Is it so time consuming to start using a new piece of mathematical software?

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Abstract: In this paper we summarize an experience carried out with pre-service teachers, students of the Master's Degree in Secondary Teacher Training (MFPES) of a public university, about their ability to self-learn the use of the computer algebra system for smartphones "Maple Calculator". It is focused on determining if they were able to find in a short session the main features of the new software and the advantages and disadvantages of its use compared to the use of "Maple" on a computer (note that even if the students handled the latter, it does not resemble the smartphone software). This research has a qualitative approach, of an ethnographic nature. The responses to the questionnaire have shown great maturity, highlighting the expected points and pointing out others.

1. Introduction

We summarize an experience focused on determining the self-learning capacity of the use of the *Maple Calculator* Computer Algebra System (CAS) for smartphones of some future teachers, students of the *Master's Degree in Secondary Teacher Training* (MFPES) of a public university in Spain. All had at least a bachelor's degree in mathematics (some had a double degree, two degrees, another master's degree, or a doctorate). All had some prior computer training and had programmed in at least one computer language. During this academic year they had used the computer language *Scratch 3* and the CAS *Maple¹ 2021* in the subject *Innovación Docente e Iniciación a la Investigación Educativa en Matemáticas (Teaching Innovation and Initiation to Educational Research in Mathematics*) of the MFPES (taught by the first author).

This experience is focused on determining if they were able, after a brief training session, to find the main characteristics of the new software and the advantages and disadvantages of its use on smartphones compared to the use of the already known software on computers.

Something very important is that, although the names are very similar, both the philosophy and the handling of both CAS are very different:

• *Maple* is a huge traditional CAS, with data entry by keyboard or through friendly menus, with a powerful programming language and enormous possibilities in very different fields.

¹ All product names, trademarks and registered trademarks are property of their respective owners.

• Meanwhile, *Maple Calculator* is a small CAS, aimed at recognizing handwritten mathematical text and solving and representing the proposed problem (it can also serve as data input for *Maple*).

In short, knowing *Maple* commands and programming does not help you learn to use *Maple Calculator*.

The result of the experience confirmed an unexpected ability to adapt to the use of the new technology by almost all of these MFPES students, who highlighted the most significant possibilities of the new software, as well as the expected differences in its use with respect to more traditional software. No significant differences were observed by complementary degrees or by gender.

2. Theoretical Framework

The first author has a long experience as a user of CAS (since the late 1980s) and has used it in teaching since the early 1990s.

In the work summarized here we have chosen constructivism as the underlying pedagogical theory, since it attempts to develop skills in students [1]. It is a theory that considers the participation of teachers and students in the teaching-learning process fundamental, a participation that is conveyed through the methodological strategies used in the classroom. In this sense, relying on [2], we distinguish three methodological strategies that support the preceding approaches:

- expository (the one in which the teacher's protagonism is greater),
- demonstrative (where the protagonism of teacher and student are similar), and
- active (in which there is greater protagonism of the student).

Likewise, each methodological strategy uses different teaching methods and techniques.

In our opinion, it is desirable that active learning be based on at least partially guided discovery, with a more or less interventionist guide from the teacher, but present [3]. In our case, we usually apply the expository strategy and, fundamentally, the active methodological strategies (the details of the session under study can be found later, in Section 4.5).

The methods we use in the expository methodological strategy are:

- oral presentation (with its lecture technique), reduced as much as possible, and
- interrogative (with its exploratory question technique), trying to encourage the attention and involvement of the students.

The methods we use in the active methodological strategy are:

- Enquiry Based Learning (EBL) –an application to mathematics teaching can be found in [4], and
- Learning through discovery, a guided discovery always focused on mathematical problems of the appropriate level.

Observe that [5] distinguishes four levels in the EBL:

- Level 1 (Confirmation Inquiry): students test a known result.
- Level 2 (Structured Inquiry): the students work on a problem proposed by the teacher, in the way proposed by him or her.
- Level 3 (Guided Inquiry): the students work, following the procedures that they determine, a problem proposed by the teacher.
- Level 4 (Open Inquiry): the students investigate topics that they formulate, in the way that they determine.

Of these four levels we use almost exclusively the second and third levels.

3. State of the Art

Given its great possibilities, there are many references on the use of CAS in the teaching of mathematics [6,7,8,9].

Regarding smartphones and their possible beneficial use in the classroom, there are works such as [10].

