Exploring Creativity and Innovation in STEM Education: With and Without Technology

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Abstract

In this work, we present a collection of geometric problems described in wooden tablets known as Sangaku. These original problems developed in Japan during the Edo period. Quadrilaterals, a subject of immense geometric richness, have been widely researched in the literature. They offer opportunities to explore constructions with ruler and compass, various representation techniques, proofs and theorems, with and without technology.

The main objective of this study is to demonstrate how Sangaku geometric problems can serve as valuable pedagogical resources, integrating different disciplines and enriching students' learning experiences. In addition to deepening the mathematical aspects of these problems, special attention was also given to the artistic elements present in the wooden tablets, stimulating students to the creative expression inherent to Geometry.

Adopting an interdisciplinary approach, the activity seeks to stimulate students' creativity, encouraging them to create visual representations of problems and explore various artistic materials, while developing their visual communication and mathematical skills.

This approach placed a strong emphasis on promoting transversal skills and attitudes, promoting the development of interdisciplinary skills. Consequently, it contributed to a broader and more integrated education in several fundamental disciplines.

1 Introduction

Geometry serves as a means to describe, understand, and interact with the space in which we live, making it the most intuitive, concrete, and reality-related branch of mathematics. However, teaching and learning geometry can be challenging due to the cognitive complexity involved in geometric activities.

Geometry provides an easy connection between the abstract and the concrete, bridging the formal description of a given geometric construction with its tangible representation [9]. Euclid's Elements of Geometry written around 300 BC, stands as a prime example of mathematical deduction. Over the course of more than two millennia, mathematical proofs have become increasingly rigorous, and the language of mathematics has evolved to be more precise and symbolic. In the 17th century, Gottfried Wilhelm Leibniz (1646–1716) proposed the development of a universal symbolic language, known as *characteristica universalis* capable of expressing all statements, not limited to mathematics alone. He also suggested a universal logical calculus, the (*calculus ratiocinator*) to decide the truth of statements expressed in *characteristica universalis*. Leibniz's work on differential calculus, as presented in *Nova methodus pro maximis et minimis, itemque tangentibus, qua nec irrationales quantitates moratur* ("A new method for maxima and minima, as well as tangents, which is not hindered by irrational quantitie") in 1684, further contributed to the advancement of mathematical concepts [3].

Meanwhile, in Japan, the Edo period (1603-1867) was marked by national isolation, leading to the interruption of negotiations with the West, including Western mathematics. Despite this isolation, Japanese mathematics, known as Wasan, experienced significant development and a unique approach to presenting mathematical problems emerged, leading to a flourishing of Euclidean geometry in the country. Wooden painted tablets, called "Sangaku" were displayed in Shinto shrines and Buddhist temples for recreational fun and religious offerings. Although many of these tablets were lost during the modernization period that followed the Edo era, around 900 have been preserved.

The Sangaku tablets frequently feature simple figures, where the creativity of the forms plays a decisive role in selecting the problems. The majority of these problems were solved using analytic geometry and algebraic methods, collectively forming what is known as the "Geometry of Japanese Temples". This collection includes simple or regular polygons and polyhedra, circles, ellipses, spheres, and ellipsoids. Various conic sections, including the paraboloid, appear as well. The cylinder primarily intervenes in creating the ellipse through intersection with a plane, and affine transformations are used to transition from the circle to the ellipse. The collection contains several optimization problems with provided answers. However, the methods used to obtain these answers remain absent. Since the works of Newton and Leibniz were unknown to Japanese mathematicians at the time, and there is no evidence of contemporary Japanese mathematicians having a formal definition of the derivative, their resolution techniques for these problems remain unsolved [4].

1.1 The Interdisciplinary

The aesthetics of the forms on the Sangaku tablets play a crucial role in selecting the problems to be solved, demonstrating a direct connection between geometry and the visual arts. Sangaku tablets frequently present optimization problems, challenging students to find efficient solutions for practical real-world situations. This can open doors to interdisciplinarity with physics, engineering or other sciences, where optimization is a valuable tool for solving complex problems. In addition, some tablets may explore properties of the materials used in temple construction, relating to materials engineering and architecture. The use of affine transformations to create geometric figures is also related to mathematics and physics.

The historical aspect of Sangaku tablets can also promote a connection with Japanese history and culture, enabling interdisciplinary approaches in the areas of history, cultural studies and anthropology. In this way, the Sangaku tablets illustrate how geometry can act as a solid foundation to embrace interdisciplinarity, connecting to other areas of knowledge and enriching learning by providing a holistic and broad view of the relationships between different disciplines. By exploring and solving problems on the Sangaku tablets, students are encouraged to integrate knowledge from different areas, seeing mathematics as a powerful tool for understanding and solving problems in the world around them. This interdisciplinary approach can arouse students' curiosity and enrich their understanding of knowledge in diverse areas, preparing them to be more informed citizens capable of facing the complex challenges of contemporary society.

