classroom at tertiary level. In the traditional classroom, the instructors will demonstrate the procedure associated with a particular concept and then having students drill the procedure. As a result, many students cannot achieve a deep understanding even when they have finished learning it. Many students found that calculus is abstract, boring and hard to learn. Besides that, a wide range of calculus concepts that cause problems for students have been identified as common problems including algebraic manipulation [11], understanding limits (see [17] and [26]), use of notations (see [12] and [16]), and function (see [5], [18] and [20]).

Modern technology, especially computer assisted instruction (CAI) is widely used in teaching and learning. Many teachers or instructors have begun to use mathematical software such as MATHEMATICA, MATLAB, MATHCAD and so on in their mathematics lessons. With computer LCD replacing the blackboard or transparency writing or the combination of both, instructors will have more time to explain mathematics and students are learning better and more effectively with deeper understanding. The review of relevant literature on the effectiveness of CAI has yielded inconsistent results. Some studies reported there technology can enrich learning environments of CAI while some studies prove that CAI does not add any favorable preference on students' mathematical achievement. [15] described CAI as designed to supplement existing forms of education rather than to replace them. In CAI, computers are used as tools and assistants in classroom to extend and contribute to what is already being offered. Meanwhile, [19] found that CAI improved mathematics achievement of regular education, special education and limited English proficient middle school students on mathematics pre – post tests when compared to traditional and teacher directed practice techniques.

A research study by [7] that compared students' achievement in computerized instruction and traditional instruction in developmental algebra course at a university. The achievement test is based on the pre and post tests. The traditional lecture based classes showed a significantly higher amount of achievement than the computer class. Meanwhile [1] compared the effectiveness computer based instruction to a traditional lecture based instruction. A pre and posttests were given to determine the effects of using geometer's Sketchpad versus instruction that utilized strictly paper and pencil activities. The report showed that participants in the experimental group scored significantly higher on the posttest than control group. A study conducted by [2] on the influence of CAI on mathematics achievement of low achieving students. They found the low achievers did not perform significantly better. [3] introduced a computer assisted instruction problem solving called

MATHCAL. The effectiveness of the system was tested through an experiment on 49 fifth grades student who were divided into experiment and control groups. Pre and posttest results indicate that
(a) Students in the experiment group showed significantly more improvement than the control group on the post test;
(b) There was a significant difference between pre and posttest results in the experimental group;
(c) The control group showed improvement but there was no significant difference between pre and posttest.

Research by [8] compared students using *Ethnomathematics* software group and traditional group showed that there is a significant difference between both groups with students using *Ethnomathematics* had higher scores.

2. Research Objectives

This paper will give a brief description about TEMACCC package which consists of teaching calculus computer courseware (TECCC) and mastering computer calculus software. An experimental test was conducted among the three groups to test the effectiveness of TEMACCC package. The first group (Group 1) followed a calculus lecture and tutorial using traditional method. For the second group (Group 2), TECCC was used for teaching calculus during lecture and tutorial class will be handled in a traditional ways. Lastly, Group three used both TECCC and MACCC package as a method to deliver lecture and tutorial class activity. This paper also discusses the effectiveness of using TEMACCC among Groups 1, 2 and 3.

3. Description of TEMACCC

The teaching package known as the *Teaching Calculus Computer Courseware* (TECCC) was developed for teaching calculus in a lecture room. TECCC was developed using the Direct Instructional Method. Meanwhile, the other package is for tutorial class activity. The *Mastering Calculus Computer Courseware* (MACCC) was developed based on the Mastery Learning Model which will help students to master difficult concepts in Calculus. The combination of this two packages were known as the Teaching and Mastering Calculus Computer Courseware (TEMACCC).

Teaching Calculus Computer Courseware.