Closer to our objective, focused on the use of CAS on smartphones, there are works such as [11], which analyses their support for the teaching of mathematics including self-assessment, or that of [12], where the use of the CAS *Maxima* on *Android* for teaching number theory and cryptography is discussed.

There are also experiences that compare the execution of exactly the same software on two different hardware, like [13] (where *Turtle Graphics* are used on computers and robots).

But we know of no work on exactly the subject of this particular work.

4. Research Process

4.1 Goal

The present experience tries to evaluate, on a small sample, if future Secondary Education teachers, familiar with at least one CAS and accustomed to using smartphones, are able in a short time to handle another of these systems on a smartphone (explaining their possibilities and the advantages and disadvantages of each CAS and each platform).

4.2 Methodology

This research is developed through a qualitative approach of an ethnographic nature, whose objective is to know the reality *from the inside*, together with those people who are involved and committed to the analysed reality [14]. We can define this type of methodology as *a systematic activity aimed at in-depth understanding of educational and social phenomena, at the transformation of socio-educational practices and scenarios, at decision-making and also at discovery* (translated by the authors from ([15], p. 123)). Under this framework, we develop our research in six phases [16]:

- 1) selection of the design to be used,
- 2) determination of the techniques to be used,
- 3) access to the identified research scenario,
- 4) selection of the main informants,
- 5) data collection and processing of the information obtained.

Our design tries to answer the following question:

Are the pre-service teachers of the MFPES able to handle well the new application (the CAS for smartphones *Maple Calculator* in this experience) in a short period of time?

We have chosen two techniques for the evaluation of the experience [16]:

- Non-participant observation, through the analysis of the students' behaviour by the teacher, without intervening.
- The interview, carried out through the formulation of a set of open questions, which have been presented in writing, allowing us to obtain the internal perspective of the participating student body.

The classroom of the MFPES is the selected research scenario, as it is the space where teaching is given to the participating students, and the teacher of the master's degree is the chosen informant.

We have used the descriptive-narrative writings of the teacher as instruments for data collection, which have been complemented with a battery of questions that have been presented to the participating students. Finally, the information is processed through the analysis of the study data, carried out through the collection instruments that we have previously indicated. The data collection process and its analysis are inextricably linked, as they are two interdependent aspects.

4.3 Participants

The group has been made up of 20 students (all with a degree in mathematics and many with some other qualification for having completed a double degree or another degree or a master's degree or a doctorate, as said above). They were all Spanish and had studied at universities in our country.

The number of applicants exceeds the number of places for this master's degree and the students who choose it do so on a vocational basis (in several cases because they descend from or are relatives of teachers).

4.4 Resources for the Intervention

Two pieces of related software (from the same company) have been chosen in this experience. Nevertheless, they have very different characteristics, not only the platforms (computer and smartphone).

CAS have two characteristics that distinguish them from other pieces of mathematical software:

- CAS work in exact arithmetic (not in floating point arithmetic, unless otherwise specified), that is, they don't approximate numerical values.
- CAS can handle unassigned variables. This represents a higher level of abstraction than that of usual computational languages and traditional computational mathematical software. It allows CAS to bring implemented, in addition to the manipulation of algebraic expressions, other issues such as:
 - equations and linear and algebraic systems solving (even including parameters),
 - symbolic differentiation and integration (computing primitives),
 - evaluation of limits,
 - operations with symbolic matrices,
 - etc. (Figure 4.4.1).

The first CASs were developed in the late 1960s: *Macsyma* (at MIT) and *REDUCE* (by the University of Utah and RAND Corporation) and were written in *LISP* (because they used lists to implement exact arithmetic), although almost modern ones are written in *C*.

Maple [17,18] is a very powerful CAS. Its first version was released in 1981 and it was developed at the University of Waterloo. There are versions available for the three most popular computer operating systems.

Maple along with *Mathematica* are currently the most powerful and widespread CAS, with hundreds of thousands of users. Their use stands out at the university level, for teaching and research [19]. For example, many Spanish universities have a campus license for one and/or the other, although there are also experiences and proposals at Secondary School level [20,21].

Maple along with *Mathematica* have the drawback compared to other CAS, like *Maxima*, *SageMath*, *REDUCE*, *AXIOM*, etc. to be paid software. In addition, they require relatively powerful computers, unlike the discontinued and remembered *DERIVE*.