STEM (Science, Technology, Engineering, and Mathematics) education, it is an interdisciplinary approach to learning that integrates these four fields to promote critical thinking, problem-solving, and creativity among students. STEM education aims to provide students with real-world applications and hands-on experiences in these subjects, preparing them for future careers in various STEM-related fields. Integrating various areas of knowledge, interdisciplinary approaches inspire students to delve into novel concepts, seek inventive resolutions, and foster creative thinking. The rich significance of Sangaku, embracing historical, religious, mathematical, and artistic dimensions, renders it a compelling model for integrating STEM education among students, bridging diverse domains effectively. In this study aims to show how Sangaku geometric problems enrich students' learning through interdisciplinary integration and artistic exploration. It emphasizes both mathematical depth and artistic elements, fostering creative expression. Through an interdisciplinary approach, the activity boosts students' creativity, fostering visual problem-solving and artistic skill development.

2 Geometric Problems

Several geometric problems involving quadrilaterals and their respective solutions will be presented. In these problems, some are easy to solve, others are more difficult to solve, for basic/secondary education. They involve concepts of both Euclidean geometry, differential calculus or even complex numbers.

Problem 1 Problem taken from a Sangaku displayed at Tagami Kannondo Shrine, Shin–Nagano, 1809 (Figure 1). In this problem, a rhombus $[ABCD]$ is given on side a (Figure 2). It is known that $\overline{BD} = 2t$, depending on t. Let $S(t)$ be the area of the rhombus minus the area of the square of the diagonal $\overline{BD}$ and side x. Determine the value of x as a function of a, when $S(t)$ is maximum [4].

Figure 1 Tagami Kannondo Shrine, n.º 2 (Nakano, 1809)[1]

Resolution:

The problem to be solved is shown below:

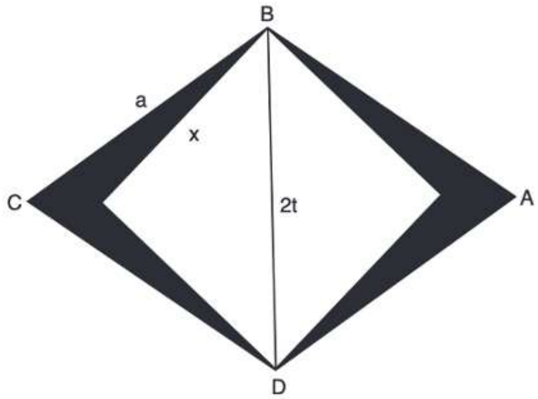

Figura 2 A Square inscribed in a Rhombus

By the Pythagorean theorem we have

$$x^2 = t^2 + t^2 \Leftrightarrow x = \sqrt{2}t, x > 0 \qquad (1)$$

The area of the rhombus $[ABCD]$, at Figure 2, is given by $A_{[ABCD]} = \dfrac{2t\sqrt{a^2 - t^2}}{2}$ and the area of the square is given by $A = x^2$.
It is $S(t)$ the difference between the area of the rhombus and the square, that is, $S(t) = \dfrac{2t\sqrt{a^2 - t^2}}{2} - x^2$, with $0 \leq t \leq a$, by (1) it comes,

$$S(t) = t\sqrt{a^2 - t^2} - 2t^2 \qquad (2)$$

To determine the maximum value, derive $S(t)$ in order to t, i.e.,

$$S'(t) = \sqrt{a^2 - t^2} - \frac{t^2}{\sqrt{a^2 - t^2}} - 4t,$$

equaling zero, $S'(t) = 0$, results,
$\sqrt{a^2 - t^2} - \dfrac{t^2}{\sqrt{a^2 - t^2}} - 4t = 0 \Leftrightarrow \dfrac{a^4}{16t^2} - \dfrac{a^2}{4} + \dfrac{t^2}{4} = a^2 - t^2$, with $t \neq 0$, we have

$$20t^4 - 20a^2t^2 + a^4 = 0,$$

doing

$$t^2 = x \qquad (3)$$

we have, $x = \dfrac{20a^2 \pm 8a^2\sqrt{5}}{40} \Leftrightarrow x = \dfrac{a^2}{2} \pm \dfrac{a^2\sqrt{5}}{5}$, replacing x por (3), it comes,

[1] http://www.wasan.jp/nagano/tanoue2.html

$$t^2 = \frac{a^2}{2} \pm \frac{a^2\sqrt{5}}{5},$$

for $S'(t)$ be maximum, $t > 0$, $t = \sqrt{\frac{a^2}{2} - \frac{a^2\sqrt{5}}{5}}$. i.e., $t = a\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{5}}$

By (1), it follows that, $\frac{x}{\sqrt{2}} = a\sqrt{\frac{1}{2} - \frac{\sqrt{5}}{5}}$. Therefore,

$$x = a\sqrt{1 - \frac{2\sqrt{5}}{5}}.$$

Problem 2 A version of problem 1 is given as follows:

Let $[BCAD]$, be a square as shown in the Figure 3.1. Consider the diagonals $[AB]$, $[CD]$ and M your point of intersection. On the line segment $[CD]$, let's take the points P, R symmetrical with respect to the point M. We obtain a rhombus $[BPAR]$. Consider also the square $[PQRS]$. For what value or values of lengths $\overline{PQ}$ (sides of squares $[PQRS]$) do the values of the areas marked in red reach their maximum?

