Bridging the Mathematics Gap Through the Use of Mathematical Apps

Ma. Louise Antonette N. De Las Peñas
mdelaspenas@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Debbie Marie B. Verzosa
dmbverzosa@usm.edu.ph
Department of Mathematics and Statistics
University of Southern Mindanao, Philippines

Maria Alva Q. Aberin
maberin@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Agnes D. Garciano
agarciano@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Juan Carlo F. Mallari
jmallari@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Jumela F. Sarmiento
jsarmiento@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Mark Anthony C. Tolentino
mtolentino@ateneo.edu
Department of Mathematics
Ateneo de Manila University, Philippines

Abstract: *During the COVID-19 pandemic, school campuses worldwide were forced to close, and students had to learn primarily from home. This sudden disruption is estimated to have caused significant learning loss among learners. This paper reports the use of mathematical applications (apps) to bridge the mathematical learning gaps in Grades 1 to 11 in the Philippines after the pandemic, as part of a project funded by a national government agency. The apps include those that strengthen foundational concepts in number and fraction sense in grade school mathematics, develop proving skills in geometry, promote mastery in algebraic and trigonometry through drill and practice, and facilitate statistical understanding and reasoning. The description of the apps, their design, and pedagogical basis are discussed. Challenges encountered in the implementation of the project are also presented.*

1. Introduction

The COVID-19 pandemic disrupted education worldwide due to the closure of schools and the shift to online and blended learning modalities. As classes have returned onsite post-pandemic, educators, teachers, and school administrators are assessing the extent of the learning loss in students, and finding means to address this. In the school year 2022-2023, the Department of Education (DepEd) in the Philippines developed the *Basic Education Learning Recovery Plan* [1] to guide schools in addressing and identifying learning gaps. This was followed by the adoption of the *National Learning Recovery Program* [2] to address these gaps and learning setbacks. The program is currently being implemented across all grade levels by local DepEd school division offices, with their external stakeholders and partners.

Under a government-funded project [3] our team collaborated with some DepEd school divisions in providing mathematical resources to partner schools. These resources (also called *Mathplus* resources) consist of mathematical applications (apps), teaching guides/instructional videos, and performance tasks. The mathematical apps were designed to address the Most Essential Learning Competencies (MELCs) prescribed by DepEd [4] as well as to help narrow the existing learning gaps on competencies that were determined by the school administrators and teachers in our partner schools. The teaching guides and instructional videos were created as support to teachers towards the implementation of the apps in the curriculum. In partnership with the schools, capacity building seminars and webinars were given to teachers to guide them on the use of these resources.

The mathematical resources are made available through the *Mathplus* website https://mathplusresources.wordpress.com. Two new frameworks for sharing information are also used that are geared towards maximizing the reach of educational materials to locations with minimal internet connectivity. For selected schools, one way of distribution of the mathematical resources is through a community LTE network [5]. The community LTE (Long Term Evolution) network used is a mobile cloud network architecture called *Educloud* that facilitates a systematic and resilient way of distributing educational content that is ideal for distant and asynchronous learning. It is low cost, has a low technology requirement and is compatible with *Mathplus* resources for distribution [6]. A second way of distribution of instructional content is using a digital datacasting framework called *RuralCasting* [7]. Under this technology, a custom *set-top box* receives usual TV programming, along with the attached data content, providing multiple users access to the content thru Wi-Fi, and return information back using alternative transmission methodologies. The content can also be updated regularly using the datacasting transmission. The contents, namely, the mathematics applications, videos, text files, images are locally stored in the set-top box. A lightweight learning management system like Moodle and Canvas but specifically for the RuralCasting set-top box, named *Edukastv* was created. In *Edukastv*, the content creators or teachers can upload their subject outline, reading modules, quizzes, etc. for the students to access.

This paper discusses a selection of mathematical apps that were designed under [3] to bridge the mathematical learning gaps as communicated by partner schools to the *Mathplus* team. These apps are, for Grades 1-6: foundational concepts on place value, fraction and number sense; for Grades 7 to 10: algebra, geometry and trigonometry; and statistics for Grades 1 to 11. The design and pedagogical basis of the apps are discussed briefly. We also discuss the challenges in the implementation of the project.

2. General Design Considerations

The use of technology in mathematics education is at times criticized for perpetuating the "digital divide" or the gap between those with access to modern technology and those who do not [8]. However, the apps described in this paper were designed in consideration of under-resourced educational systems such as the Philippines where students and even teachers have limited access to the latest technology. First, the apps were meant to run on Android devices with basic specifications. Second, the apps were made freely available even for schools with little or no budget for purchasing software. Third, the portability and convenience of using mobile devices allow students to access educational materials more frequently [9], even when they are in their homes or when they are not under the guidance of their teachers.

The following key elements were incorporated in the design of the apps: the existing literature on mathematical learning, the benefits of technology in mathematics education and game-based learning principles.

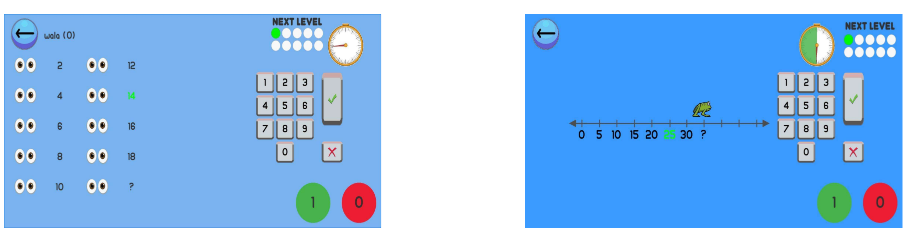
An important strategy that has been adapted in the aforementioned project is the use of game-based learning. Thus, several of the apps developed in the project are games that have clearly defined learning outcomes. The use of games for learning may lead to increasing students' motivation and engagement while allowing them opportunities for graceful failures and to customize their playing experiences according to their own situation [10]. Thus, such games may have a stronger potential to help with bridging learning gaps in mathematics, which many students find difficult and/or unenjoyable.

The design of our game-based apps is guided by Shi and Shih's Game-based Learning (GBL) Design Model, which includes macro-design concepts and 11 interrelated game-design factors: game goals, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery [11]. The central game-design factor is game goals, which naturally affect the other factors of the game's design. In the case of our apps, the game goals always involve the learning or development of different mathematics learning competencies. The game mechanism is then usually guided by pedagogical research that promotes a more effective achievement of the learning competencies. To increase engagement, the games are designed to be interactive and responsive while allowing some freedom for players to customize their playing experience. The game fantasy, which includes its narrative and sensation, is mostly influenced by the game mechanics and the target grade levels for the game. In most cases, simple narratives are used and are complemented with colorful graphics and appropriate music and sound effects. To enhance the game value, most of the games have different modes and levels that contribute to the game's mystery while also setting up a progression according to the challenge or difficulty level. Usually, this progression is also aligned to the progression of the learning content. Since the apps are also intended for independent learning, they are mostly single-player games; however, opportunities for social interactions through the game can be possible in the classroom or virtual meetings.

