Analysis of Progressive Casino Game Betting Systems

D. Cole Payne
paynedc33@gmail.com
Department of Mathematical Sciences
Appalachian State University
Boone, North Carolina, USA

Richard E. Klima
klimare@appstate.edu
Department of Mathematical Sciences
Appalachian State University
Boone, NC, USA

Neil P. Sigmon
npsigmon@radford.edu
Department of Mathematics
Radford University
Radford, VA, USA

Abstract: *This work is primarily the product of the first author (who is also the presenting author), a student who completed the work under the direction of the secondary authors. We analyze three progressive betting strategies, each applied to three casino games, aimed at identifying optimal strategies after a given number of bets. The strategies analyzed are Martingale, Paroli, and Fibonacci, each of which are applied to the casino games blackjack, roulette, and craps, with bets placed that pay 1:1. The purpose of this work is not to try to discover methods for beating the house, which are known to not exist, but rather to search for methods for advancing gameplay through a maximum number of bets while retaining the possibility of earning a profit. Programming in the computer algebra system Maple will be used for the calculations.*

1 Introduction

Casino gamblers have been attempting to beat the house since organized gambling was introduced between the 7th and 10th centuries during the Chinese Tang dynasty [6]. Players have attempted to increase their odds of winning through both legal and illegal means, using playing strategies such as choosing when to make a particular decision or how much to wager on a specific outcome. The motivation for this paper came from a run of bad luck by one of the authors at a casino over several months. The hope was that by using some mathematical analysis, we would be able to see how to use betting strategies to skew outcomes in our favor over the short or long term.

Please note that we do not wish to give the impression that anything can be done to change the odds to players or the house. Rather, we only wish to try to identify a betting strategy that allows players to limit risk while retaining the potential for success. With a focus on games with around 50% probability of success, or, as it is known, "1:1 (one-to-one) odds," by applying the betting systems, we propose that a player could play a chosen number of bets with only a minimal loss or even a potential profit. The three games which will be played through simulation are blackjack, craps, and roulette, to which the three betting systems Martingale, Paroli, and Fibonacci will be applied.

2 The Games

2.1 Blackjack

Blackjack has a convoluted origin, with researchers unsure of its inception. The most common theory gives credit to early 18th century France with the invention of the game *Vingt-et-Un*, which translates to *Twenty-One*. Under the reign of King Louis XV, the game was played at French Royal Court, and by the early 19th century it had made its way into the streets of New Orleans, though played with a slight variation on the rules as compared to today. When gambling became legalized in Nevada during the 20th century, the game began being hosted by casinos and gambling halls. Its name was changed to blackjack with the hopes of growing its popularity, and additional payouts were given to players who won with a black Jack (the Jacks of clubs or spades) or the Ace of spades. These bonus payouts eventually became less common after the game did indeed grow in popularity [3].

A few variations exist for the rules of blackjack, although the odds of winning change only a very small amount. The standard rules for blackjack and the parameters for our analysis are:

1. All cards are assigned point values. Cards numbered 2–10 are assigned the number of points that match their face value, while Kings, Queens, and Jacks are all assigned 10 points each. Aces can be assigned either 1 or 11 points, as chosen on each deal by the player, who presumably chooses whichever option benefits them the most.

2. The dealer deals the player two cards, while the dealer receives two cards with one facing up. The player then decides whether to "hit," meaning take an additional card, or "stand," meaning do not take an additional card. If the player hits, then they decide again whether to hit or stand, and they continue in this manner until they either stand or "bust," meaning they have more than 21 total points. The player loses if they bust. If the player stands with fewer than 21 points, then the dealer repeatedly draws cards until they either beat the player or bust. In this scenario the house loses only if the dealer busts.

3. For our analysis, we will assume eight full decks of cards are used, and the dealer stands on a soft 17, meaning that if the dealer has any combination with an ace in their hand that could be counted as either 7 or 17, the dealer stands. Also, if the player's first two cards are a "blackjack," meaning any Ace and any other card worth 10 points for a total of 21 points on the deal, the payoff to the player is 3:2, meaning any bet earns a profit of 150%. Finally, the player will follow the basic strategy of the betting systems analyzed, which dictate gameplay based on scenarios. Although electronics cannot be used at blackjack tables, basic strategies can be printed and used at tables legally.

Following these rules, the odds are in the dealer's favor, but by a margin of only 0.43096% [5]. That is, the dealer wins on average 50.21548% of the time, while the player wins 49.78452% of the time.

2.2 Craps

Craps is a dice game also with a convoluted origin. The most common origin story for craps is that it was invented during the Crusades, when it was known as *Hazzard*, and grew in popularity during the gambling boom in 17th century France. As with blackjack, craps came to the United States via New Orleans as a street game called *street craps*. The term *craps* comes from the French word *crapaud*, meaning *toad*, which a person tended to resemble as they crouched over to play the game on a floor or

sidewalk. The American version of craps saw a few minor rule changes over the years, but the game has basically been played in a very similar manner for many years.