From the figure on the left, already described in the statement, the diagonal $[AB]$, $[CD]$ are symmetry axes and, therefore, the proposed problem is solved by determining what is the length value of $\overline{PQ}$ for which triangle $[PAQ]$ has maximum area.

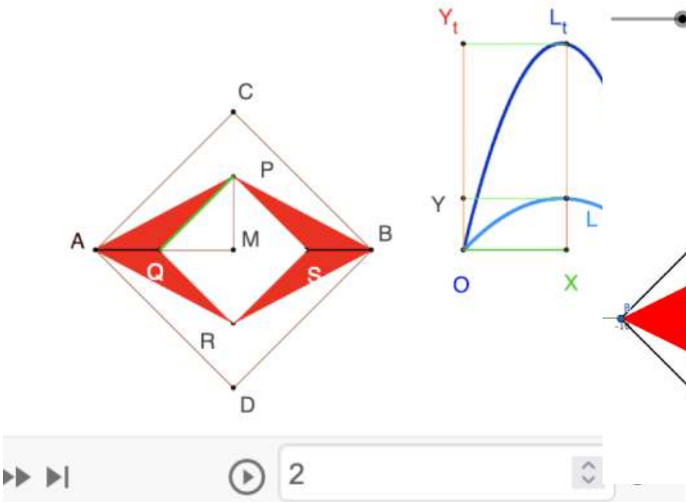

Figure 3.1 Rhombus[2]

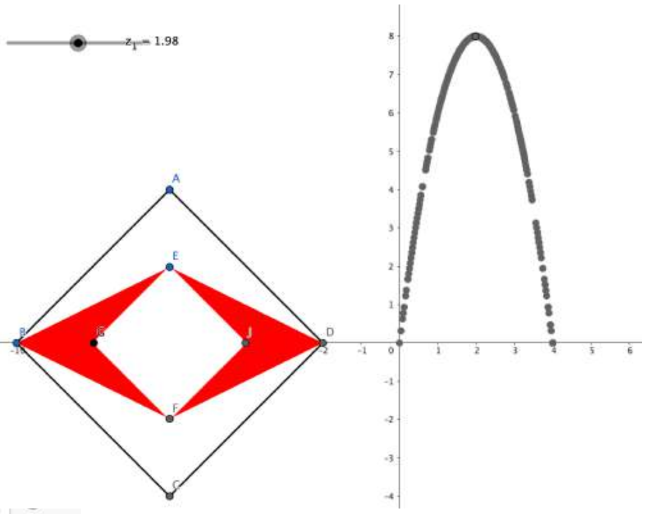

Figure 3.2 Construction in GeoGebra of the Rhombus (author)

Resolution[3]**:**

Using Figure 3.1, what we are going to do is study the dependence of the values $y = \overline{OY}$ of the areas of triangle $[APQ]$ as a function of the values of the lengths of the sides $x = \overline{OX} = \overline{PQ}$ of square $[PQRS]$. The diagonals of the squares are equal $[AB] = [CD]$, $[PR] = [QS]$, bisect each other $[MQ] = [PM]$ perpendicularly $C\hat{M}A = P\hat{M}Q = 90^{\circ}$, which is why

[2] *https://www.geogebra.org/material/iframe/id/nyfTHgGr/

[3] https://tinyurl.com/areaproblem

$\overline{PQ}^2 = \overline{PM}^2 + \overline{MQ}^2 \Leftrightarrow \overline{PQ}^2 = 2\overline{PM}^2 \underset{x=\overline{PQ}}{\Leftrightarrow} x = \sqrt{2}\ \overline{PM} \Leftrightarrow \overline{PM}^2 = \dfrac{x^2}{2}$ and, designating by $2a$ the fixed length of $[AB]$, and by $2d$ the value of the variable lengths of the diagonals of $[PQRS]$, over the area y of the triangle $[PAQ]$ which is equal to the triangle $[PAM]$ subtracted from the triangle $[MPQ]$, we can write

$$y = \frac{a \times d}{2} - \frac{d^2}{2} = \frac{\sqrt{2}ax}{2} - \frac{\frac{x^2}{2}}{2} = \frac{2\sqrt{2}ax - x^2}{4}$$

When P takes the position of M, $P \equiv M \equiv Q \dots$ then $x = 0$. The largest value that $x = \overline{PQ}$ can reach is when $P = C$ and $Q = A$ then $\overline{PQ} = \overline{AC} = \sqrt{2}a$. For our problem, x can take all values between 0 and $\sqrt{2}a$:

$$0 \leq x = \overline{OX} \leq \overline{AC} = \sqrt{2}a$$

and therefore as,

$$y = \frac{2\sqrt{2}ax - x^2}{4} = \frac{-(x^2 - 2\sqrt{2}ax + 2a^2) + 2a^2}{4} = \frac{1}{4}(2a^2 - (x - \sqrt{2}a)^2)$$

polynomial function of the second degree where x^2 has a negative coefficient $-\dfrac{1}{4}$

$$y = \frac{1}{4}(2a^2 - (x - \sqrt{2}a)^2) = 0 \Leftrightarrow x = 0 \vee x = \sqrt{2}a$$

y reaches its maximum value for the average x value of $[0, \sqrt{2}a]$ which is $\dfrac{\sqrt{2}a}{2}$.

