Scissors, Cardboard and GeoGebra: Technology as instruments, not only as artefacts

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***Abstract**: This paper presents an example of technology integration by modifying an optimisation problem as an initial activity of the calculus course in the last year of high school. This proposal articulates three ideas; the importance of the instruments used for the mathematical activity, the concrete, pictorial and abstract model, and the notion of instrumental genesis. Technological tools appear in this proposal initially as an artefact, but this design generates that they become an instrument for learning progressively. Experimentally noted that the activity generates a high level of commitment, causing students to put in play and refine their ideas about working with functions to model reality and make better decisions. This model for technology integration into the mathematics classroom by modifying existing tasks could encourage teachers to integrate technology easily and more meaningfully.*

1. Introduction

> The computational capacity of technological tools extends the range of problems accessible to students. It enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualising and modelling. [13]

Within mathematic teachers' communities, there is a consensus about the need to include technology in the classroom, but there is no agreement on how to do it. Our work started within Uruguay's mathematics teachers' community, a country that could be considered a pioneer in the inclusion of technology in education. But even there, with large-scale support for these proposes and teachers eager to include technology [2], research shows that the mathematics teachers did not fully consider the pedagogical implications of incorporating such technologies (see [17],[18],[22]). In our experience as teachers and teacher trainers, we have seen that most mathematics teachers in our community struggle, believing they must master the tool before including it in a classroom activity. Intended to help knock out that -from our perspective- misconception is that we present this design and the concepts supporting it.

As a starting point, we subscribe to instrumental approaches in mathematics education and technology. In that direction, [10] underlines the tool's impact on the link between the user and de phenomenon, shaping how students think about mathematics. We introduce an activity using technology and an inter-tools approach, sewing the construct *Humans-with-Media* [6] and the *Concrete, Pictorial, and Abstract* approach [7].

This work aims to develop a view on the use of technology in mathematics classrooms that could help teachers consider the pedagogical and technological aspects involved in the process through the design and implementation of classroom activities. In that sense, we agree that mathematics education "presupposes a specific didactic approach that integrates different aspects into a coherent and comprehensive picture of mathematics teaching and learning and then transposing it to practical use in a constructive way" ([23], p. 88). For this, we took a classic activity of every precalculus and calculus course. We modified it to integrate technology and, consequently, develop new ways for students to think about the involved mathematics.

2. Methodology

This work was conceived as the first step of design-based research [16]. Two mathematic teachers designed the tasks and structure presented in the *design* section considering the theoretical framework. We aim to validate and improve the proposed sequence and formulate a way of introducing technology in a meaningful way through modification of existing analogue tasks. The activity was applied to a group of 15 students in the last year of high school as the initial activity of the calculus course. The characteristics of the group are varied; none had substantial experience in using GeoGebra nor in working with modelling activities. We observed the implementation and took field notes in every application of the sequence. The observations were aimed at three elements: The process of technology utilisation, the modelling process, the school content emerging from the modelling process and the interaction with the symbolic and material artefacts.

3. Theoretical framework

Humans-with-Media [6] proposes that mathematical knowledge and the thinking to be developed are conditioned by the means used to represent, communicate, and produce mathematical ideas. Moreover, [21] add that two pillars underpin this framework: cognition is a work of a collective and non-individual nature; cognition includes devices, tools, artefacts and means that produce knowledge. According to this theoretical construct, the subject of knowledge is not only made up of the human being but also the means -that are a fundamental part of it- so it makes no sense to consider them separate.

From a different perspective within instrumental approaches, *Instrumental Genesis* describes the interaction between a *subject* and an *artefact* as the subject increases their experience and practice using it [15]. An artefact can be physical or symbolic; as in our case, the commands in GeoGebra software are symbolic artefacts, and cardboard is the material. The interaction between a subject and an artefact has a physical and a psychological component, for example, in interpreting the information received by the subject and making active decisions on the artefact. [14] introduces the *instrument* to identify the assimilation by the subject of some characteristics of the artefact whose domain allows him to achieve the objective. An instrument is formed by an artefact and by *schemes of use* resulting from the interaction of the subject with the artefact. These schemes may have been elaborated by the subject or have been appropriate.