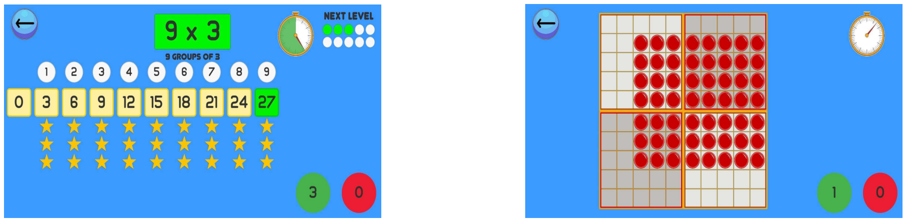
3. Apps for Grades 1 to 6

Remediation on the four fundamental operations has been continuously carried out in elementary schools in the Philippines even before the pandemic. Multiplication, for example, is one of the least learned competencies of students that has been emphasized by one of our partner school division offices. The *Multiplication Game* is one of the apps that we have designed to address this need.

The *Multiplication Game* facilitates mastery of the multiplication table through research-based representations such as repeated addition (modeled by equal groups), arrays and skip counting (modeled by a number). The range of representations is necessary because some representations are more accessible than others in promoting multiplicative thinking [12]. Further, illustrating multiplication using these representations is one of the MELCs prescribed by the DepEd [4] in Grades 1 and 2, The *Multiplication Game* addresses this, starting with the *Beginner* level where students carry out repeated addition by 2's, 5's, and 10's (Figure 3.1(a)) and in the *Advanced I* and *II* levels where students perform skip counting by 3's, 4's, 6's, 7's, 8's and 9's. The sequence of levels follows the learning trajectory and the official DepEd curriculum [13] which starts with multiples of 1, 2, 5, and 10, in Grades 1 and 2, followed by the other multiples, starting in Grade 3.



(a) (b)



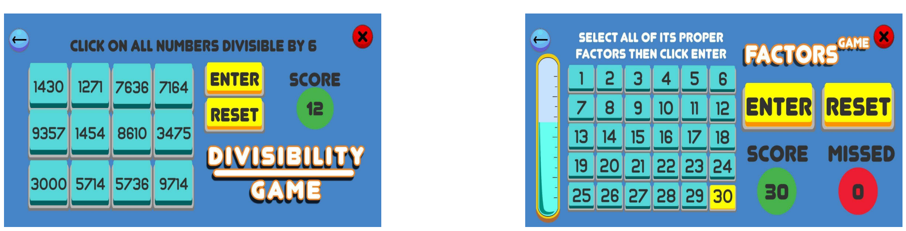
(c) (d)

Figure 3.1 Repeated addition, skip counting and use of arrays in the *Multiplication Game*

Skip counting is the recommended strategy by DepEd for Grade 4 learners [13] to identify the multiples of a given number up to 100. The game illustrates this using equal jumps in the number line. It is recommended that teachers first write multiplication equations on the board that match the repeated jumps on the number line. Using the *Multiplication Game*, students use skips of 5 (Figure 3.1(b)), for example, to help them multiply by 5. Students input the missing multiple in the number line. When students reach the last part of the *Beginner* and *Advanced* I and II levels, they write a multiplication equation using repeated addition and an array. As shown in Figure 3.1(c), students give the product of "9 x 3" and then the game will show that this is the same as "3 groups of 9". Consequently, the concept of commutativity is also reinforced by the *Multiplication* game. Lastly, in the *Expert I* and *II* levels (Figure 3.1(d)), students are shown red dots on an array and players must input the number of red dots. As the number of red dots becomes bigger, students will refrain from counting each dot one by one but will rather use a multiplication equation to get the number of dots.

After mastering the multiplication table by playing the *Multiplication Game*, students practice using divisibility and factoring rules by playing the *Divisibility Game* and *Factors Game*. Grade 5 learners are expected to use divisibility rules to find for 2, 5, and 10; 3, 6, and 9; 4,8,12, and 11 to find the common factors of numbers. In the *Divisibility Game* (Figure 3.2(a)), students can choose the dividends (3 or 4 digits), the divisors, for example 3, 6, 9, and the number of items (9, 12, or 16). Then the students click on the numbers that are divisible by the divisor. On the other hand, in the *Factors Game* (Figure 3.2(b)), the student first selects a number and then all its factors. The game helps students of varying grade levels in determining factors as well as multiples of a given number. This is helpful in providing them with a firm understanding of prime and composite numbers [14].

Some apps and games were also designed to study fractions, one of the least learned competencies for Grades 1 to 6. The learning of fractions may start with early experiences of 1/2 and 1/4, with the eventual aim of conceptualizing fractions as a number; that is, as a point on the number line [15].



(a) (b)

Figure 3.2 The *Divisibility Game* and *Factors Game*

The apps designed for learning fractions provide various representations that are intended to help students connect fractions to the number line model. The first one we will mention here is the *Moving Fractions* game (Figure 3.3(a)). The following concepts can be visualized in a game-like setting: 1/2