In GeoGebra, we can click on the animation button, in Figure 3.1, and visualize the traces of the abscissa points x between 0 and $\sqrt{2}a$.

L which has as ordinate $y = \overline{OY}$ the value associated with the area of the triangle $[PAQ]$ corresponding to each value of $x \dots$ L_t which has as ordinate $y_t = \overline{OY}_t$ the value associated with the area of the entire red surface $y_t = 4y = 4[PAQ]$ corresponding to each x length of the side of the square $[PQRS]$.

Problem 3 Given two rhombuses $[ABCD]$ and $[AECF]$ (Figure 4). Let P be the area $A_{[ABCD]} - A_{[AECF]}$, $\overline{AD} = q$ and $\overline{AF} = r$. Determine the diagonal $\overline{AC}$ in terms of P, q, and r [6].

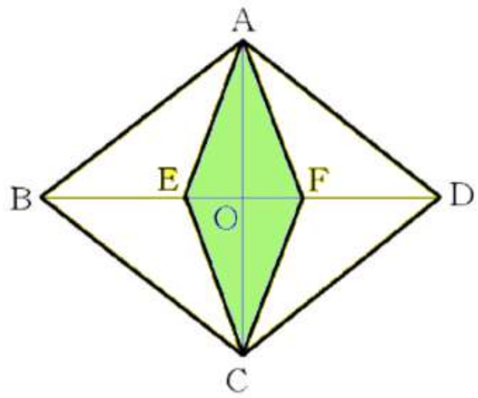

Figure 4. Rhombus

Resolution:
Let $\overline{OD} = a$, $\overline{OF} = b$ and $\overline{AC} = x$.
Knowing that $A_{[AFD]} = \dfrac{(a-b)\frac{x}{2}}{2}$ and that $P = A_{[ABCD]} - A_{[AECF]}$, it follows that, $\dfrac{P}{4} = \dfrac{(a-b)\frac{x}{2}}{2}$, then

$$b = a - \frac{P}{x} \tag{4}$$

By the Pythagorean Theorem we know that,

$$q^2 = \frac{x^2}{4} + a^2 \tag{5}$$

and

$$r^2 = \frac{x^2}{4} + b^2 \tag{6}$$

we have,

$$a = \sqrt{q^2 - \frac{x^2}{4}}, \text{with } a > 0 \tag{7}$$

$$b = \sqrt{r^2 - \frac{x^2}{4}}, \text{with } b > 0 \tag{8}$$

By (1) and (4),

$$b = \sqrt{q^2 - \frac{x^2}{4}} - \frac{P}{x} \tag{9}$$

By (3) and (6), it follows that, $r^2 = \dfrac{x^2}{4} + \left(\sqrt{q^2 - \dfrac{x^2}{4}} - \dfrac{P}{x}\right)^2$

Simplifying a few steps, we see that:
$x^4[P^2+(q^2-r^2)]-2P^2x^2(q^2+r^2)+P^4=0$, solving for x, with $x>0$, then,

$$x=P\sqrt{\frac{q^2+r^2\pm\sqrt{4r^2q^2-P^4-q^4}}{P^2+(q^2-r^2)^2}}.$$

Therefore,

$$\overline{AC}=P\sqrt{\frac{q^2+r^2\pm\sqrt{4r^2q^2-P^4-q^4}}{P^2+(q^2-r^2)^2}}.$$

Problem 4 It was written on a tablet hanging in a shrine which reads as follows:

Let four points be A, B, C and D (Figure 5) on the same circle (the points are said to be concyclic). If H, I, J and K are respectively the centers of the circles inscribed in the triangles $[ABC]$, $[BCD]$, $[CDA]$ and $[DAB]$, show that the quadrilateral $[HIJK]$ is a rectangle.

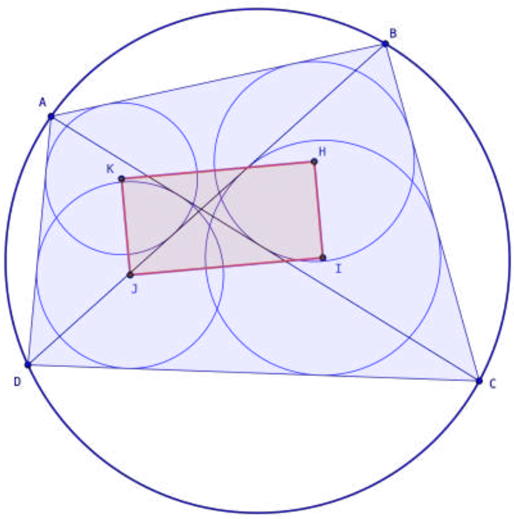

Figure 5. Cyclic quadrilateral - Construction in GeoGebra[4]

Resolution:
To solve this problem we will need the following definition:

[4]https://www.geogebra.org/m/suuwu4wj

Definition 1: A quadrilateral inscribed in a circle, that is, its four vertices are on the same circle, is called a cyclic quadrilateral.