Craps consists of 27 bets that can be played in prediction of the sum of two dice rolled by a *shooter* [4]. A few bets are inverses of each other, so playing both during the same roll would not be logical, but any other combination of bets is reasonable. The parameters for our analysis are:

1. The player will only play the *Field* bet, meaning the sums 3, 4, 9, 10, and 11 are 1:1 winners, and the sums 2 and 12 are 2:1 winners since they each result from the roll of two dice with only one possible combination.

The odds for the Field craps bet are in the house's favor by a margin of 5.56% [4].

2.3 Roulette

Roulette is a game played with a small white ball, called the *pellet*, spinning around a horizontal wheel, on which it eventually settles into one of the following 37 or 38 numbered positions: ones labeled 1–36 split between red and black with 18 positions for each, one labeled 0, and, sometimes, also one labeled 00. The roulette wheel was the accidental invention of the famed French mathematician Blaise Pascal when he was attempting to create a continuous motion mechanism requiring no outside force to stay in motion [8]. While Pascal's intended experiment failed, he nonetheless succeeded in creating the roulette wheel. As with craps, there are many different ways one can bet on roulette, including betting on individual numbers, splitting a bet between two or four numbers, or dividing the table into thirds by betting on 1–12, 13–24, or 25–36. The parameters for our analysis are:

1. The player will make the same even-money bet on each spin, which can be the even/odd where a player choosing even wins if a nonzero even number hits and a player choosing odd wins if an odd number hits. The player can also bet the high/low, which pays for either 1–18 or 19–36. The player can also bet on black or red, which pays if the correct color is chosen. All of these bets pay 1:1.

2. The board has 38 spaces, including both 0 and 00 positions, since this is the most common board used today. With this board, all even-money bets will have probability of success of $\frac{18}{38}$.

The odds for an even-money roulette bet are in the house's favor by a margin of 5.26% [5].

3 The Betting Systems

3.1 The Martingale System

The Martingale system, which was introduced by the French mathematician Paul Pierre Lévy in the 18th century, is likely the most common progressive betting system used today. Legend states that the system derives its name from an early 18th century London casino owner named John H. Martindale, who encouraged patrons to apply it in his own casino, which ironically led, as claimed by some, to its bankruptcy. The Martingale system has transcended standard casino gambling though, and is now used in many nontraditional forms of gambling, including investment banking, stock brokering, and sports betting. The rules of the Martingale system are straightforward; with each loss, one doubles their bet, until they win. Although starting bet sizes of 5% of one's bankroll are more common in practice, we will use 1% in our analysis in order to hopefully more clearly see long-term trends [2].

The following table shows an example of the Martingale system in practice, with a beginning bankroll of $100, and a starting bet size of $1.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Loss	$ 99
2	$2	Loss	$ 97
3	$4	Loss	$ 93
4	$8	Win	$101
5	$1	Win	$102

Under the Martingale system with a beginning bankroll of $100, after six consecutive losses it would be not possible to make a seventh bet. Nonetheless, even though Martingale is indeed a high risk system, it is also true that for one whose beginning bankroll is sufficient to fund the next bet after n consecutive losses, a win on that next bet would return the beginning bankroll plus the amount of the first bet. Exhausting one's bankroll is not the only drawback to the system though. Casinos also often have betting limits, so one might reach a point where doubling after a loss would not be allowed.

The following code written for the computer algebra system Maple is for analysis of Martingale applied to blackjack, stopping if 25 rounds are reached. The code is unique to Maple and Martingale, but could easily be altered for any programming language and any game as long as the player knows the house edge, the probability of winning a round, and the maximum number of rounds to be played.

```
randomize():
with(Statistics):
Success := Vector[row](1 .. 25):
Running_Total := Vector[row](1 .. 25):
End_Value := Vector[row](1 .. 1000):
Num_Turns := Vector[row](1 .. 1000):
for j to 1000 do
    for k to 25 do
        r := rand(1.0 .. 2.0):
        if r() < evalf(1 + 0.4978452) then Success[k] := "W"
           else Success[k] := "L"
        fi
    od:
    initial_value := 500:
    bet := 5:
    for i to 25 do
        if Success[i] = "W" then initial_value := initial_value + bet:
                bet := 5: Running_Total[i] := initial_value:
           else initial_value := initial_value - bet: bet := 2*bet:
                Running_Total[i] := initial_value:
           if initial_value < bet then break
           fi:
        fi
    od:
```

```
    Num_Turns[j] := i - 1:
    End_Value[j] := Running_Total[i - 1]:
    if Num_Turns[j] < 25 then End_Value[j] := Running_Total[i]:
       else End_Value[j] := Running_Total[i - 1]
    fi:
od:
Num_Turns; End_Value; Mean(Num_Turns);
Mean(End_Value); max(End_Value); min(End_Value);
```

The following table shows the result of applying 1000 trials of the Martingale betting system to each game, stopping if 25, 50, or 100 rounds are reached, with a beginning bankroll of $500 and initial bet size of $5 for each. Within each cell of the table, the first number is the average number of rounds played before either the indicated number of rounds was reached or the next bet could not be made, and the second number is the average final bankroll.