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> diff(x*cos(a*x^3),x);	(5)
> int(cos(x)^2,x);	(3)
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Meanwhile, *Maple Calculator* is a free mobile application with text recognition and its own small CAS, which can be used autonomously (Figure 4.4.2). It can also communicate with the *Maple* parent system, both to continue performing other calculations not available for this smartphone application, or to act as a complement to *Maple* to enter scanned input. *Maple Calculator* is available for the two most common smartphone operating systems.

Maple Calculator was previously called *Maple Companion* and was released in 2019. Its capabilities as a stand-alone application are like those of *Photomath* [22]. *Maple Calculator* is a very widespread application, mentioning on its website that more than three million downloads have already been made [23].

The experience was carried out in a classroom with two large screens and two white boards. All the students had a laptop (or tablet), on which they installed *Maple* (our university has a *Maple Campus License*), and a smartphone, on which they installed *Maple Calculator*.

4.5 Timing and development

It should be noted that, before the key experience of this study, there was a 3 hours *Maple* session. It began with the basic handling of this CAS and was oriented to its applications in mathematical problems of the level of which these students will be teachers, insisting, when possible, on the visualization of the different processes (like: solving quadratic equations, solving linear systems, symbolic differentiation and integration, etc.). Specific issues of interest were even detailed, such as the commands on calculation and graphical representation of lower and upper Riemann sums from the *Student [Calculus1]* package.



Figure 4.4.2 *Maple Calculator* screenshot showing an example the exact solving of a quadratic equation (introduced using handwriting recognition and the camera of the smartphone), its graphical representation and the derivative of the corresponding polynomial. As shown, the application is multilingual. The image captured is shown in Figure 4.4.3.



Figure 4.4.3 The image captured by *Maple Calculator* camera as input to produce the output in Figure 4.4.2. The handwriting recognition works pretty well.

In this first session, a hybrid model was followed, interspersing expository and, more frequently, active methodological strategies (EBL method, Level 2).

That session was followed by a 1-hour free personal work with *Maple Calculator* on their smartphones, at the beginning of which they were given the survey. Although they could comment on their discoveries and consult with their classmates, they had to write the answers individually. This session was very well received.

In this second session, an active methodological strategy was applied (EBL method, Level 3 since the students worked freely). On some few occasions, the teacher interacted with a student or

group of students (at their request), solving a specific question, or giving a clue to solve it. However, most of the questions referred to what the questions posed in the survey asked for.

4.6 Assessment

To complement the descriptive-narrative notes taken by the teacher and derived from his nonparticipant observation, a final structured survey was given to the participants in the experience, as it was the core of its evaluation. Specifically, pre-service teachers were asked to answer the following questions in writing:

- 1) Describe the most important possibilities of *Maple Calculator* or what has caught your attention that you have discovered that *Maple Calculator* does (use the space you want).
- 2) Describe the advantages that you consider working with *Maple Calculator* on a smartphone compared to working with *Maple* on a computer (use the space you want).
- 3) Describe the disadvantages that you consider working with *Maple Calculator* on a smartphone compared to working with *Maple* on a computer (use the space you want).

The first question has the purpose of checking the level reached in handling the new software, while the second and third require a solid knowledge of the CAS *Maple* (used in the previous session) to be able to compare them properly (several students opened *Maple* on their laptops to see if it did exactly what they remembered).

Apart from the convenient handwriting recognition, the most outstanding possibilities of *Maple Calculator* we had considered in advance were:

- linear equations and systems solving,
- algebraic equations and systems exact solving (some) (Figure 4.6.1),
- graphic representation of 2D and 3D functions (Figure 4.6.2),
- symbolic differentiation and integration (Figure 4.6.1),
- evaluation of limits (Figure 4.6.3),
- operations with matrices, determinant, inverse matrix (Figure 4.6.4),
- eigenvalues and eigenvectors, and
- ODE solving (limited),
- step by step explanation of some processes (Figure 4.6.5).

As advantages of the smartphone application, we had considered:

- portability,
- gratuity,
- the visual interface, very intuitive and friendly,
- the possibility of recognizing mathematical text with the camera of the smartphone, and
- to directly offer various possibilities for the input.

And as drawbacks of the smartphone application, we had considered:

- the small screen,
- that it doesn't include a programming language, and
- that it has far fewer mathematical possibilities than *Maple*.

Note that we did not try to evaluate how many of the characteristics that we had detected were mentioned by the students, in the style of an exam correction, but to verify that they reported a good proportion of them, by way of a discretization of the type of those of fuzzy logic.