The Teaching Calculus Computer Courseware TECCC comprised five modules which are teaching and learning module, tutorial module, and quiz module (see Diagram 1). TECCC was developed based on the direct Instruction Model. Direct instruction is a general term for the explicit teaching of a skill-set using lectures or demonstrations of the material (see [9]). It involves some explication of the skill or subject matter to be taught and may or may not include an opportunity for student participation or individual practice. Direct instruction is a method of classroom management that begins with review of material, and then proceeds with briskly paced presentation of new material to maintain student attention, guided practice, and independent practice.
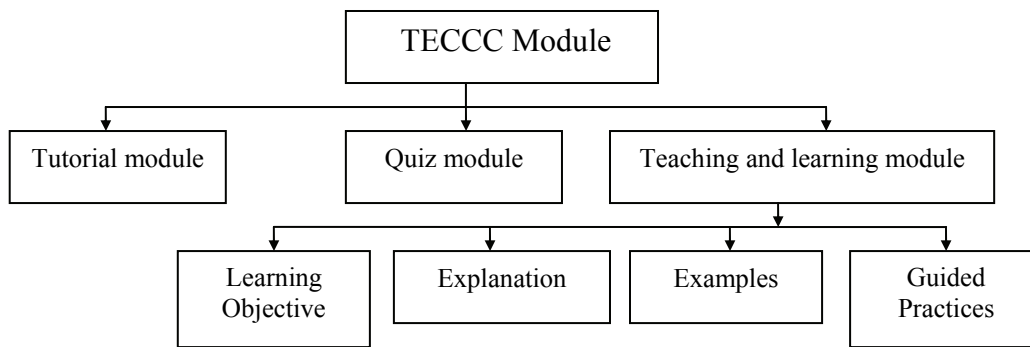
Diagram 1 : TECCC Module

TECCC module consists of three main chapters in calculus which are limits, derivative and integrations and each chapter has several subtopics.
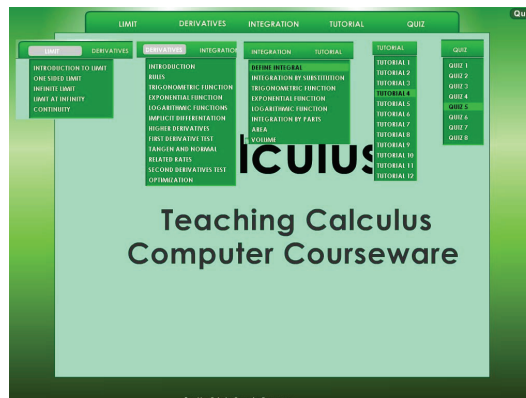