Let the triangle be $[BAD]$.

$$B\hat{K}A = 90^{\text{o}} + \frac{B\hat{D}C}{2} \tag{10}$$

Likewise the triangle $[CBA]$

$$B\hat{K}A = 90^{\text{o}} + \frac{B\hat{C}A}{2} \tag{11}$$

However, as the quadrilateral is cyclic (Definition 1) the $B\hat{D}A = B\hat{C}A$. Thus, $B\hat{K}C = B\hat{H}C$.
The quadrilateral $[BHKA]$ is cyclic, like this

$$B\hat{A}K + B\hat{H}K = 180^{\text{o}} \tag{12}$$

Similarly, the quadrilateral $[CIHB]$ is cyclic, so

$$B\hat{C}I + B\hat{H}I = 180^{\text{o}} \tag{13}$$

(3) and (4) implies that

$$B\hat{H}K + B\hat{H}I = 360^{\text{o}} - \frac{B\hat{A}D}{2} - \frac{B\hat{C}D}{2}.$$

Therefore, $I\hat{H}K = 90^{\text{o}}$. The other angles are treated similarly.

Problem 5 Another version of the previous problem is the following:

Let be the cyclic quadrilateral $[ABCD]$ (Figure 6.1). Let r_A, r_B, r_C and r_D be the radii of the circumferences of the circles inscribed in the triangles $[DAB]$, $[ABC]$, $[BCD]$ and $[CDA]$, respectively. Show that $r_A + r_C = r_B + r_D$ [1] .

For the resolution the following theorem and lemmas [4] will be needed:

Ptolemy's Theorem: In a cyclic quadrilateral, the sum of the products of the lengths of the opposite sides is equal to the product of the lengths of the diagonals.

Lemma 1 The area of a triangle $[ABC]$ can be expressed as $S_{ABC} = rs = \dfrac{abc}{4R}$, where a,b and c are the sides of the triangle; $s = \dfrac{a+b+c}{2}$ is the semiperimeter of the triangle $[ABC]$; and r

[5] `https://www.wasan.jp/kanagawa/matubara1.html`

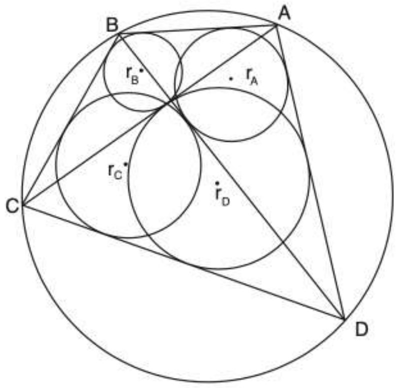

Figure 6.1 Cyclic quadrilateral

Figure 6.2 Cyclic quadrilateral - Matsubara Shrine (Odawara) [5]

and R are the radii of the inscribed and circumscribed circles of the triangle (see Figure 6.1). This lemma follows directly from two elementary theorems: 1) that the area of a triangle $[ABC]$ can be written as $S_{ABC} = rs$, where r is the radius of the triangle's inner circle and is its semiperimeter; 2) the radius R of the circumcircle is $R = abc/4S_{ABC}$.

Lemma 2 $\overline{AB} \cdot \overline{CD} + \overline{AD} \cdot \overline{BC} = \overline{AC} \cdot \overline{BD}$. This is known as Ptolemy's theorem. Note: From this point on we will use the following designations: $\overline{AB} = a$, $\overline{BC} = b$, $\overline{CD} = c$, $\overline{AD} = d$, $\overline{AC} = e$, $\overline{BD} = f$. So Ptolemy's theorem for this problem becomes $ac + bd = ef$.

Lemma 3 Let the designation be $abe + cde = bcf + adf$. From Figure 6.1 we see that if S is the area of the quadrilateral, then $S = \frac{1}{2} \cdot a \cdot d \cdot sinA + \frac{1}{2} \cdot b \cdot c \cdot sinC$. But $\hat{A}$ and $\hat{C}$ are supplementary and so $S = \frac{1}{2}(ad+bc) \cdot sinA$. Likewise, $S = \frac{1}{2} \cdot a \cdot b \cdot sinB + \frac{1}{2} \cdot c \cdot d \cdot sinD = \frac{1}{2}(ab + cd) \cdot sinB$. On the other hand, the triangle $[ABD]$ is inscribed in the circle R, so by lemma 1, $S_{ABD} = adf/4R$. But also $S_{ABD} = \frac{1}{2} \cdot a \cdot d \cdot sinA$, giving $sinA = f/(2R)$. Likewise, $sinB = e/(2R)$. Eliminating $sinA$ and $sinB$ from the previous two expressions immediately results in $f(ad + bc) = e(ab + cd)$.

Remembering also the following theorem:

Japanese Theorem: Triangulate a cyclic polygon by lines drawn from any vertex. The sum of the radii of the triangles' inner circles is independent of the chosen vertex [2].