Rounds	Blackjack	Craps	Roulette
25	23.59 / $509.10	22.76 / $482.60	23.33 / $502.25
50	42.95 / $504.99	40.96 / $473.82	40.56 / $475.34
100	72.56 / $498.88	65.69 / $442.77	66.99 / $441.38

As we see, blackjack shows the highest likelihood of success, with almost 2% in expected profit with at most 25 rounds, and less than 0.2% in expected loss with at most 100 rounds. The following graphs show the distribution of the 1000 trials for all three games with at most 25 rounds, all of which show a bimodal distribution with left skewness and a majority of trials ending in a profit.

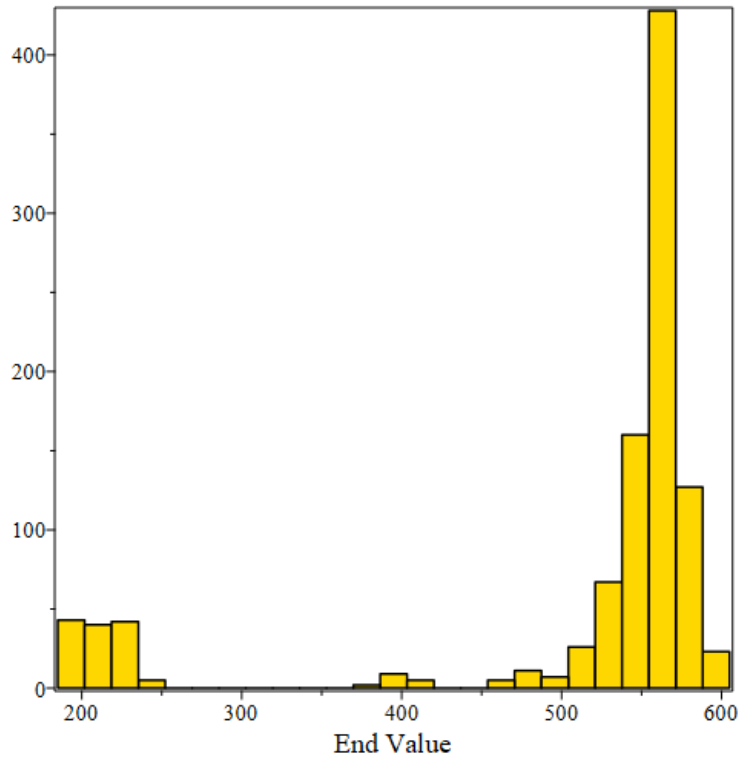

Figure 1: Martingale Frequency of Ending Values for Blackjack With at Most 25 Rounds