Instrumental genesis has two components: *Instrumentalization* concerns the emergence and evolution of the components of the artefact that are part of the instrument: selection, regrouping, production and institution of functions, the transformation of the artefact into structure and operation and extending the initial conception of the artefacts. *Instrumentation* refers to the emergence and evolution of the schemes of use: their constitution, their operation, their development, and the assimilation of new artefacts to already constituted schemes [1].

Mathematical modelling appears as the third component in this approach. This is "the process of translating between the real world and mathematics in both directions" ([4], p.45). We look at this process through the modelling cycle presented in [5], as shown in Figure 1. The reason behind our particular attention to modelling processes is that:

> Mathematical models and modelling are everywhere, often connected to powerful technological tools. Preparing students for responsible citizenship and participation in societal developments requires them to build modelling competency. ([4], p.47)

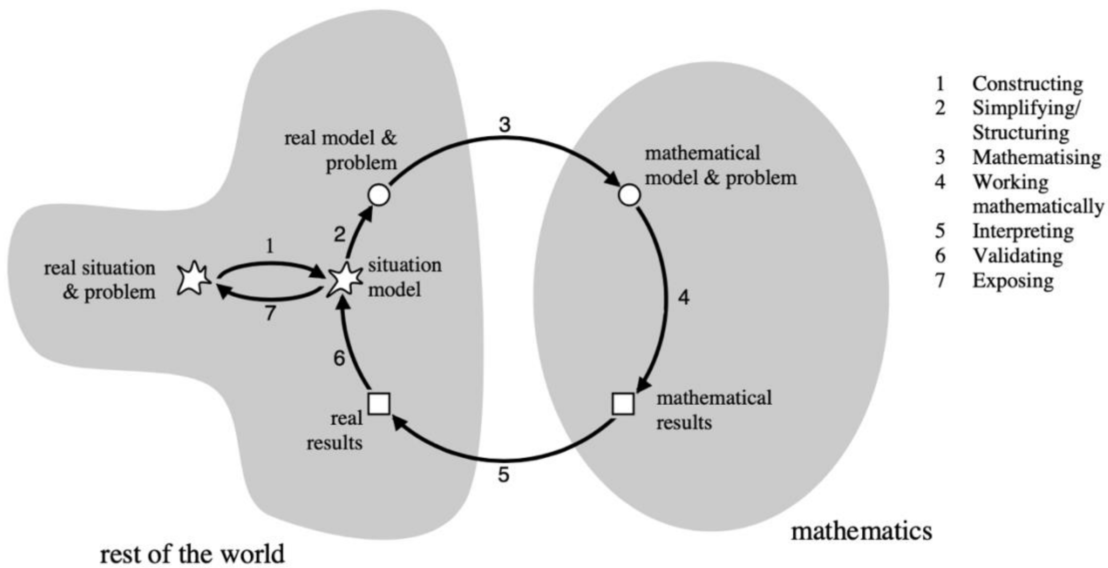

Figure 1. Modelling Cycle. [5]

[11] stated that "Several studies highlight the positive effect of the design and implementation of a multi-representational approach to exploring 3D objects using crafts, computer technology, and paper-and-pencil methods". In our design, we introduce an approach utilising digital and material tools for the visualisation of different registers of representation of the problem, allowing students with this opportunity to approach the problem from different perspectives. GeoGebra will enable students to create a virtual 3D model of the problem to experience it dynamically. But also, the arithmetic data visualisation to analyse the covariation of the quantities involved in the situation. And on top of that, the link between these two (or three if we consider the 2D Geometrical model) representations to perform the conversion process [9]. As stated in [21], the dynamic nature of GeoGebra's representations helps the process of conversion and treatment [9], creating an almost unique representation combining dynamically graphical, analytical, and numeric representations.

4. Design

For the design, we start with an activity, usually part of the optimisation unit of a regular high school calculus course. It asks students to decide the most effective way to manufacture a box from a metal plate, trimming squares from corners and folding the plate to obtain the square-based prism with the largest possible area. See Figure 2.