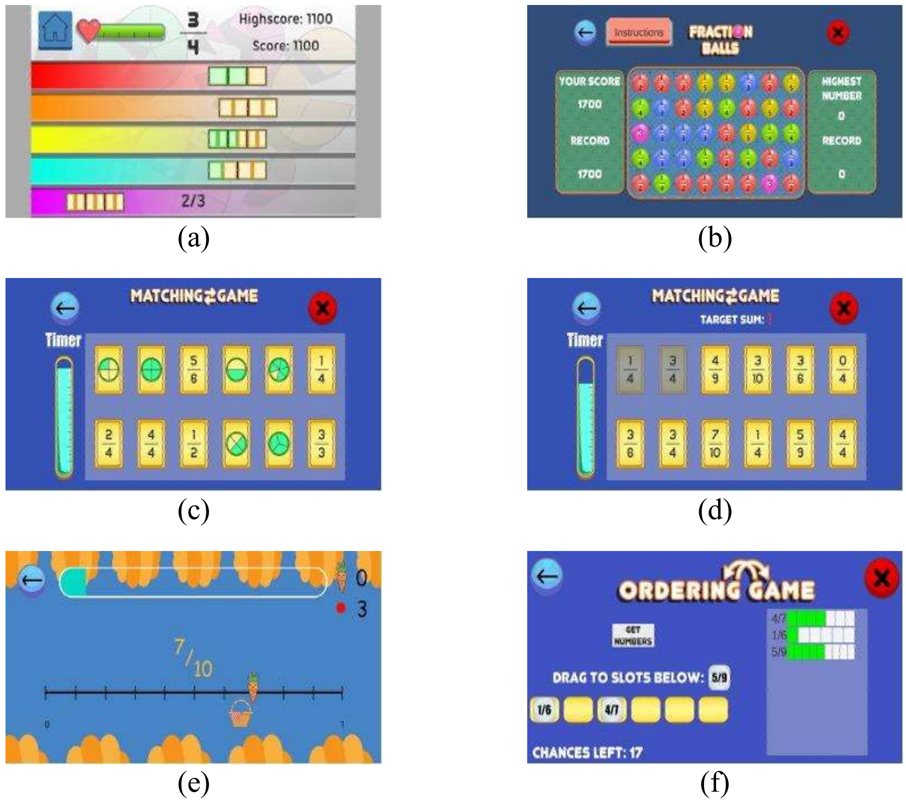


Figure 3.3 Apps and games focusing on fractions

and 1/4, unit fractions, proper fractions, improper fractions, and mixed numbers. Students can envision how to add (join) or subtract fractions. This game is discussed in detail in [16]. Another game is the *Fraction Balls* game (Figure 3.3(b)) which shows addition (through joining) and subtraction (through missing addend) of similar fractions with colored visual supports. The user has the option to work with sums that convert to the lowest terms or not. In the *Matching Game*, a set of cards is shown (Figure 3.3(c)) and the student must match fractions and their corresponding pictures. Another option of this game asks students to match two fractions that add up to a certain target sum (Figure 3.3(d)). Students also have an option to play with whole numbers or decimals in the *Matching Game*. In the *Catch the Carrot* game, a random fraction and a number line are shown on screen (Figure 3.3(e)). The student must estimate where the fraction is located on the number line. The student's aim is to situate as many numbers as possible within the time given. Other options available in the *Catch the Carrot* game are whole numbers, decimals, integers or irrationals. The article [17] presents the design of C*atch the Carrot*. It also discusses the pedagogy of the use of number lines in developing the estimation and number sense skills of students. In the *Ordering Game*, a fraction is generated at random (Figure 3.3(f)), and the learner drags it to a row. The goal of the game is to fill up the entire row, so that the fractions appear in an ascending order, within the allowed number of chances, indicated at the bottom part of the screen. The game becomes more difficult as it progresses because it becomes more likely that a randomly selected number cannot be correctly placed in any of the remaining slots [18].

To summarize, a list of apps that are used by learners in different grades levels, based on their math curricula, are given in the table below.

Table 1. Applicable apps for different grade levels

Level	Apps	Level	Apps
Grade 1	Multiplication Game, Matching Game, Moving Fractions, Catch the Carrot	Grade 4	Multiplication Game, Factors Game, Moving Fractions, Fraction Balls
Grade 2	Multiplication Game, Matching Game, Moving Fractions. Ordering Game, Catch the Carrot	Grade 5	Multiplication Game, Factors Game and Divisibility Game, Moving Fractions, Fraction Balls, Ordering Game, Catch the Carrot
Grade 3	Multiplication Game, Matching Game, Moving Fractions. Ordering Game, Fraction Balls, Catch the Carrot	Grade 6	Moving Fractions, Fraction Balls, Ordering Game, Catch the Carrot

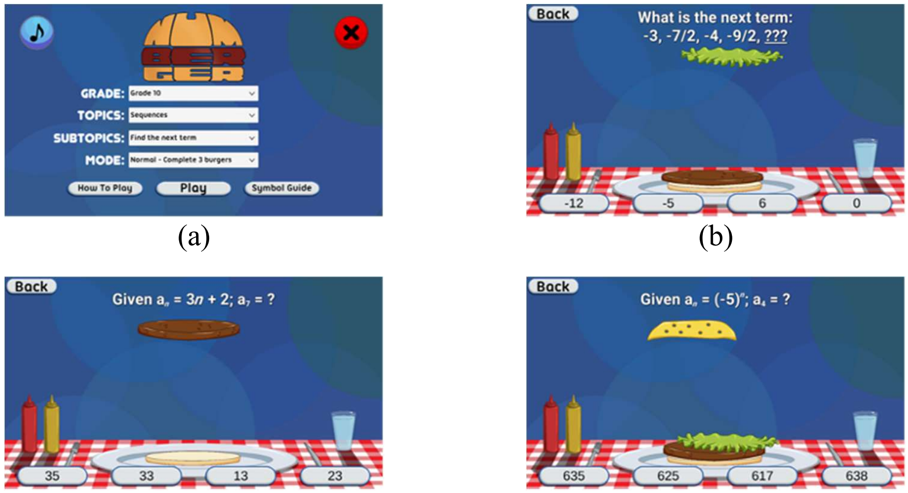
4. Apps for Grades 7 to 10

A selection of the mathematical apps developed for use in Grades 7 to 10 is meant to engage students in regular practice and drills on topics which the students have difficulty with. These apps, designed with game-like settings, are accessible, easy to play with and provide immediate feedback to students. This helps direct the students to either proceed to advanced levels in the app or simply give them opportunities to practice their math skills. Some mathematical skills can also be learned through deliberate practice [19]. Deliberate practice involves the learning of higher-order skills; drill and practice, meanwhile, is often associated with procedural skills [19]. Features of deliberate practice include well-defined goals to address weak points as determined for instance, by a teacher. Thus, the apps designed using the deliberate practice model can be used for remediation activities to provide students the opportunity to work on their weaknesses.

An important skill in mathematics is the ability to recognize number patterns. To this end, the authors designed an app called *Numberger* (Figure 4.1(a)). The app covers different topics: number identification (integers, rational numbers etc.), equivalent fractions, domain and range of functions and sequences. It aims to give opportunities for students to practice various math skills. The objective of *Numberger* is to let the user make burgers (hence the name "Numberger") by being able to choose the correct answer to a given question. One of the topics included in this app is "Sequences". There are two subtopics under "Sequences": "Find the Next Term" and "Find the Term". In the first subtopic, a student is shown some terms of a sequence and is asked what the next term is (Figure 4.1(b)). For this he needs to see the pattern or rule to be able to answer the question correctly. The questions are shown one at a time at the top part of the screen while an ingredient to make a burger is initially shown moving back and forth from left to right on the screen. Some options for the answer are shown at the bottom part of the screen and when a correct answer is chosen, the ingredient falls on the plate found at the bottom of the screen. In the second subtopic, an expression for the nth term, a_n, of a sequence is given and the student is asked to give the value of a particular term. Here the focus is on how to evaluate a term of a sequence, hence the sequence given may be an arithmetic sequence (Figure 4.1(c)) or a geometric sequence (Figure 4.1(d)).

There are two modes for playing the app: Normal Mode-Complete 3 Burgers or Endless-Play without end (Figure 4.2(a)). A player who chooses the Normal Mode receives a "Good Job" message after completing three burgers (Figure 4.2(b)) and can choose to go play again or end the game.