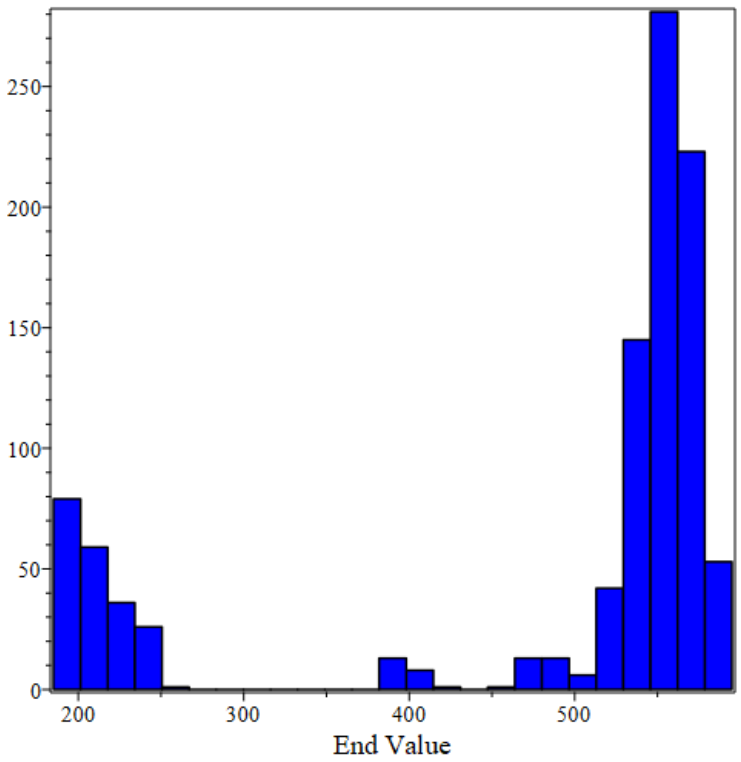

Figure 2: Martingale Frequency of Ending Values for Craps With at Most 25 Rounds

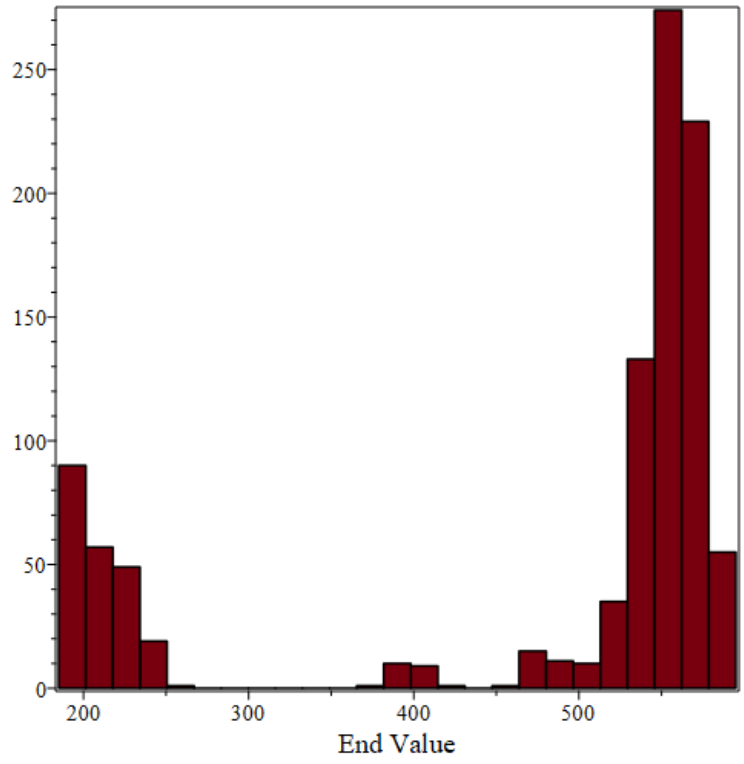

Figure 3: Martingale Frequency of Ending Values for Roulette With at Most 25 Rounds

3.2 The Paroli System

The Paroli betting system derives its name from the Latin word *par*, which carries the meaning *to break even*. This system has been credited as a 17th century invention of the French mathematician Blaise Pascal, who we have noted is also credited with inventing the roulette wheel. While Martingale is a high-risk betting system, the Paroli betting system, also known as "anti-Martingale," allows players to set a maximum amount of money they are willing to lose with each play [1]. Under the Paroli system, players double their bet after each round until they have achieved either three consecutive wins or a loss, after which they return to their initial bet amount [5]. For example, the following table shows an example of the Paroli system in practice, with a beginning bankroll of $100, and a starting bet size of $1.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Win	$101
2	$2	Win	$103
3	$4	Win	$107
4	$1	Loss	$106
5	$1	Win	$107

For a player with a starting bet size of $1, note that under the Paroli system, three consecutive wins results in a profit of $7. In gambling terminology, we would thus say that under the Paroli system, three consecutive wins pays "7 to 1." Since each round offers a chance of winning of approximately $\frac{1}{2}$, it follows that the probability of winning three consecutive rounds is approximately $\frac{1}{2^3} = \frac{1}{8}$. That is, under the Paroli system, players are taking a chance on earning a profit of 7 to 1 against odds of winning of 8 to 1.

The following code written for Maple is for analysis of Paroli applied to roulette with 50 rounds. The code is unique to Maple and Paroli, but could easily be altered for any programming language and any game as long as the player knows the house edge, the probability of winning a round, and the number of rounds to be played.

```
randomize():
with(Statistics):
Success := Vector[row](1 .. 50):
Running_Total := Vector[row](1 .. 50):
End_Value := Vector[row](1 .. 1000):
Num_Turns := Vector[row](1 .. 1000):
bet_placed := Vector[row](1 .. 50):
for j to 1000 do
    for k to 50 do
        r := rand(1.0 .. 2.0):
        if r() < evalf(1 + 18/38)
           then Success[k] := "W"
           else Success[k] := "L"
        fi
    od:
    initial_value := 500:
```

```
    bet := 5:
    for i to 50 do
        if bet > 20 then bet := 5
        fi:
        if Success[i] = "W"
           then bet_placed[i] := bet:
                initial_value := initial_value + bet:
                bet := 2*bet:
                Running_Total[i] := initial_value:
           else bet_placed[i] := bet:
                initial_value := initial_value - bet:
                bet := 5:
                Running_Total[i] := initial_value:
                if initial_value < bet then break
                fi:
        fi
    od:
    Num_Turns[j] := i - 1:
    End_Value[j] := Running_Total[i - 1]:
od:
Num_Turns;
End_Value;
Mean(Num_Turns);
Mean(End_Value);
max(End_Value);
min(End_Value);
```

The following table shows the result of applying 1000 trials of the Paroli betting system to each game, stopping when 25, 50, or 100 rounds are reached, with a beginning bankroll of $500 and initial bet size of $5 for each. Within each cell of the table, the number is the average final bankroll after the indicated number of rounds was played.

Rounds	Blackjack	Craps	Roulette
25	$501.79	$488.11	$489.81
50	$500.77	$476.43	$478.01
100	$495.32	$455.11	$455.39

In comparison with the Martingale system, the Paroli system rarely returns a large profit. However, under Paroli players can guarantee that they would be able to play any specified number of rounds, since the most they could lose on any single round would be the value of their initial bet. For example, if a player wanted to guarantee that they would be able to play 25 rounds, then they could start by betting 4% of their bankroll, since this would require 25 consecutive losses for them to lose their entire bankroll.