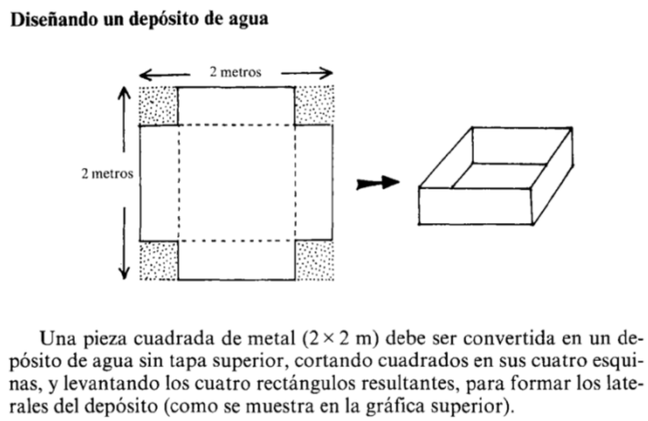

Figure 2. Original activity ([19], p.190)

Based on the idea that each instrument generates a different approach to mathematical activity [6] and, therefore, a different kind of mathematical work, we adopt the CPA model [7], deciding to modify the activity to tour those three moments. Being clear that it will not be a linear sequence of work and that the three components of the work will alternate so that students can make successive approaches to the problem and develop different strategies. Moreover, [8] explored problem-solving heuristics using digital and 3D printed manipulatives, stating that additional material supports for the activity produce differences in problem-solving strategies.

Usually, at this level, working with models to solve optimisation problems has a robust abstract component with plenty of formal and algebraic approaches and few of the other two parts presented by [7]. To introduce the other moments, we add to the activity two artefacts that, mediated by the context of the problem, go through an *instrumental genesis* process, becoming instruments. The first artefact to be introduced is cardboard. Students will receive the instructions (Figure 2) next to a four-square-meter cardboard square and will be told that this is the material they will have to generate the final prototype. This will have the effect of having students manipulate the problem, live it as a close and actual situation and therefore get deeply involved with the activity. This is part of the *rest of the world* stage and guides the steps of *constructing* and *structuring* the modelling cycle, see Figure 1.

The second artefact is GeoGebra; each team will receive computers initially as a support tool. It is expected that in need to design the solution before the prototype's development, they will use GeoGebra to make representations and calculations that allow them to build ideas on possible solutions to the problem. This is the mathematisation stage of the modelling cycle. At this time, the students do not have good management of the artefact, so it is necessary for the teacher's intervention to feed the instrumentalisation process.

Work is driven on GeoGebra from multiple representational registers [9]. First, developing a flat model of the original cardboard square will be suggested to observe the relationship between the trimmed squares' dimensions and the resulting box's dimensions. The results could then be tabulated in the software to begin numerically visualising the covariation between the magnitudes. And with this, start the jump to the abstract moment. It is expected that working with different instruments and the possibility offered by GeoGebra in the link of different registers of semiotic representation [3] enhance students' work and problem-solving strategies [8]. And so, students will be able to develop their arguments for the development, justification, and presentation of the prototype.

5. Application

Teams collaborated, independently prototyping and addressing the initial problem from different perspectives, sharing ideas and offering help when needed. Some groups developed scaled physical models on paper to generate an intuition about the covariation of the magnitudes to be analysed, using a *guess and check* strategy [8]. At the same time, other teams worked with the computer to calculate volumes of cases they considered relevant, as in *systematic experimentation* [8].

Later, teams were suggested to visualise the possible cuts on the cardboard plate by elaborating a representation in GeoGebra. At this stage, students lack experience working with GeoGebra and modelling activities became clear because of the need for the suggestion and the need for the teacher to be present and help with the basic commands of the software. From these episodes, we observed the process of instrumentalisation, as students managed to construct a utilisation scheme of GeoGebra to obtain the desired result, as shown in the example file[1]. This moment showed that it is needed for the teacher to have primary control of GeoGebra and experience dealing with the problem to be able to foster the instrumentalisation process [1,15].

After creating the geometric representation, students worked on the models developed to tabulate and graph the data obtained in GeoGebra (Figure 3). Then, they conjecture the possibility of a quadratic model. Still, with arguments in the graphical and tabular records about the non-symmetry of the results, they discarded the conjecture, using the *guess and check* strategy. The experimental work led to developing an analytical model for the problem and its introduction in GeoGebra to contrast it with the data obtained and determine the optimal solution. The possibility to experiment on GeoGebra helped students develop a new utilisation scheme that gives them unique views of the problem and the mathematics involved in what can be interpreted as part of the instrumentation process [1, 15].