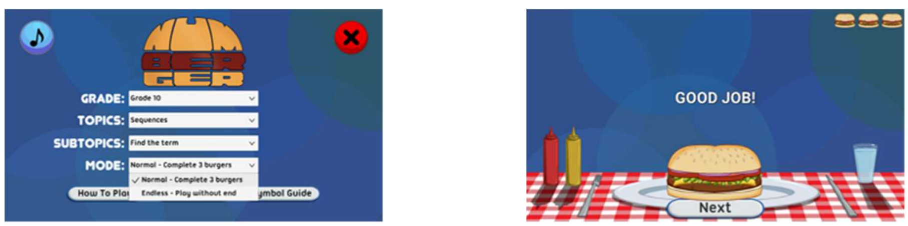
Another app that employs the deliberate practice model to hone algebraic skills is *Just Keep Solving.* The app focuses on helping students become more proficient in solving linear equations and inequalities. There are two topics: *Linear Equations* and *Linear Inequalities*. A sample opening screen of this app is shown in Figure 4.3(a) where the topic chosen is *Linear Inequalities*.



(a) (b)

(c) (d)

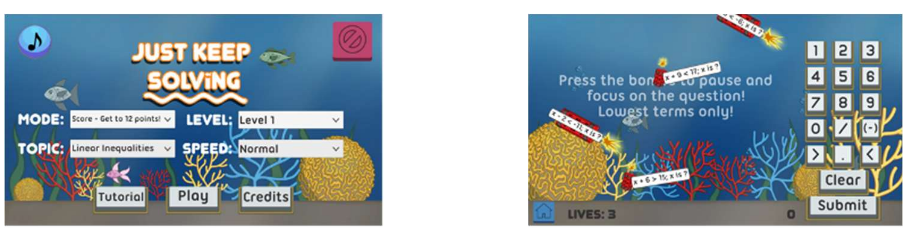
Figure 4.1 (a) Numberger Topic: Sequences; (b) Finding the Next term; (c) Finding a term (arithmetic sequence); (d) Finding a term (geometric sequence)



(a) (b)

Figure 4.2 (a) Modes: Normal or Endless; (b) Message after completing 3 burgers

When a student presses the play button, a screen such as the one shown in Figure 4.3(b) appears. The display is an underwater scene and bombs containing inequalities to be solved fall from the top of the screen. The goal is to be able to solve the inequalities before the bombs reach the ocean floor and cause destruction of the corrals. The mechanics for the topic of *Linear Equations* are like that of *Linear Inequalities*. More details about this app are given in the paper [20].

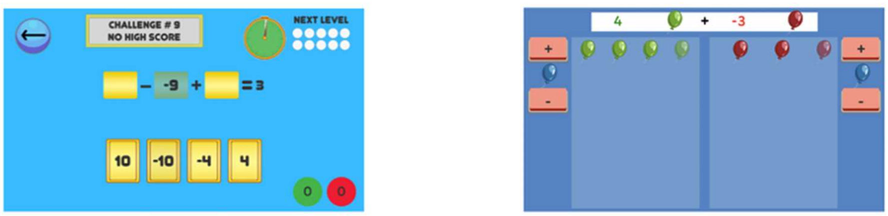


(a) (b)

Figure 4.3 The *Just Keep Solving* app

Disruption to education systems in the pandemic has led to many students experiencing learning loss and learning gaps as they enter high school. For instance, Grade 7 students have not mastered numeracy skills involving fractions, decimals, and integers. Some of the apps developed during the project were helpful in a high school remediation program to address this problem. The *Number puzzles* app, for instance, provides numerous opportunities for practice involving addition, subtraction, and multiplication. *Algeops* incorporates three models: objects, numerals, and number line in a visualization of integer operations. The app incorporates both the neutralization and number line models [21] which offer a more holistic representation of integers. *Moving Fractions and Fraction Balls* were also very helpful in addressing the difficulties of the students in fractions.

Students and teachers were given a survey through Google Form with the goal of evaluating the apps used in the remediation program in three (3) different categories: (a) usability; (b) game-like experience; and (c) alignment with learning objectives. Results indicate a positive response from students and teachers. Most of the students (73%) had no difficulty using the apps. The apps helped 74% of the students comprehend the lesson and its solutions. The apps also helped them find the answers easily and quickly. About 70% of the students noted that the apps provided a lot of opportunities for practice and expressed that with continued use, their math skills would improve. Moreover, the teachers found the apps useful in helping the students stay motivated and engaged throughout the lesson. They also commented that the numerous exercises provided by the apps helped the students master the foundational skills in mathematics.

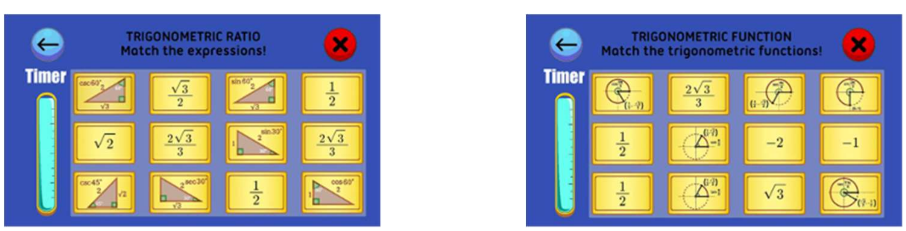


(a) (b)

Figure 4.4 The (a) *Number Puzzles* and (b) *AlgeOps* apps

One of the high school math subjects that requires the development of algebra and geometry abilities is trigonometry. These abilities are crucial for studying pre-calculus and calculus. In the course of our work in [3], teachers reiterated the need for technological tools involving trigonometry.

Trigomatch employs an interactive game with interactive feedback to help users master basic trigonometric concepts like reference angles, trigonometric ratios and trigonometric functions. It presents visual representations of these concepts to allow students to extend their understanding of trigonometry from the right triangle to the unit circle approach. When visual representations of trigonometry are presented using interactive technology, they are more accurate and allow the learners to proceed at their own pace [22]. Additionally, the accuracy of these presentations can reinforce images in a person’s memory. This is not the case for hand-drawn images which tend to be distorted. Figure 4.3(a) and 4.3(b) display the *Trigonometric Ratio* and *Trigonometric Function* set-up, respectively. The objective is for the learners to match two equivalent representations of the trigonometric ratio and function within the allotted time.

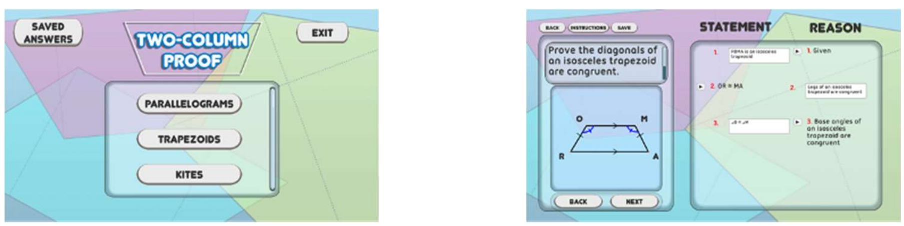


(a) (b)

Figure 4.5 (a) *Trigonometric Ratio* Level; (b) *Trigonometric Function* Level of *Trigomatch*

In high school geometry, one of the topics teachers find very challenging to teach is proving geometric statements. Students struggle with writing proofs and find the process abstract and uninteresting. This situation was exacerbated during the time of the pandemic when it was much more difficult to teach proving remotely. When teaching proofs, teachers rely on textbooks or modules, students' opportunities to learn proofs are limited to what is provided in these instructional materials. Two apps to meet the needs of teachers and students on proving are *Two Column Proof* and *ProveIt.* The design of both apps is grounded on the use of visualization and imagery in mathematics [23].