By lemma 1, $S_{ABD} = rs = \frac{adf}{4R} = r_A \frac{a + d + f}{2}$, with similar expressions for S_{BCD}, S_{ACD} and S_{ABC}. Like this,

$$r_A = \frac{adf}{2R(a + d + f)}; \quad r_B = \frac{abe}{2R(a + b + e)}; \quad r_C = \frac{bcf}{2R(b + c + f)}; \quad r_D = \frac{cde}{2R(c + d + e)} \tag{14}$$

We have,

$$2R(r_A + r_C) = \frac{bcf(a+d+f) + adf(b+c+f)}{(b+c+f)(a+d+f)} \tag{15}$$

The numerator of the second member is $(a+d)bcf + (b+c)adf + f^2(bc+ad) = f[ab(c+d+e) + cd(a+b+e)]$, where,

$$f^2(bc+ad) = ef(ab+cd) \text{ by lemma 3.}$$

The denominator is $f^2 + f(a+b+c+d) + (b+c)(a+d)$, but by lemmas 2 and 3,

$$(b+c)(a+d) = (ac+bd) + (ab+cd) = ef + (f/e)(ad+bc),$$

and so,
the denominator is $\frac{f}{e}[ef + e(a+b+c+d) + e^2 + bc + ad] = \frac{f}{e}[(a+b+e)(c+d+e)]$, where lemma 2 was used again to obtain the second equality. Equation (12) then becomes

$$2R(r_A + r_C) = \frac{e[ab(c+d+e) + cd(a+b+e)]}{(a+b+c)(c+d+d)} = \frac{abe}{a+b+e} + \frac{cde}{c+d+e}$$

or by equation (11) the following result is obtained,

$$r_A + r_C = r_B + r_D$$

3 Sangaku as Activity

The proposed activity involves concepts of Euclidean geometry, were given to students from Basic Education, focusing on quadrilaterals, more specifically the cyclic quadrilaterals. In the class there is a student who is blind[6] and the challenge of Ptolemy's Theorem were given, while the other colleagues worked on another challenge, the Japanese Theorem.

Students constructed a cyclic quadrilateral with ruler and compass. We can verify some of these constructions obtained in Figure 7.

[6]The student with severe low vision has visual acuity less than 1/10 (scale of Wecker) and needs Braille to read

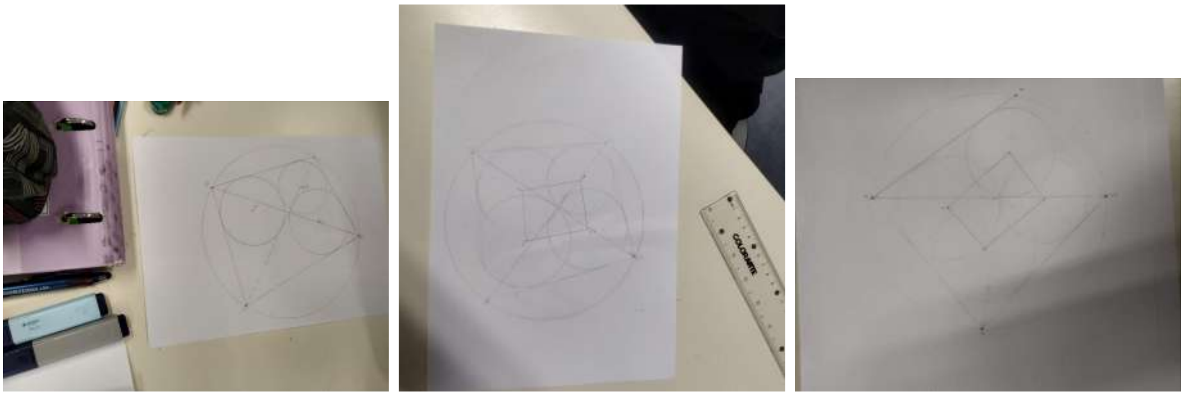

Figure 7 Students' construction

After construction, students would have to answer two questions. The questions were as follows:

- Joins the points K, H, I and J. What conclusion did you come to?
- Measures the length of the radii of the smallest circles. Does it check for the following: $r_H + r_J = r_I + r_K$?

Students would have to come to the conclusion that they were facing a rectangle and that equality was verified. We can verify by Figure 8 that the students reached the intended conclusions.