Figure 4.6.1 *Maple Calculator* menu offered when the input is the equation of the spherical surface $x^2+y^2+z^2=1$.



Figure 4.6.2 *Maple Calculator* plot of $x^2+y^2+z^2=1$.



Figure 4.6.3 A limit computed by Maple Calculator using L'Hôpital's rule.

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Figure 4.6.4 *Maple Calculator* computation of the inverse of a parameter depending 3×3 matrix.

< Compute the Limit
$\lim_{x\to 0} \frac{x}{\sin(x)}$
▶ 1. Apply L'Hôpital's Rule lim _{x→0} 1/cos(x)
> 2. Apply the power rule $\frac{1}{\lim_{x\to 0} \cos(x)}$
 3. Evaluate the limit of cos(x) 1

Figure 4.6.5 Maple Calculator step by step explanation of the limit computation in Figure 4.6.3.

5. Some comments about mathematical symbolic applications for smartphones and symbolic calculators

An overview of the evolution of the portable devices running computer algebra systems, that is symbolic calculators (Figures 5.0.1 and 5.0.2) and smartphones, can be found in [24] The underlying software is similar in both cases, so their capabilities are similar.



Figure 5.0.1 An early calculator of the *TI-92* range, showing its qwerty keyboard (the 1995 *TI-92* was the first symbolic calculator). The screen of the calculator shows in this case the following computations: complex roots of x^3 -1 (in exact arithmetic); factorization of x^3 -1 in R[x]; and limit of sin(x)/x in 0 (calculated using L'Hôpital's rule).



Figure 5.0.2 An early *TI-89* symbolic calculator, showing a classic "calculator look". On its screen the plot of $x^{(2/3)}$ is shown.

However, the hardware of calculators and smartphones is not comparable. Let us consider, for instance, the excellent range of *Texas Instruments* calculators with CAS capabilities shown in [25]: *Voyage 200, TI-89 Titanium, TI-Nspire CAS* and *TI-Inspire CX CAS*. Only the latter has a backlit colour screen. The biggest user available memory is 100 MB in this last case and is much smaller for the other calculators. *HP* and *Casio* also have symbolic calculators comparable to *TI*'s top of the line: the *HP Prime Graphic Calculator* [26] and the *Casio ClassPad II fx-CP400* [27].

A mid-range smartphone, meanwhile, has 3 or 4 GB of memory, close to that of a laptop, and a high-resolution backlit colour touchscreen. The computation speed is higher. The features of smartphones screens are similar to those of high-end calculators.

As mentioned above, some software pieces developed for smartphones (such as *Maple Calculator* or *PhotoMath*) offer data input using the smartphone's camera. However, the keyboard is the only way to introduce data in other CAS for smartphones such as *MaximaOnAndroid* [28] or *Symbolic* [29] (an *Android* version of the CAS *REDUCE*).

On the other hand, the smartphone must split the screen (output / keyboard) whenever the keyboard must be used, while the calculators have a comfortable separated keyboard. Moreover, the calculator is more resistant, especially to shocks. Another drawback for the use of smartphones in the classroom is that they are frequently forbidden in schools and high schools.

The software of some calculators like the *TI-Nspire CX CAS* is completely compatible with available computer software, so intensive computations can be effortlessly transferred to a standard computer, as happens with the application for smartphones *Maple Calculator*.

Summarizing, in our opinion, the choice depends on the age of the student and the availability and usability of the hardware. For instance, if all students carry a laptop to class, or if all mathematics classes take place in a computer lab, it makes no sense to use portable devices such as calculators or smartphones for performing symbolic computations. But that is not always the case. So, the symbolic calculator is possibly the best option for high school students. The situation could be completely different for university students: at least in our country almost a 100% of the students own smartphones and laptops, and take the smartphones with them everywhere (even to the classroom) and many take their laptops to the classroom too.

6. Results

Table 1 summarizes the responses given by the 20 students to the questions posed in the survey, categorized into three types of response:

- Good: detailed and correct answer.
- Fair: correct answer in general, although not complete or with something incorrect.
- Poor: very little detailed response or with serious errors or serious inaccuracies.