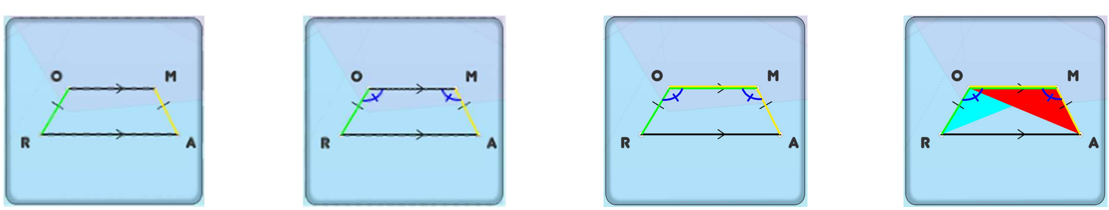
The *Two Column Proof* app is structured to support the students in writing a two-column proof, where a statement or reason in the proof is supported by a visualization element. The intent is to highlight properties and congruence relations of sides, angles and triangles through animation which will serve as visual clues to the students when completing the line of a proof. The visualization elements not only strengthen the student's understanding of the proof process but also stimulate their interest. In the Philippines, Grade 9 lessons on writing two column proof begin with properties of parallelograms. Much time is utilized discussing the first proving topic on parallelograms so oftentimes proving properties of other quadrilaterals such as kites and trapezoids are left for the students to work on individually. We designed the *Two Column Proof* app to include proving geometric concepts on parallelograms, kites, and trapezoids (Figure 4.6(a)).



(a) (b)

Figure 4.6 (a) Main Menu of *Two Column Proof;* (b) Proving exercise on trapezoids

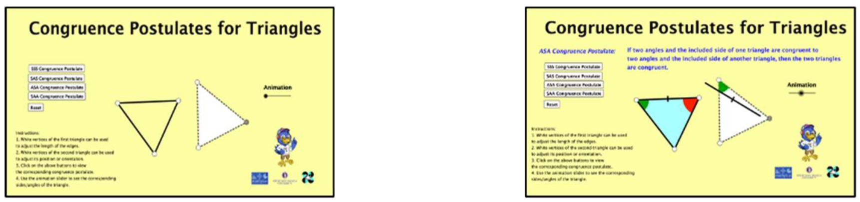
One proving exercise given in the *Two Column Proof* app is to prove the diagonal isosceles trapezoid congruency (Figure 4.6(b)). An example of a task is to write a mathematical statement in the "Statement" column where the justification in the "Reason" column is given. To guide the student, an animation is provided to visualize the property of a trapezoid or a congruence property in the "Reason" column. For instance, to visualize "Base angles of an isosceles trapezoid are congruent", the congruent base angles (highlighted in blue) of the isosceles trapezoid are flashed repeatedly (Figure 4.7(b)). Next, to visualize the Side-Angle-Side (SAS) congruence, an animation is provided in the following sequence: a pair of sides OR and MA from two different triangles (Figure 4.7(a)), a pair of congruent angles ROM and AMO (Figure 4.7(b)), the common side OM (Figure 4.7(c)). Then the two triangles are highlighted: triangle ROM (light green) followed by triangle AMO (red) (Figure 4.7(d)).



(a) (b) (c) (d)

Figure 4.7 Animations in the *Two Column Proof* app

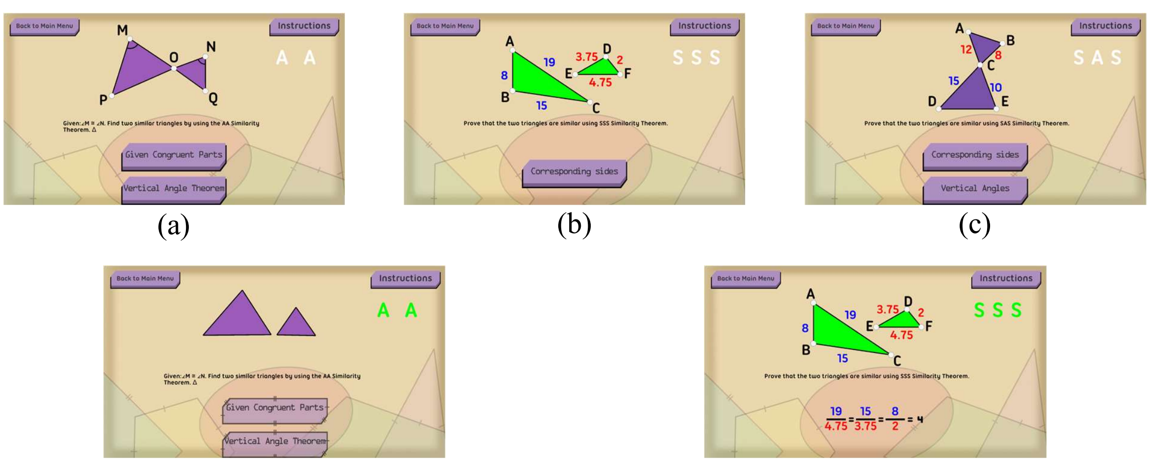
One of the essential tools when formulating proofs is knowledge of previously taught mathematical concepts that are needed in formulating the proof. For instance, in proving concepts on parallelograms, kites or trapezoids, one needs a firm understanding of the congruence postulates for triangles. Unfortunately, students either forget these concepts that have been taught earlier in high school, or these were not taught at all in the time of Covid. We also designed *GeoGebra* apps on triangle congruence for this purpose (Figures 4.8(a)-(b)). One advantage of *GeoGebra* is one can create applications in a much shorter time (than math apps that require some extensive programming) to fill in the learning gap, when necessary.



(a) (b)

Figure 4.8 GeoGebra applets on Triangle Congruence

Prove It is another app that aims to facilitate students' proving competency. The first version of *Prove It* for the topic of triangle congruence is given in [24]. In [3], the app was extended to include the topic of triangle similarity, as a need to bridge the learning gap. A task requiring the Angle-Angle (AA) Similarity Theorem (Figure 4.9(a)), Side-Side-Side (SSS) Similarity Postulate (Figure 4.9(b)), or SAS Similarity Postulate (Figure 4.9(c)) is initially shown. The corresponding text AA, SSS, and SAS on the screen provides guidance to help students draw their attention to the relevant parts of the triangles. For AA, students need to select two pairs of congruent angles. For SSS, students need to select three pairs of proportional sides. For SAS, students need to select two pairs of proportional sides and one pair of congruent angles (which should be the included angle between the two proportional sides). Feedback is also provided to guide students towards selecting the correct pair. Once the student successfully identifies the correct pairs, an animation shows that the two triangles are similar (Figure 4.9(d)); for SSS and SAS, a corresponding ratio is shown (Figure 4.9(e)).