Diagram 2 : TECCC Chapters and Subtopics

For the teaching and learning module, students will be representing the learning objective first. This is for the student to know what they should achieve at the end of the lesson. The explanation phase is where instructors will represent the course material. Each explanation phase start either with a video clip, every day life scenario or examples that would help to develop students' background knowledge. Most of the concept will be explained using animation. By using animation, it would help students' process information on the concept and have better understanding. The next process is by giving guided practices and also exercises for the students to practice during the lecture.
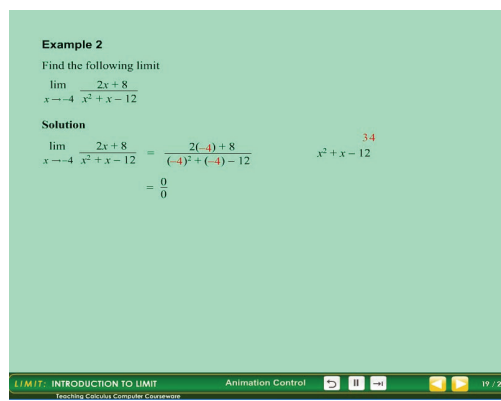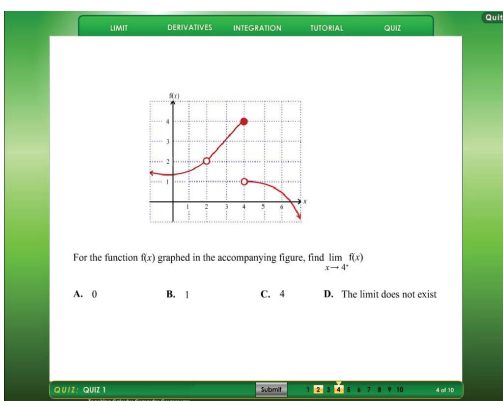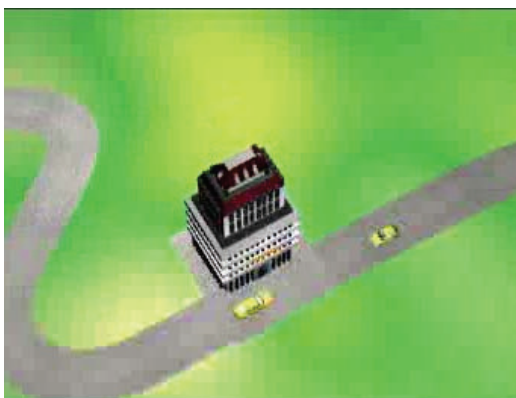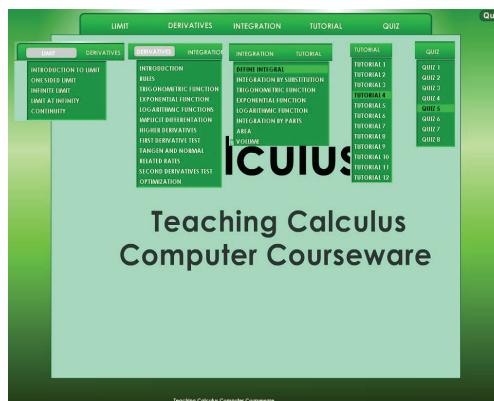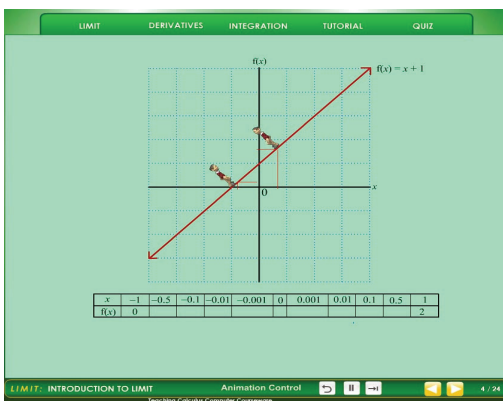
Diagram 3 : Screen Snapshots of TECCC

Mastering Calculus Computer Courseware

Mastering Calculus Computer Courseware (MACCC) was developed by the researchers. MACCC is a tutoring courseware to help student's mastery of calculus concepts.   MACCC, which incorporates mastery learning concepts and strategies, is designed to take into account individual difference among learners in such a way to help student's master the concepts that they have not learned yet. For the purpose of this study, two topics in calculus were used. Those topics are limit and derivative. MACCC consists of two types of courseware, which is Mastering in Limit (MAIL), which covers topics in limit, and Mastering in Derivatives (MAID) for topics in derivative. Both

MAIL and MAID consists of five modules, which are the diagnostic module, explanation module, exercise module, and exploration module (refer to Diagram 4).
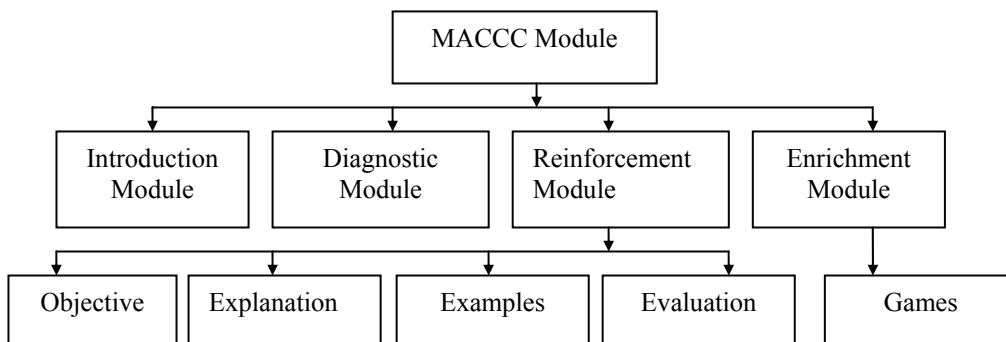