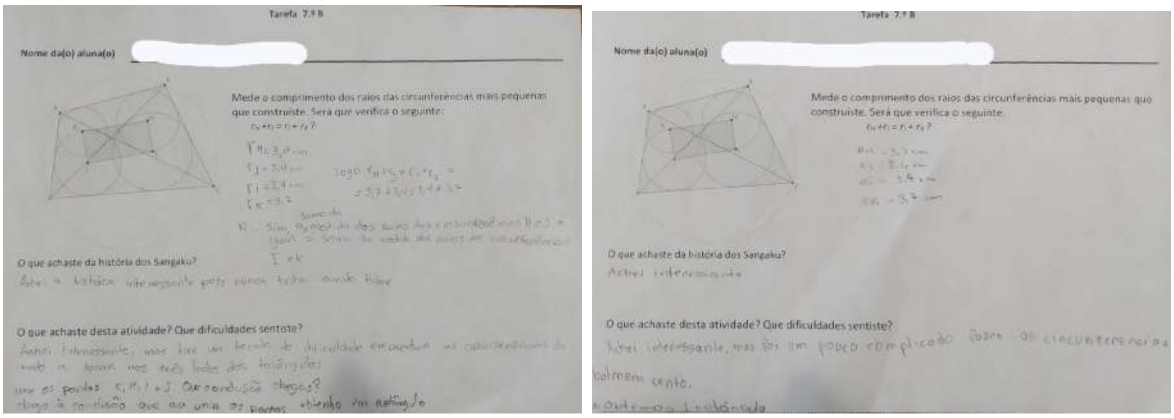

Figure 8 Students' responses

The blind student performed an alternative activity using Ptolemy's Theorem because the compass she had available did not allow for "erasing" the auxiliary lines, and it could be very confusing to read on paper. She approached the task differently, using thick paper and a metallic compass adapted with accessories (toothed wheels) and a graduated ruler that was also adapted for her needs (Figure 9).

Figure 9 Adapted material

With the adapted material, the student drew a circle and marked four points on it with a metallic pen. Then, with a pen with a toothed wheel, she traced the quadrilateral and its diagonals (Figure 10).

Finally, with the ruler she measured the lengths of the line segments to show the following: $\overline{AB} \cdot \overline{CD} + \overline{AD} \cdot \overline{BC} = \overline{AC} \cdot \overline{BD}$.

Verified that $\overline{AB} \cdot \overline{CD} + \overline{AD} \cdot \overline{BC} = 281.75$ and that $\overline{AC} \cdot \overline{BD} = 280.5$, despite not using materials with great precision the result is very close.

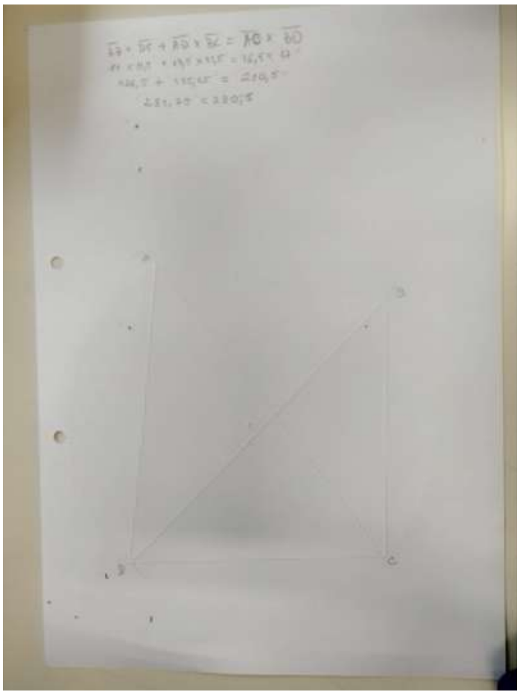

Figure 10 Blind Students' construction

In the last part of the activity, the students were asked to paint their "work of art". The results of the drawings were quite diverse, resulting in a variety of creative and original images. In Figure 11, we can observe three students who chose to use different colors.

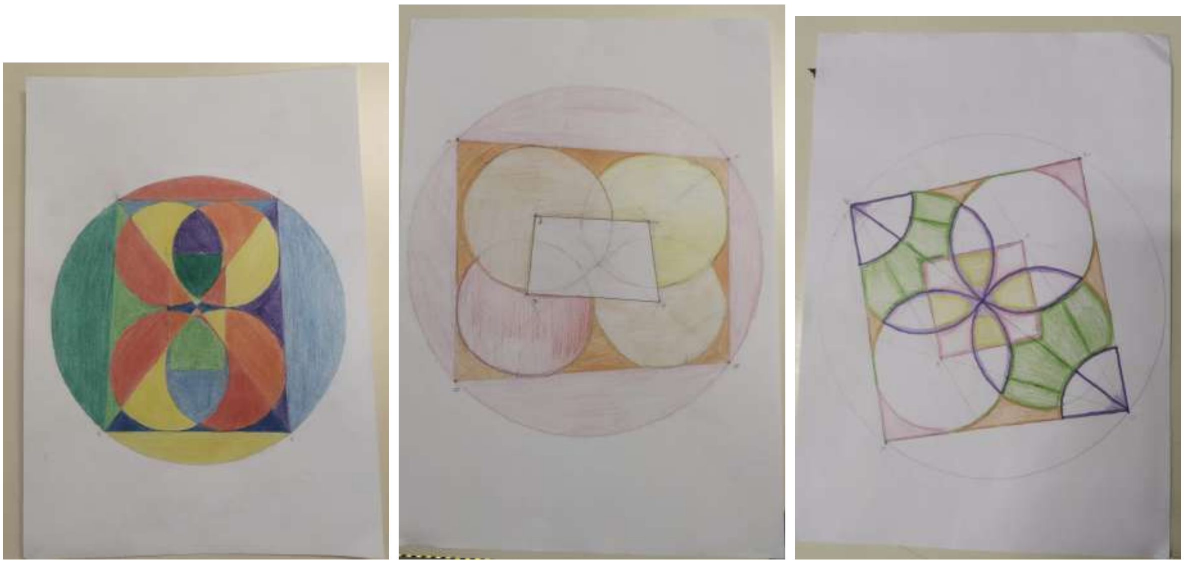

Figure 11 Students' Sangaku

The blind student managed to paint (Figure 12), without help, her construction, the only help was the choice of colors, as she has difficulty distinguishing between dark colors and between light colors.