(a) (b) (c)

(d) (e)

Figure 4.9 The app *ProveIt* featuring tasks on Similarity

In the third quarter of the school year 2022-23, a short study was conducted to assess the effectiveness of the *Two Column Proof* app. Two pairs of control-experimental classes in a public high school in Quezon City, Philippines were given pretests and posttests covering the following competencies: (i) proves theorems on the different kinds of a parallelogram (rectangle, rhombus, square), (ii) proves theorems on trapezoids and kites. For the first pair of classes, the control class (38 students) recorded a mean gain score of 30.94% while the experimental class (40 students) recorded a mean gain score of 44.90%. The gap between mean gain scores was wider for the second pair of classes: 25.73% for the control (25 students) and 79.29% for the experimental (27 students). These results indicate the potential of the *Two Column Proof* app to help students learn geometry, particularly proving topics, better.

5. Probability and Statistics Apps

Probability and Statistics is identified as one of the five content strands in the Philippines' K to 12 Basic Education Curriculum [13]. The focus of the strand is on "developing skills in collecting and organizing data using charts, tables, and graphs; understanding, analyzing and interpreting data; dealing with uncertainty; and making predictions about outcomes" [13, p.5]. Competencies in probability and statistics primarily comprise the fourth quarter of the mathematics subject in Grades 1 to 10, except 9. Additionally, Grade 11 students take the core subject *Statistics and Probability* during the third and fourth quarters of the school year [25].

Even before the pandemic, there have already been some challenges in ensuring that students develop the learning competencies in probability and statistics. As the corresponding topics are scheduled to be covered during the last quarter of each school year, class delays or cancellations (e.g., due to typhoons, other disasters, or unexpected holidays) usually result in some of these topics being taught quickly or neglected altogether [26, pp. 300-301]. Another challenge in statistics education in the Philippines is the lack of resources [27]. Particularly, there is a need for accessible and available materials that can support or enhance the teaching and learning of probability and statistics that are also compatible with independent learning.

To bridge the learning gap in probability and statistics, we have developed different apps as part of [3]. These apps have been deliberately designed so that these can be used not only in actual classroom settings but also for remote or independent learning activities.

As emphasized in the Pre- K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II), "the focus on variability in data, the importance of context associated with the data, and the questioning of data, sets statistics apart from other mathematical sciences and makes it particularly relevant for all fields of study" [28, p.8]. Thus, it is important to have different data sets with clear contexts and with which learners can deal with variability. Further, a classroom study by Pfannkuch [29] has found that the contexts of data sets have the potential to assist students, up to a certain extent, in developing informal statistical inferential reasoning. Rossman and Chance [30], in their proposed introductory course for statistics, included activities where students are tasked to do probabilistic experiments such as flipping a coin. This is in alignment with Cobb [31], who has suggested using simulation- and randomization-based methods to develop statistical inference.

Given the above, we have developed the Android/PC app *Probability Simulator* and the web-based app *Senso Eskwela Pilipinas* (*SEP*). The *Probability Simulator* app (Figure 5.1) allows for the simulation of different probabilistic experiments (e.g., rolling a fair or unfair die, drawing a card from a deck). The number of trials of the experiments can be set in the app. The outcomes are displayed in the app or can be saved as a spreadsheet file for further processing or analysis.

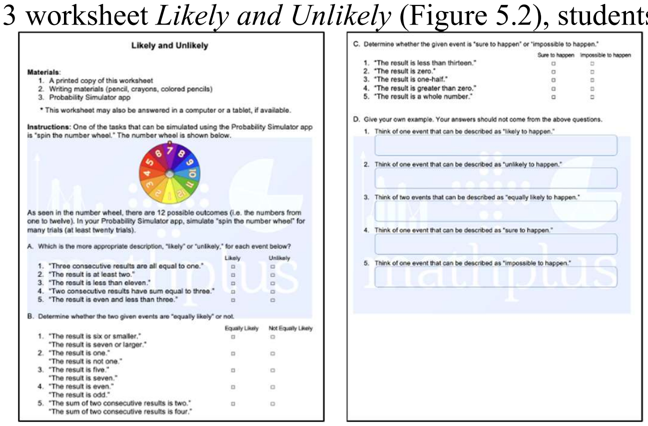
(a) (b)

Figure 5.1 Screenshots of *Probability Simulator*'s (a) home screen and (b) outcomes screen

SEP [32, 33] is an online platform that is based on the Census-at-schools program [34]. Students get access to *SEP* through their teachers and, once logged in, they can anonymously answer a 31-question survey that consists of varied questions that can appeal to young Filipinos' interests and experiences. The students' responses then become part of the *SEP* database that is accessible to *SEP* users. Teachers and students may access their own class data while any *SEP* user may access random samples from the database. Thus, *SEP* becomes a readily accessible source of authentic data that is relevant and relatable to the students. Initial data on the potential benefits of *SEP* with respect to students' learning of statistics are presented in [33].

With *Probability Simulator* and *SEP*, students have ready and easy access to data sets that are otherwise difficult, timely, or tedious to obtain. Moreover, these contextually clear data sets exhibit the variability that students need to handle when learning probability and statistics. Finally, both apps can be used with minimal supervision so these are ideal for independent learning and can be repeatedly used for learning activities across different competencies in probability and statistics.

Student worksheets have also been developed to complement the apps mentioned above. Each student worksheet corresponds to a learning activity; the materials needed, and step-by-step instructions are provided in the worksheet. The worksheets are also formatted in a way that students can readily write their answers on the worksheets, whether on a printout or electronically. For example, in the Grade 3 worksheet *Likely and Unlikely* (Figure 5.2), students are asked to use the



Likely and Unlikely

Materials:
1. A printed copy of this worksheet
2. Writing materials (pencil, crayons, colored pencils)
3. Probability Simulator app

* This worksheet may also be answered in a computer or a tablet, if available.

Instructions: One of the tasks that can be simulated using the Probability Simulator app is "spin the number wheel." The number wheel is shown below.

As seen in the number wheel, there are 12 possible outcomes (i.e. the numbers from one to twelve). In your Probability Simulator app, simulate "spin the number wheel" for many trials (at least twenty trials).