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                        │  MACCC Module   │
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  │Introduction│  │ Diagnostic │  │ Reinforcement │  │ Enrichment │
  │  Module    │  │  Module    │  │    Module     │  │  Module    │
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Diagram 4 : MACCC Module

MACCC starts with users choosing what topic they want to master. For example, in MAIL courseware there are four sub topics like introduction to limit, one sided limit, infinite limit and continuity. MACCC starts with a diagnostic test. Users have to answer several questions to test their knowledge in each and every concept related to the topics chosen. After answering all the questions, the system will analyze the responses given by the users.

Diagram 5 : Diagnostic Test Module

Users without the mastery of certain concepts (70% correct) need to do a reinforcement module while the enrichment module is for those who had mastered all the concepts in the tutorial selected. Enrichment module consists of questions related with the tutorial. The purpose of this module is to help students to try out more challenging questions. In MACCC, the enrichment module is built based on game concept (see Diagram 6)
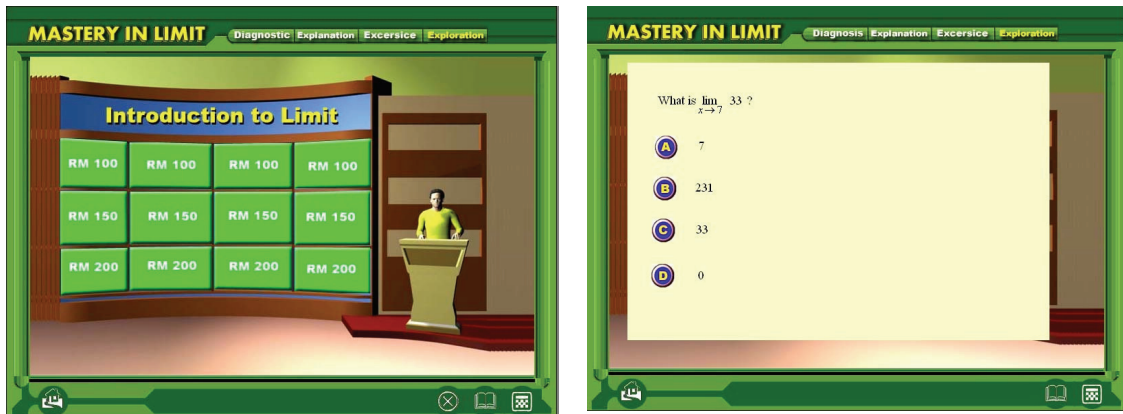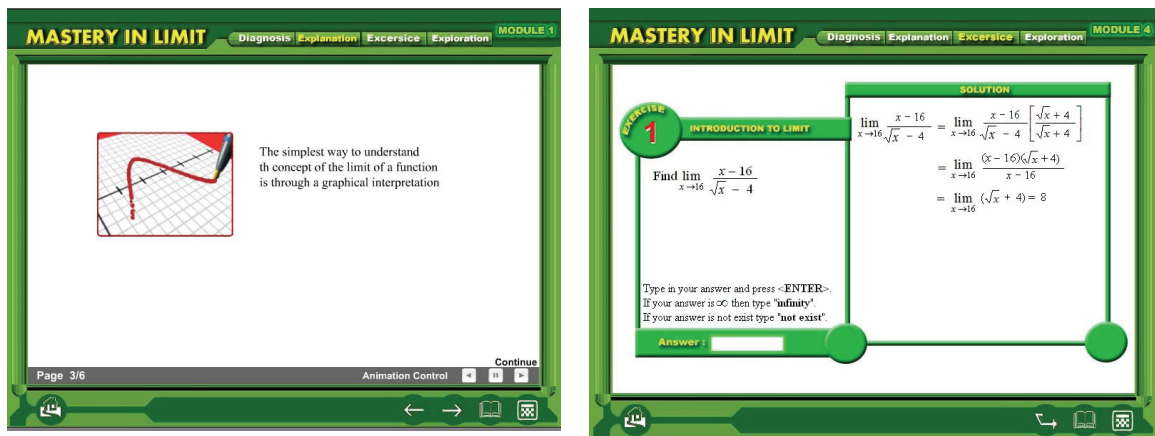
Diagram 6 : Enrichment Module