The following graphs show the distribution of the 1000 trials of the Paroli system for all three games with 100 rounds. All of the graphs show a more normal distribution for the Paroli system than was seen with Martingale, with more predictability and less volatility. In particular, note that the ending values rarely dip below $300.

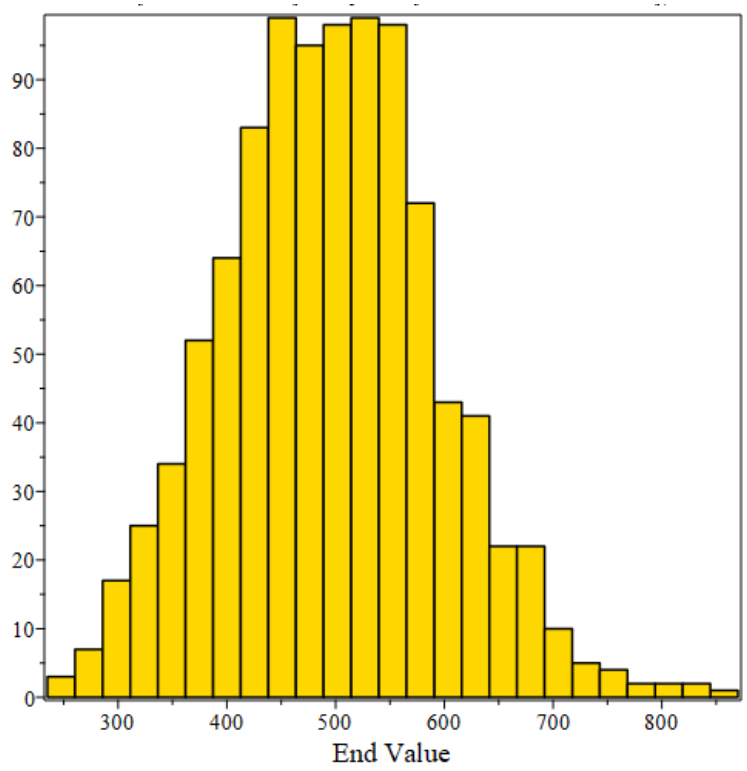

Figure 4: Paroli Frequency of Ending Values for Blackjack With 100 Rounds

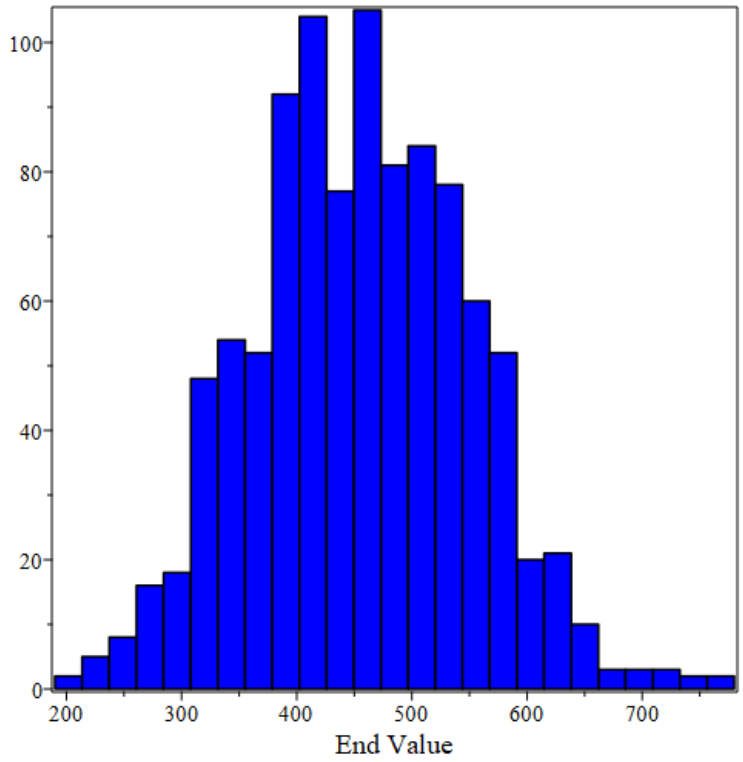

Figure 5: Paroli Frequency of Ending Values for Craps With 100 Rounds

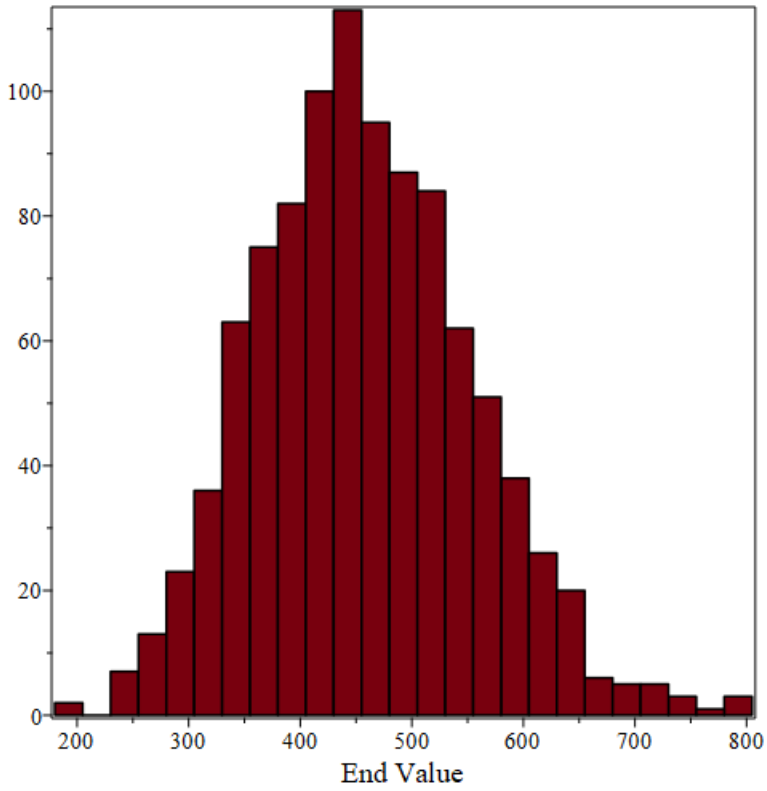

Figure 6: Paroli Frequency of Ending Values for Roulette With 100 Rounds

3.3 The Fibonacci System

The Fibonacci betting system connects naturally with a number sequence first studied by mathematicians in India going as far back as at least 200 BCE. This number sequence was brought to Western Europe by an Italian mathematician named Fibonacci, who included it in his 1202 publication *Liber Abaci* [7], after which it became one of history's most prolifically studied number sequences. In the 19th century the sequence began being referred to as the *Fibonacci sequence*. The Fibonacci sequence begins with the pair of terms $F_0 = 0$ and $F_1 = 1$. Subsequent terms F_i after these first two are the sum of the previous two terms: $F_n = F_{n-1} + F_{n-2}$ (for $n > 1$). That is, the Fibonacci sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Given the myriad ways in which the Fibonacci sequence appears in nature, it only makes sense that it should be applied to casino games and nontraditional forms of gambling. That being said, there is no certainty that the Fibonacci system has ever actually been used in practice. The Fibonacci sequence does lend itself perfectly to a progressive betting system though, since the goal of any such system should be to make up for as much loss as possible with a single win.

In the Fibonacci betting system, each bet a player makes is the starting bet multiplied by a multiplier. For the first bet, players multiplying their starting bet by the number in the third position of the sequence ($F_2 = 1$). After each loss, the bet multiplier moves forward one position in the sequence (from F_i to F_{i+1}). After each win, the bet multiplier moves back two positions in the sequence (from F_j to F_{j-2}), though never moving past the third number in the sequence [5]. For example, the following table shows an example of the Fibonacci system in practice, with a beginning bankroll of \$100, and a starting bet size of \$1.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Loss	$ 99