A. Which is the more appropriate description, "likely" or "unlikely," for each event below?

	Likely	Unlikely
1. "Three consecutive results are all equal to one."	□	□
2. "The result is at least two."	□	□
3. "The result is less than eleven."	□	□
4. "Two consecutive results have sum equal to three."	□	□
5. "The result is even and less than three."	□	□

B. Determine whether the two given events are "equally likely" or not.

	Equally Likely	Not Equally Likely
1. "The result is six or smaller." "The result is seven or larger."	□	□
2. "The result is one." "The result is not one."	□	□
3. "The result is five." "The result is seven."	□	□
4. "The result is even." "The result is odd."	□	□
5. "The sum of two consecutive results is two." "The sum of two consecutive results is four."	□	□

C. Determine whether the given event is "sure to happen" or "impossible to happen."

	Sure to happen	Impossible to happen
1. "The result is less than thirteen."	□	□
2. "The result is zero."	□	□
3. "The result is one-half."	□	□
4. "The result is greater than zero."	□	□
5. "The result is a whole number."	□	□

D. Give your own example. Your answers should not come from the above questions.
1. Think of one event that can be described as "likely to happen."
2. Think of one event that can be described as "unlikely to happen."
3. Think of two events that can be described as "equally likely to happen."
4. Think of one event that can be described as "sure to happen."
5. Think of one event that can be described as "impossible to happen."

Figure 5.2 Screenshots from the Grade 3 student worksheet *Likely and Unlikely*

Probability Simulator so that they can develop their sense of events that are likely, unlikely, sure to happen, or impossible to happen. Another example is a Grade 11 worksheet *Discrete Random Variables,* which outlines an activity in which students use an *SEP* question and data set to define a discrete random variable then construct its probability distribution and histogram. The student worksheets can be used for in-classroom activities and are also ideal for independent learning activities. Thus, together with the apps, these can be used to continue or to reinforce the learning of probability and statistics competencies that can no longer be fully accommodated within the last quarter of the school year.

6. Challenges in Implementation

Several challenges were encountered during the project [3] in the implementation of the use of technological tools in schools to bridge the learning gaps in mathematics.

Students' access to technology (smartphones, tablets, computers, etc.) remains one of the most challenging problems in the Philippines. Some schools under DepEd school divisions get support from the local government where each student is provided with a tablet, but this is not true for all schools. In this respect, it was difficult for students who did not have their own gadgets to have access to the apps and to use these to help in their math learning. Other students had to rely on the gadgets of their parents or older siblings but could only use these at certain times during the week.

Lack of teachers has also been a main concern. With the lack of teachers there are missed opportunities for a more focused and nurturing approach in addressing the least learned competencies of students. In a class for example, the learning gaps are different for each student, and the choice of mathematical app to remedy a particular learning competency may vary. Moreover, even if school administrators would want to conduct remediation programs outside of class hours, there are not enough teachers to carry out these programs.

Classroom shortage has also been a concern. There are insufficient classrooms that could be venues for remedial classes or learning spaces where students work on the mathematical apps during their free time in school.

Finally, families play a critical role in successfully integrating technology into learning. It has not been easy, for example, for teachers to collaborate with parents with regards to knowledge on the mathematical apps adopted in class so that their contribution at home could be more effective. For one, there is difficulty on the part of the parents understanding how the gadget and the mathematical apps work. Particularly for low-income families, there is really no time left for parents to learn the technology, as they must prioritize work to support their families. This can enhance the learning inequality further, particularly for young learners. For some students, there are no opportunities to carry out learning activities with the mathematical apps as parents must rely on them to carry out tasks at home.

7. Conclusion and Future Direction

The use of technological tools to help address the issue of major learning gaps in students after the pandemic is not unique to the Philippines. In India for example, the World Economic Forum launched the new 'Education 4.0 India Report' where digital tools will be utilized to enhance foundational literacy and numeracy skills [35]. The implementation of our project [3] to schools in collaboration with DepEd school division offices, is one step towards addressing this problem in the Philippines. In this paper, we showed the different mathematical apps that we have developed to address learning gaps in several content areas in the DepEd curriculum that have been identified by teachers and administrators in our partner schools. The design of the apps has been based on pedagogical principles and game design elements, as well as what is informed in the literature as benefits of technology in mathematical learning. The apps are free and run primarily on low-end mobile devices for accessibility. It is hoped that other educators can find these apps helpful in their work on the use of technological tools in bridging gaps in mathematics learning outcomes.

There is a lot of work to be done in terms of the mathematical apps-more updates can still be done in terms of adding questions and levels of difficulty. Collaboration is still ongoing with DepEd school division offices in terms of assessing the effectiveness of these apps in student learning. Furthermore, there are ongoing efforts soliciting support from NGOs and private sectors to bring the mathematical resources to more schools in the country.

Acknowledgments This paper is one of the outputs of the Ateneo *Mathplus* Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors would like to thank the Department of Science and Technology- Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the project [3]. We thank our app developers Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, Sparksoft Solutions, Inc and our artist Mr. Amiel Damian F. Justiniani.

References

[1] Department of Education (DepEd) Region I. (2022). *Regional Memorandum No. 664, s. 2022: Basic Education Learning Recovery Plan in light of the COVID 19 Pandemic*. Retrieved 7/20/2023 from depedro1.com/wp-content/uploads/2022/06/rm0664s2022.pdf

[2] Department of Education. (2023). *Adoption of the National Learning Recovery Program in the Department of Education.* Retrieved 7/3/2023 from www.deped.gov.ph/wp-content/uploads/DO_s2023_013.pdf

[3] De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B., and Mallari, J. C. (2023). *Terminal Report (Department of Science and Technology-Philippine Council for Industry, Energy and Emerging Technology Research and Development): Mathematical Resources for Distance Learning Utilizing Community LTE Networks and Television Frequencies*.

[4] Department of Education. (2020). *K to 12 Most Essential Learning Competencies with Corresponding CG Codes*.

[5] De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Mallari, J.C.F., Sarmiento, J.F., Tolentino, M.A.C., and Verzosa, D.M.B. (2022). Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network. In W.-C. Yang, M. Majewski, D. Meade, and W.K. Ho (Eds.), *Proceedings of the 27th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.

[6] Villanueva, J.C.P.M., Melendres, M.A.V., Lagunzad, C.G.B., and Libatique, N.J.C. (2021). Design and Deployment of a Mobile Learning Cloud Network to Facilitate Open Educational Resources for Asynchronous Learning. In M.M.T. Rodrigo, S. Iyer, A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. Asia-Pacific Society for Computers in Education.

[7] De Las Peñas, M. L. A. N., Kwong, J. C. T., Banzon, P. A. B., Martinez, P. A., Lapada, W. I. G., Eballa, J. N., Sebastian, J. E. C., Asido, M. D., San Juan, J. D. M., Verzosa, D. M. B., Sarmiento, J. F., Garciano, A. D., Tolentino, M. A. C., Aberin, M. A. Q., and Mallari, J. C. F. (2023). Mathematical Mobile Apps via Rural Casting. In P. Kommers, I. A. Sánchez, and P. Isaías (Eds.), *Proceedings of the International Conferences on e-Society 2023 and Mobile Learning 2023*. International Association for Development of the Information Society.