Diagram 7 : Reinforcement Module

During the reinforcement module, users will be given explanation of the related concepts. These include using animation and also examples to provide more understanding to the users. After finishing the explanation module, users will be given questions related to the concepts. Again, users have to achieve scores of 70% to proceed to the next concept, which they have not mastered or go directly to the exploration module. If the users cannot achieve the mastery level, they have to repeat the reinforcement module. This process will continue until users have reached the mastery level of the concepts.

4. Research Methodology

Samples

Participants involved in this research were diploma college students who were enrolled in a fundamental calculus course. These students are categorized as low achievement students based on their low performance of mathematics in the schools. This study was conducted using the quasi experimental design which comprised three groups from three different colleges consisting of the control group labeled as Group 1 (n= 28 students) and two treatment groups which is labeled as Group 2 (n=24 students) and Group 3 (n=24 students). According to [5] and [6], the minimum number of samples needed for experimental studies is fifteen. A Mathematics test was administered

on the participants in order to obtain the achievement scores. The test consists of 10 questions from the two topics.

Research procedures

The duration of the experiment was nine weeks. In the first week, students from both groups were given a diagnosis test. This diagnosis test was to identify the present knowledge of students in mathematics. The diagnosis test comprised questions in the manipulation of algebra, functions, derivative and integration. The questions are based on the mathematics and additional mathematics at the school level. Each groups received the same amount of contact hours which is two hours of lectures and tutorial class. For the control group, the lecturer handled the tutorials and lectures in the lecture hall. Meanwhile, for the treatment group, lectures were conducted with the aid of a computer and LCD and the tutorial sessions were at the computer laboratory. The achievement test was conducted after the experiment session. A lecturer from the Mathematics Department corrected the achievement test of every student.

5. Findings

The achievement test is based on the score that every student achieved at the end of the experiment. The means and standard deviation for the achievement test of all three groups are shown in Table 1.

Table 1 : Mean and Standard Deviation For All Three Groups

Group	Numbers of Students	Mean (M)	Standard Deviation (SD)
1	28	30.64	11.56
2	24	39.67	18.49
3	23	57.74	22.35

Referring to Table 1, Group 3 scored the highest mean (M = 57.74, SD = 22.35) followed by Group 2 (M = 39.67, SD = 18.49). Group 1 scored the lowest mean (M = 30.64, SD = 11.56) compared to the other two groups.

A one-way ANOVA analysis was used to compare the mean scores of the achievement test among the three groups. The results are as in Table 2. Table 2 indicates that there is a significant difference of test achievement scores among the groups [F(2,72) = 15.130, p = .000].

Table 2 : ANOVA test Results

	Sum of Squares	DF	Mean Square	F	Sig
Between Group	9437.883	2	4718.942	15.130	.000
Inner Group	22456.197	72	311.892		
Total	31894.080	74			

Post hoc comparisons using Scheffe test was conducted to identify specifically the groups which differed for the achievement test. The results are shown in Table 3.

Table 3: Scheffe test results

Group (I)	Group (J)	Sig
1	3	.000
2	3	.003

*The mean difference is significant at the .05 level.

Table 3 shows that Group 3 (Min = 57.74, SP = 22.35) scored significantly higher than those in Group 1 (Min = 30.64, SP = 11.56) and Group 2 (Min = 39.67, SP = 18.49). However, there is no significant difference between Groups 1 and 2. The findings indicate that students who were exposed to the TEMACCC package performed better than those who were exposed to the TECCC as well as those in the traditional class.