Figure 12 Blind Student's Sangaku

4 Conclusion

Problems written in Sangaku demonstrate a surprisingly high standard in mathematics in Japan during the Edo period, especially in geometry and calculus. It is all the more surprising that such a high level was achieved independently of the West due to the strict isolation policy. Some interesting problems have been posed by many people from various socio-economic backgrounds. This fact may indicate that mathematics is attractive to many people, regardless of class, economic, ethnic, age and gender boundaries.

When comparing Euclid's work with a Sangaku, the two styles are very different, and therefore terms like 'proof' and 'theorem' should be used with caution when describing the Japanese tradition. The Sangaku authors did not employ proof by diagrams or axiomatics in the sense of Greek and Western mathematicians. Sangaku authors did not seem to enunciate theorems that were then proved with reasoning or logical argument. The tablets only contain a problem, an answer and a formula, which provide little information about how it was obtained from the figure and no reasoning about why it worked [5].

Integrating Sangaku into contemporary education offers a holistic and dynamic learning experience that enhances students' critical thinking, problem-solving, and creative skills while connecting them with historical and cultural dimensions.

Two geometric activities were presented: the Japanese Theorem and Ptolemy's Theorem for cyclic quadrilaterals. The challenges found in the wooden tablets, known as Sangakus, consisted of one problem, one answer, and one formula. However, these solutions offered little information about how they were obtained from the figures, lacking reasoning about their underlying principles [5].

The students were then challenged to work on a practical project, exploring different forms of expression and perceiving Mathematics as a tool for artistic creation and symbolic communication. Throughout the project, the students demonstrated autonomy and responsibility in its execution. The activity not only promoted critical thinking but also fostered creativity through interdisciplinary collaboration.

Considering the diverse range of possible activities from wooden tablets to Sangakus, the proposals were carefully selected to match the students' educational level. This particular work stands out for its originality in involving basic school students. Furthermore, the presence of a blind student in the class posed an additional challenge. However, through inclusive design, the objective of involving all students in the activity was successfully achieved. This inclusion of all students, including the blind student, highlights the significance of accessibility and equal opportunities in the educational process.

Employing an interdisciplinary approach, the activity stimulated students' creativity, motivating them to create visual depictions of the problem and interact with artistic materials. This process concurrently simultaneously their abilities in visual communication and mathematics.

Incorporating creativity and innovation into STEM education, whether through technology-driven approaches [7] or non-technological means [10], creates a dynamic learning environment that prepares students for the challenges and opportunities of the modern world. By fostering creativity, encouraging innovation, and promoting computational thinking, we empower the next generation with the essential skills to shape a future driven by technology [8].

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References

[1] Ahuja, M., Uegaki, W., & Matsushita, K. (2004). Japanese theorem: A little known theorem with many proofs–part I. *Missouri Journal of Mathematical Sciences*, 16(2), 72-81.

[2] Ahuja, M., Uegaki, W., & Matsushito, K. (2006). In Search of "The Japanese Theorem". *Missouri Journal of Mathematical Sciences*, 18(2), 87-94.

[3] Boyer, C. B. (1974). História da Matemática. São Paulo: Edgar Blücher.

[4] Hidetoshi, F., & Rothman, T. (2008). Sacred Mathematics: Japanese Temple Geometry, Princeton: Princeton University Press, pp. 145-190. `https://doi.org/10.1515/9781400829712-011`.

[5] Hosking, R. J. (2016). Sangaku: A mathematical, artistic, religious, and diagrammatic examination (Doctoral dissertation, PhD dissertation. University of Canterbury, New Zealand).

[6] Ito, E., Namura, E., Kokayashi, H., tanaka, H., Kitahara, I., Otani, K., Nakamura, N., Yanagisawa, R., & Sekiguchi, T.(2003) Japanese Temple Mathematical Problems in Nagano Pref. Japan. Japan: Kyoikushokan.

[7] Pinheiro, M. M., & Santos, V. (2021, November). STEM Learning in Digital Higher Education: Have It Your Way!. In *International Conference on Innovative Technologies and Learning* (pp. 567–578). Cham: Springer International Publishing.

[8] Santos, V., Vaz-Rebelo, P., Bidarra, G., Stettner, E., Guncaga, J., & Korenova, L. (2022, August). Teacher Training in the Fields of STEAM: From Physical to Digital Tools. In *International Conference on Technology and Innovation in Learning, Teaching and Education* (pp. 171–177). Cham: Springer Nature Switzerland.

[9] Santos, V., & Quaresma, P. (2021). Exploring Geometric Conjectures with the help of a Learning Environment-A Case Study with Pre-Service Teachers. *The Electronic Journal of Mathematics and Technology*, 2(1).

[10] Vaz-Rebelo, P., Bidarra, G., Costa, C., de Almeida Evangelista, S., Santos, V., & Thiel, O. (2022). Heads-on, hands-on, hearts-on: Educators' perceptions on ViduKids contributions to integral education. *International Journal of Developmental and Educational Psychology*