[8] Sacristan, A.I., Parada, S.E., and Miranda, L. (2011). The Problem of the Digital Divide for Mathematics Teachers in Developing Countries. In M. Joubert, A. Clark-Wilson, and M.

McCabe (Eds.), *Proceedings of the 10th International Conference for Technology in Mathematics Teaching*.

[9] Norris, C., Hossain, A., and Soloway, E. (2011). Using Smartphones as Essential Tools for Learning. *Education Technology*, 51(3), 18-25.

[10] Plass, J.L., Homer, B.D., and Kinzer, C.K. (2015). Foundations of Game-Based Learning, *Educational Psychologist*, 50(4), 258-283.

[11] Shi, Y.-R. and Shih, J.-L. (2015). Game Factors and Game-based Learning Design Model. *International Journal of Computer Games Technology*, 2015, 1-11.

[12] Bolden, D., Barmby, P., Raine, S., and Gardner, M. (2015). How Young Children View Mathematical Representations: a Study Using Eye-tracking Technology. *Educational Research*, 57, 59-79.

[13] Department of Education (2016). *K to 12 Curriculum Guide: Mathematics*. Retrieved 7/20/2023 from www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

[14] De Las Peñas, M.L.A.N., Verzosa, D.M.B., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., and Tolentino, M.A.C. (2021) Designing Performance Tasks in Mathematics Using Technological Tools. In W.-C. Yang, D. Meade, and M. Majewski (Eds.), *Electronic Proceedings of the 26th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.

[15] Wu, H., (2011). *Understanding Numbers in Elementary School Mathematics.* Providence, RI: American Mathematical Society.

[16] Verzosa, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Sarmiento, J.F., Mallari, J.C.F., and Tolentino, M.A.C. (2022) Development of an App and Videos to Support the Fraction Learning Trajectory from Grades 1-7. In Iyer, S. et al. (Eds.), *Proceedings of the 30th International Conference on Computers in Education. Asia-Pacific Society for Computers in Education*. Asia-Pacific Society for Computers in Education.

[17] Verzosa, D.M.B., Aberin, M.A.Q., De Las Peñas, M.L.A.N., Garciano, A.D., Sarmiento, J.F., and Tolentino, M.A.C. (2021). Development of a Gamified Number Line App for Teaching Estimate and Number Sense in Grades 1 to 7. In M. M. T. Rodrigo, S. Iyer, and A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. Asia-Pacific Society for Computers in Education.

[18] Verzosa, D.M.B., De Las Peñas, M.L.A.N., Sarmiento, J.F., Aberin, M.A.Q., Tolentino, M.A.C., and Loyola, M.L. (2021). Using Mobile Technology to Promote Higher-order Thinking Skills in Elementary Mathematics. In I.A. Sánchez, P. Kommers, T. Issa, and P. Isaías (Eds.), *Proceedings of the International Conferences on Mobile Learning 2021 and Educational Technologies 2021*. International Association for Development of the Information Society.

[19] Lehtinen, E., Hannula-Sormunen, M., McMullen, J., and Gruber, H. (2017). Cultivating Mathematical Skills: from Drill-and-practice to Deliberate Practice. *ZDM Mathematics Education*, 49, 625-636.

[20] Garciano, A.D., Verzosa, D.M.B., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Mallari, J.C.F., Sarmiento, J.F., and Tolentino, M.A.C. (2023) Practice through Play Using Mobile Technology. In P. Kommers, I. A. Sánchez, and P. Isaías (Eds.), *Proceedings of the International Conferences on e-Society 2023 and Mobile Learning 2023*. International Association for Development of the Information Society.

[21] Stephan, M. and Akyuz, D. (2012). A Proposed Instructional Theory for Integer Addition and Subtraction. *Journal of Research on Mathematics Education,* 43, 428-464.

[22] Jenkins, R. (2022). *Impact of Interactive Computer-aided Instruction in Learning Trigonometry in a High School Precalculus Course.* [Doctoral dissertation, Franklin University]. https://fuse.franklin.edu/ docpub/71
[23] Presmeg, N. (2006). Research on Visualization in Learning and Teaching Mathematics. In A. Gutierrez and P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 205-235). Rotterdam, the Netherlands: Sense Publishers.
[24] Verzosa, D.M.B., De Las Peñas, M. L.A. N., Aberin, M.A.Q., and Garces, L.P.D. (2018). App-based Scaffolds for Writing Two-Column Proofs. *International Journal of Mathematical Education in Science and Technology*, 50(5), 1-13.
[25] Albert, J.R.G. et al. (2016). *Teaching Guide for Senior High School: Statistics and Probability*. Commission on Higher Education (Philippines).
[26] Batilantes, S.P. (2021). Project VLOGI (Video Lectures on Giving Instructions): Effects on Learners' Performance in Probability and Statistics. *International Journal of Educational Studies in Mathematics*, 8(4), 299-315.
[27] Reston, E. (2023). Statistics Education in the Philippines: Curricular Context and Challenges of Implementation. In G.F. Burrill, L. de Oliveria Souza, and E. Reston, E. (Eds.), *Research on Reasoning with Data and Statistical Thinking: International Perspectives. Advances in Mathematics Education.* Springer, Cham.
[28] Bargagliotti, A., Franklin, C., Arnold, P., and Gould, R. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II).* American Statistical Association.
[29] Pfannkuch, M. (2011). The Role of Context in Developing Informal Statistical Inferential Reasoning: A Classroom Study. *Mathematical Thinking and Learning*. 13, 27-46.
[30] Rossman, A. J. and Chance, B. L. (2014). Using Simulation-based Inference for Learning Introductory Statistics. *Wiley Interdisciplinary Reviews. Computational Statistics*, 6(4), 211-221.
[31] Cobb, G.W. (2007). The Introductory Statistics Course: a Ptolemaic Curriculum? *Technology Innovations in Statistics Education*, 1, 1-15.
[32] De Las Peñas, M.L.A.N, Verzosa, D.M.B., Sarmiento, J.F., Tolentino, M.A.C., and Loyola, M.L. (2020). Designing Mobile Apps to Promote Numeracy and Statistical Reasoning. In W.-C. Yang and D. Meade (Eds.), *Electronic Proceedings of the 25th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.
[33] Tolentino, M.A.C., De Las Peñas, M.L.A.N., Aberin, M.A.Q., Garciano, A.D., Loyola, M.L., Sarmiento, J.F., Mallari, J.C.F., and Verzosa, D.M.B (2022). Engaging Learners through Data: Senso Eskwela Pilipinas. In W.-C. Yang, M. Majewski, D. Meade, and W.K. Ho (Eds.), *Proceedings of the 27th Asian Technology Conference in Mathematics*. Mathematics and Technology, LLC.
[34] Connor, D. (2002). CensusAtSchool 2000: Creation to Collation to Classroom. In *Proceedings of the Sixth International Conference on Teaching of Statistics*. International Statistical Institute.
[35] Poddar et al. (2022). *Education 4.0 India Insight Report*. World Economic Forum.