6. Discussion

The findings of this study show that students who were exposed to TEMACC benefited the most as they have better scores than students in the other groups.   As mentioned earlier, the participants who were involved in this study were those with low achievement and the finding of this research has suggested that exposing students to TEMACCC can help students score better in Mathematics. This finding shares similar findings as those by [1] and [8]. However, a research conducted by [2] found that using computers did not have any influence on students with low achievement.   In summary, this finding indicates that instructors could use TECCC only as a supplemental tool to help diversify their teaching approach. Alternatively, students who need additional assistance to understand mathematics calculus and use it for extra practices can use the MACCC.

7. Suggestion

The results from this study had been positive on the use of mathematics courseware in teaching and learning calculus. Findings indicate that by using CD courseware in lecture and tutorial class can help students score better in calculus. However, further studies need to be conducted in order to gain more information about the benefit of using TEMACCC. In this study, sample involved were students with low performance of mathematics in schools.   This student did not have good mathematics knowledge and skills which might have an effect to the findings. Further research can be conducted to students with moderate and high performance of mathematics at school. This study is also limited based on score of the achievement test. Further analysis can be conducted to explore more on students' mathematic development such as the problem solving process, students' procedural and conceptual knowledge and also their mathematics visualizations. The findings from this aspect will show the potential of using TEMACCC in teaching and learning calculus at tertiary level.

References

[1] Baharvand, M. (2001). A comparison of effectiveness of computer assisted instruction versus traditional approach of teaching geometry. *MAI. 40(03)*, 552.

[2] Baker, S., Gersten, R., & Lee, D. (2002). A Synthesis of Empirical Research on Teaching Mathematics to low-achieving students. *Elementary School Journal*, *103(1)*, 51-73.

[3] Chang, K. E, Sung, V. F., & Lim, S. F. (2006). Computer Assisted Learning For Mathematical Problem Solving. *Computers & Education*, *46(2)*, 140 – 151.

[4] Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, *24*, 94 - 116.

[5] Gay, L.R. (1985). *Educational evaluation and measurement.* Columbus :Merrill.

[6] Green, S.B., Salkind, N.J. & Akey, T.M. (2000). *Using SPSS for windows: Analyzing and understanding data (2nd Edition).* New Jersey : Prentice Hall.

[7] Hamtini, T.M. (2000). A comparison study of computer facilitated instruction versus non-computer facilitated instruction in developmental mathematics at a university: Student's attitude and achievement. *DAI, 61(04A)*, 1368.

[8] Magallanes, A.M. 2003. Comparison of student test score in a coordinate plane unit using traditional classroom techniques versus traditional techniques coupled with an ethnomathematics software at torch middle school. Tesis Master, National University.

[9] Magliaro, S. G., Lockee, B. B., & Burton, J. K. (2005). Direct Instruction Revisited: A Key Model for Instructional Technology. *Educational Technology Research & Development, 53(4)*, 41-55.

[10] National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, V.A: Author.

[11] Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics 14(3)*, 235-250

[12] Orton, A. (1986). Introducing calculus : an accumulation of teaching ideas? *International Journal of Mathematics Education in Science and Technology. 17(6)*, 659-688.

[13] Rochowicz, J. A. (1996). The impact of using computers and calculators on calculus instruction: Various perceptions. *Journal of computers in Mathematics and Science teaching, 15(4)*, 423 – 435.

[14] Rosenshin, B. (1995). Advances in Research on Instruction. *The Journal of Educational Research, 88(5)*, 262-271.

[15] Schwartz, J.E & Beichner, R.J. (1999). *Essential of Educational Technology.* Massachusetts : Allyn and Bacon.

[16] Tall, D. (1985). Understanding the calculus. *Mathematics Teaching. 11*, 49-53

[17] Tall, D & Vinner, S. (1981). Concept Image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics. 12*, 151 – 169.

[18] Traynor, P. (2003). Effects of Computer Assisted Instruction on Different Learners. *Journal of Instructional Psychology*, *30(2)*, 137 – 151.

[19] Tall, D & Bakar, M. (1992). Students' Mental Prototypes for Functions and Graphs. *International Journal of Mathematics Education Science and Technology. 23(1)*, 39 – 50.

[20] White, P. (1993). Differential Calculus. *Reflections. 18(4),* 31-34.