2	$2	Loss	$ 97
3	$3	Loss	$ 94
4	$5	Win	$ 99
5	$2	Win	$101

Note that in this example, after three consecutive losses, it took two consecutive wins in order for a profit to be earned. As with many other systems, the drawback to the Fibonacci system is that several consecutive losses can very quickly lead to very large bets. For this reason, literature on the system often recommends that users start by betting between only 1% and 5% of their beginning bankroll.

The following code written for Maple is for analysis of Fibonacci applied to craps, stopping if 100 rounds are reached. This code is unique to Maple and Fibonacci, but could easily be altered for any programming language and any game as long as the player knows the house edge, the probability of winning a round, and the maximum number of rounds to be played.

```
randomize():
with(Statistics): with(combinat):
Success := Vector[row](1 .. 100):
Running_Total := Vector[row](1 .. 100):
End_Value := Vector[row](1 .. 1000):
Num_Turns := Vector[row](1 .. 1000):
bet_placed := Vector[row][1 .. 100]:
for j from 1 to 1000 do
    for m  from 1 to 100 do
        r := rand(1.0 .. 2.0):
        if r() < evalf(1 + 0.4722) then Success[m] := "W"
           else Success[m] := "L"
        fi
    od:
    initial_value := 500: bet := 5: k := 1:
    for i from 1 to 100 do
        if k < 1 then k := 1:
        fi:
        if Success[i] = "W" then bet_placed[i] := bet:
                bet := 5*fibonacci(k + 1):
                initial_value := initial_value + bet: k := k - 2:
                Running_Total[i] := initial_value:
           else bet_placed[i] := bet:
                initial_value := initial_value - bet:
                bet := 5*fibonacci(k + 1): k := k + 1:
                Running_Total[i] := initial_value:
                if initial_value < bet then break
                fi:
        fi
```

```
    od:
    Num_Turns[j] := i - 1:
    End_Value[j] := Running_Total[i - 1]:
    if Num_Turns[j] < 100 then End_Value[j] := Running_Total[i]:
       else End_Value[j] := Running_Total[i - 1]
    fi:
od:
Num_Turns; End_Value; Mean(Num_Turns);
Mean(End_Value); max(End_Value); min(End_Value);
```

The following table shows the result of applying 1000 trials of the system to each game, stopping if 25, 50, or 100 rounds are reached, with a beginning bankroll of $500 and initial bet size of $5. Within each cell, the first number is the average number of rounds played before the indicated number of rounds was reached or the next bet could not be made, and the second is the average final bankroll.