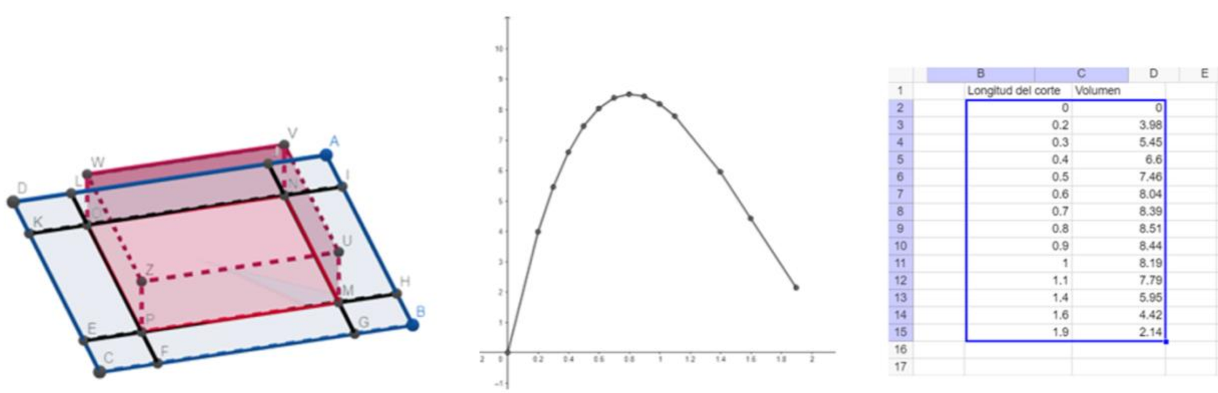

Figure 3. Three-dimensional model and data representation in GeoGebra.[1]

[1] Example file of students' construction: https://www.geogebra.org/m/sxytmst9

As a final step, the teams developed the prototype (Figure 4) and a brief justification for their design decisions based on the data obtained. The process is expected to engage students enough to generate solid mathematical arguments for the presentations.

Figure 4. The prototype was developed by one of the teams.

6. Discussion

During the application, we noticed that students managed to get out of their usual position to search through their mathematical knowledge for a method to respond to a specific teacher's proposal and pursued a genuine response to the problem in contraposition with the regular classroom attitude [2]. The degree of involvement with the activity and the results exceeded our expectations, even going so far as to work spontaneously outside the classroom. We believe this is because the need to build a solution to the problem took the initial focus of mathematical procedures placing mathematics at the service of the solution and not at the centre of the activity. The concrete nature of the initial task [7] fostered students' engagement. Even the moments of elaborating the first *pictorial* representation in GeoGebra and the construction of paper models were lived almost as a game, and there was a level of collaboration among students that is not usual.

We have observed the contribution of the activity to the development of students' mathematical thinking. Mainly the elaboration and work with different types of representations and the connection between these [9] mediated by technological instruments and the concrete element of the prototyping. Making these representations work almost as a single representation [21]. The mathematical modelling skills of students were fostered as students were led through the modelling cycle, mathematising, working with the created models and constructing a solution for a real open-ended problem.

> Too often in mathematics lessons, students take the numbers without regard for the context and process them mathematically using a previously learned procedure. Open-ended problems in real-world contexts can prevent students from doing so, for example, by providing more or less information than is needed for solving the task. ([12], p. 40).

The discussion about the use of technology in the classroom has been going on for many decades and has been going through various moments. Nowadays, we are away from the moment when technology was believed to be an educational panacea. Having come here knowing that technology will not improve education on its own and living in a time where it is ubiquitous positions us in a place of privilege to use it with judgment and realise its full potential. That is why we believe -and we hope we have done so in the design of this activity- that technology should not be at the heart of the design. Still, it should come and contribute its possibilities to a task with an educational objective and not be the objective. In this example, the options for dealing with multiple representations and the dynamic nature of GeoGebra helped to develop a modelling task from a usually algorithmic one.

In the future, we aim to develop this idea further by taking regular classroom activities and integrating them with the possibilities of the available technology, such as DGS, 3D printing and Augmented Reality, to develop new and rich tasks that foster the mathematical knowledge of students. This could open a door for teachers to integrate technology in their classrooms safely and meaningfully.

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