Maple Calculator	Good	Fair	Poor
Possibilities	14	5	1
Advantages	16	3	1
Drawbacks	12	4	4

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An example of a Good answer to the first question (Possibilities), literally translated, is:

"As expected, with *Maple Calculator* you can do any basic operation of a calculator like add, subtract, multiply, etc. The differences come when you see that you can begin to represent functions, from the most basic to trigonometric functions and even functions defined piecewise. You also have the option of writing matrices and operating on them, as well as calculating their inverse, determinants, ranks and other operations on matrices (the transpose, for example). Most surprising for me was the ability, through the mobile camera, to be able to read mathematical notation handwritten on a piece of paper. From there, I was testing and discovering many more *Maple Calculator* features with every math thing I could think of trying to read. It is capable of perfectly solving systems of equations, factoring polynomials, computing derivatives and integrals, it also knows how to calculate definite and indefinite summations and products and knows how to work with trigonometric identities. Finally, something that I did not expect and that I discovered by chance is that it is capable of making 3D representations, thus being able to see the representation of a plane in the space."

The previous answer was written by one of the best students.

An example of a Poor answer to the second question (Advantages), literally translated, is:

"It seems to me a great advantage that it is on the mobile, because as everyone has one, it becomes a resource with better accessibility than a computer. For students, who are generally addicted to mobile phones, it can be a great tool to foster their interest in mathematics."

This last answer is so general that we wonder if he or she had really worked with *Maple Calculator* or was doing something else instead.

An example of a Fair answer to the third question (Drawbacks), literally translated, is:

"Efficient use: If you know what you're doing and master the program, working on a computer with a large screen, a keyboard and a mouse will always be more efficient than working on the touch screen of a smartphone. Additional functionality: *Maple Calculator* can do a lot of things, but obviously *Maple* on a computer offers a better functionality."

The assertions of the previous answer are correct and are logical consequences of testing the two pieces of software, each one on its corresponding platforms. However, they lack technical details: mentioning that one "can do a lot of things" but the other software "offers a better functionality" is not enough.

Obviously, the classification into the crisp Good/Fair/Poor classes is subjective, but we've tried to be as accurate as possible.

Summarizing the data that appears in Table 1:

- the proportion of responses considered Good is: (14+16+12)/60 = 70%,
- that of those considered Fair is: (5+3+4)/60 = 20 %,
- and that of those considered Poor is: (1+1+4)/60 = 10%

(curiously, as there are $20 \times 3=60$ answers and this number has many divisors, round percentages have been obtained). In addition, 9 of the 20 students responded correctly to all three questions (45%).

In summary, after only one hour of autonomous work, most of the questions, almost three quarters, are answered correctly, and almost half of the students responded correctly to all the questions simultaneously. In our opinion, it is a high enough percentage to assert the ease with which these students have reached a good level in handling the new CAS and show a sufficient understanding of its fundamental possibilities (although it cannot be guaranteed that they have fully mastered the new app).

6. Conclusions and Future Work

As a result of the non-participant observation carried out and the answers provided to the questionnaire, we found that, curiously, the course activities that aroused the greatest enthusiasm among these MFPES students were those carried out with smartphones and another one carried out with programmable robots (in both of them the EBL method with Level 3 was followed).

The high level demonstrated by these future teachers in handling the new smartphone application after only one hour of working with it has been very striking to us.

The enthusiasm shown by the participants in the activities mentioned could be due to the novelty of working more autonomously or with devices not usually used in the classroom. This enthusiasm could have influenced the positive result of the experience. However, we believe that it has been proven that, at least at this postgraduate level, the introduction of new mathematical software does not have to be burdensome for the normal development of the course.

The way is opened to design and carry out future experiences both at other levels and with other mathematical software, to determine if really starting to handle another mathematical software can be satisfactory in a short space of time.

Acknowledgements This work was partially supported by the research projects PGC2018-096509-B-I00 and PID2021-122905NB-C21.

The authors would also like to thank the anonymous reviewers for their comments and suggestions.