Rounds	Blackjack	Craps	Roulette
25	24.73 / $504.47	24.54 / $496.19	24.41 / $485.67
50	47.49 / $513.69	46.32 / $467.82	46.67 / $479.24
100	90.22 / $530.51	85.08 / $450.14	86.13 / $461.16

As we see, blackjack shows a profit for every number of rounds. The Fibonacci system also shows similar results to the Paroli system, with even with a better chance of higher profit. However, unlike Paroli, the Fibonacci system does not allow players to guarantee that they would be able to play any specified number of rounds, and could also allow bets to become very large. The following graphs show the distribution of the 1000 trials for all three games with at most 50 rounds.

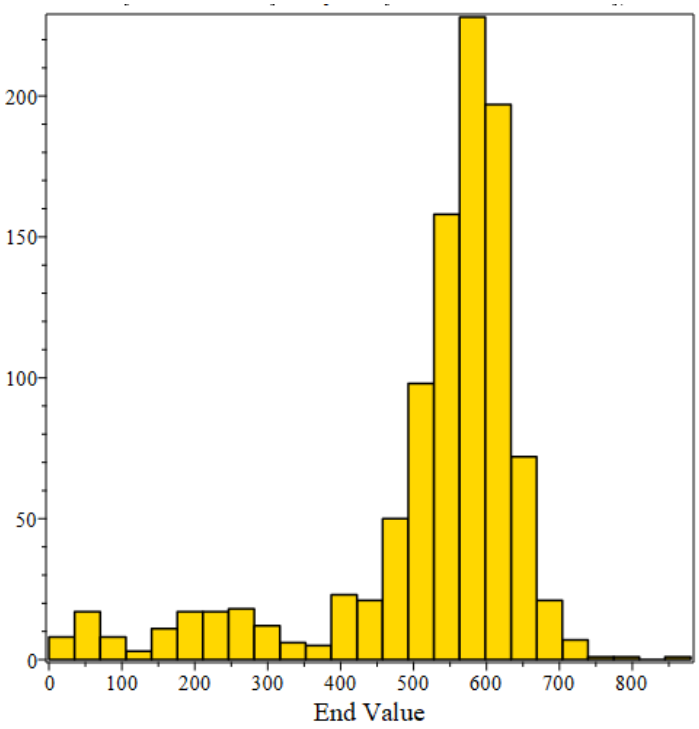

Figure 7: Fibonacci Frequency of Ending Values for Blackjack With at Most 50 Rounds

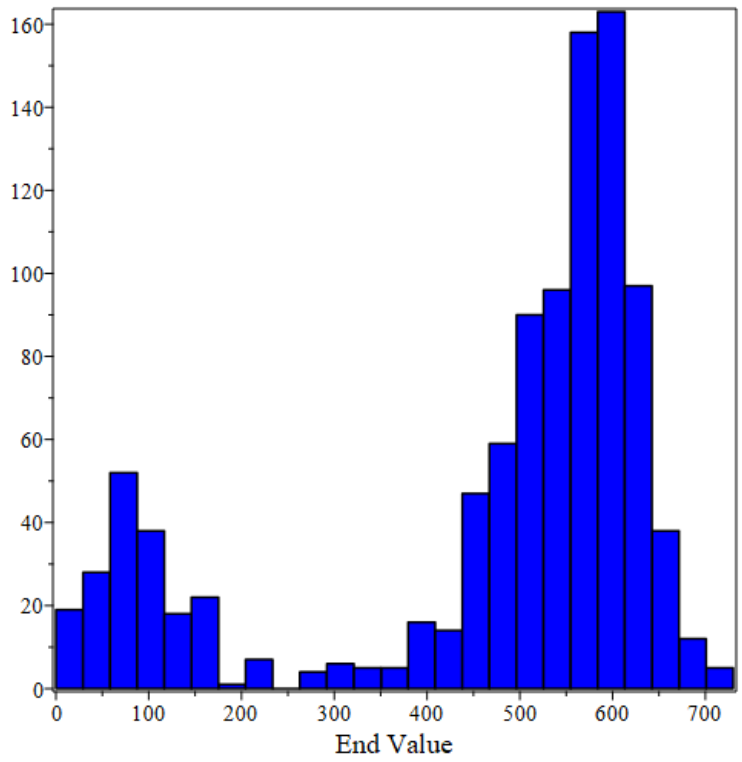

Figure 8: Fibonacci Frequency of Ending Values for Craps With at Most 50 Rounds

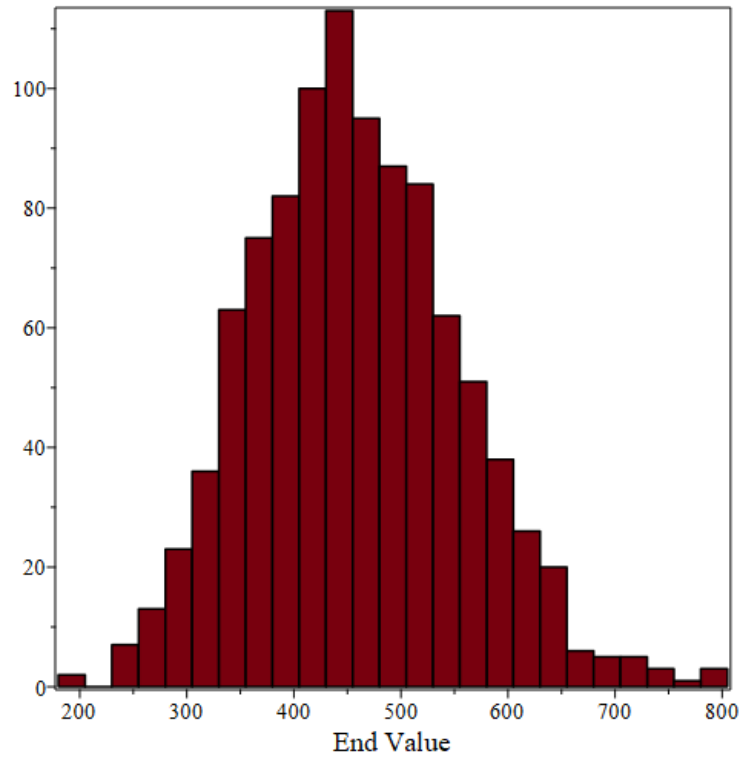

Figure 9: Fibonacci Frequency of Ending Values for Roulette With at Most 50 Rounds

4 Conclusions and Ideas for Future Work

As part of this project, simulations were also done using other number sequences such as Pi and Euler which show more randomness. These systems did not show promise from a gambling perspective though, as the key to a progressive betting system is the ability to regain losses as much as possible with a minimal number of wins.

For the analyses described in this paper, the Martingale betting system shows the most promise for potential profit in the short term, unless the probabilities of success on a given round are approximately 50%. However, under the Martingale system, if a player began by betting 1% of their beginning bankroll, then they would only be able to survive six consecutive losses before being unable to make the next bet. Even worse, if a player began by betting 5% of their initial bankroll, then they would only be able to survive four consecutive losses before being unable to make the next bet. As such, the player would need to decide what it is really worth to them to have a profit potential. An intelligent strategy for the Martingale system would be for a player to identify how much profit they would need before leaving the game. For example, if a player had the goal of earning $25 profit in at most 25 rounds, it would only take five wins when betting $5, and as soon as the player reached that goal, they could retire.

Contrary to the Martingale betting system, the Paroli betting system allows players to guarantee a specific number of rounds that they wish to ensure they will be able to play. More specifically, if the player chose to bet a certain percentage of their initial bankroll, then they would be able to predict a minimum number of rounds they would be guaranteed of being able to play. For example, starting by betting 1% of their beginning bankroll would guarantee that a player would be able to play a minimum of 100 rounds, and starting by betting 5% of their beginning bankroll would guarantee that a player would be able to play a minimum of 20 rounds (assuming the player completed 100 or 20 rounds, respectively, without three consecutive wins). Another favorable property of the Paroli betting system is that players can double their bets until they earn two wins in a row or four wins in a row or however many wins in a row they want to achieve. Recall though that regardless of however many consecutive wins a player wants to achieve in order to perceive the result as a success, the odds of reaching that number of wins would still need to be higher than the payoff. For example, two consecutive wins would result in a 3 to 1 payout, while the odds of winning two games in a row would be approximately 4 to 1.

Finally, recall that the Fibonacci betting system, like Martingale, allows bets to potentially become extremely high if players experience many consecutive losses. So, similar to Martingale, with the Fibonacci system it would be better for players to start by identifying a profit amount as the goal rather than a number of rounds to be played. The fact though is that all betting systems have benefits and drawbacks, and so it ultimately comes down to the goals of the player. This project only aims to investigate systems with the end goal of allowing players to play for as long as possible and to reach a reasonable number of rounds with the hope of spending a pleasant evening in a casino. The system that seems to best fit that criterion is Paroli, given that it is not for high-risk gamblers, and players can guarantee being able to play any specified number of rounds.

Further investigations could include developing an algorithm through which a player could choose a betting system from among the three presented in this paper (and possibly other systems as well), choose a game from among the three presented in this paper (and possibly other games as well), and then have the algorithm return to them the expected outcome of playing their chosen game under their chosen betting system. Understanding that no system can guarantee a profit, the player could at least make better choices that lead to a more pleasurable gaming experience.

References

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