References

- [1] Perrenoud, P. (1999). *Construir competencias desde la escuela*. Caracas Montevideo Santiago de Chile: Dolmen Ediciones.
- [2] Bernal, A., Fernández-Salinero, C. & Pineda, P. (2019). *Formación continua*. Madrid: Síntesis.
- [3] Mayer, R. E. (2004). Should There Be a Three-Strikes Rule Against Pure Discovery Learning? The Case for Guided Methods of Instruction. American Psychologist, 59(1), 14–19. https://doi.org/10.1037/0003-066X.59.1.14
- [4] Artigue, M. & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. ZDM Mathematics Education, 45, 797–810. https://doi.org/10.1007/s11858-013-0506-6
- [5] Bianchi, H & Bell, R. (2008). *The many levels of Inquiry*. Science and Children, 46(2), 26-29.
- [6] Berry, J., Monaghan, J., Kronfellner, M. & Kutzler, B. (1997). *The State of Computer Algebra in Mathematics Education*. Bromley, UK: Chartwell-Bratt.
- [7] Díaz, A., García, A. & de la Villa, A. (2011). An example of learning based on competences: Use of Maxima in Linear Algebra for Engineers. International Journal of Technology in Mathematics Education, 18, 177-181.
- [8] Fedriani, E. M. & Moyano, R. (2011). Using Maxima in the Mathematics Classroom. International Journal of Technology in Mathematics Education, 18, 171-176.
- [9] Gayoso Martínez, V., Hernández Encinas, L., Martín Muñoz, A. & Queiruga Dios, A. (2021). Using Free Mathematical Software in Engineering Classes. Axioms, 10, 253. https://doi.org/10.3390/axioms10040253
- [10] Brazuelo F. & Gallego D. J. (2011). *Mobile learning: Los dispositivos móviles como recurso educativo*. Sevilla: MAD Eduforma.
- [11] Barzel B., Ball L. & Klinger M. (2019). Students' Self-Awareness of Their Mathematical Thinking: Can Self-Assessment Be Supported Through CAS-Integrated Learning Apps on Smartphones? In G. Aldon y J. Trgalová (Eds.), Technology in Mathematics Teaching. Mathematics Education in the Digital Era. Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-030-19741-4_4
- [12] Ipiña R, Vallejo J. A. (2018). Number theory and cryptography in your smartphone. Electronic Journal of Mathematics & Technology, 12(1), 1-17.
- [13] Roanes, E. & Fernández-Salinero, C (2021). La actitud de futuros profesores de Secundaria ante el uso de robots programables en la clase de matemáticas. In P. D. Diago, D. F. Yáñez, M. T. González-Astudillo y D. Carrillo (Eds.), Investigación en Educación M
- [14] Eisenhart, A. (1988). The ethnographic research tradition and mathematics education research. Journal for Research in Mathematics Education, 19, 99-114
- [15] Sandín, M. P. (2003). Investigación cualitativa en Educación. Fundamentos y tradiciones. Madrid: McGrawHill.
- [16] Bisquerra, R. (Ed.). (2009). *Metodología de la investigación educativa*. Madrid: La Muralla.
- [17] The Essential Tool for Mathematics (n.a.). https://www.maplesoft.com/products/Maple/
- [18] Maplesoft (2021). Maple User Manual. Waterloo, Canada: Waterloo Maple Inc.
- [19] Camacho Machín, M. (2011). Investigación en didáctica de las matemáticas en el Bachillerato y primeros cursos de universidad. In M. Marín Rodríguez, G. Fernández García, L. J. Blanco Nieto y M. Palarea Medina (Eds.), Investigación en Educación Matemática XV (pp. 195-224). Ciudad Real: SEIEM.

- [20] Delgado Pineda, M. (1998). *Maple en la Enseñanza Secundaria*. Gaceta de la RSME 1(1), 114-120.
- [21] Méndez Contreras, J. A. (2001). Utilización de Maple como apoyo a la Matemática en el Bachillerato. Badajoz, Spain: FESPM.
- [22] Photomath (n.a.). https://photomath.es/
- [23] Maple Calculator (n.a.). https://www.maplesoft.com/products/Maplecalculator/
- [24] Roanes-Lozano, E. (2017). An overview of the evolution of the devices running computer algebra systems and their educational implications. Electrical & Electronic Technology Open Access Journal, 1(1), 7-11.
- [25] ¿Cuál es la calculadora ideal para mí? (n.a.). https://education.ti.com/es/product-resources/graphing-course-comparison
- [26] *HP Prime Graphic Calculator* (n.a.). https://hpofficesupply.com/product/hp-prime-graphing-calculator/#
- [27] *ClassPad II fx-CP400* (n.a.). https://www.casiointl.com/asia/en/calc/products/ClassPadIIfx-CP400/
- [28] *Maxima* on *Android* (n.a.). https://play.google.com/store/apps/details?id=jp.yhonda&hl=es&gl=US
- [29] Egger, D. *Symbolic* (n.a.). https://play.google.com/store/apps/details?id=de.dieteregger.symbolic&hl=en_US&gl=US