Ordering Question with Clue in Moodle

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Abstract

The ordering question is one of the third-party question types for the quiz activity in Moodle. It displays several draggable items in a random order, that may include a couple of mathematical expressions rendered by MathJax or KaTeX. The students are required to rearrange/drag them in the correct order specified in the question text. This question type is very useful to evaluate or drill the both of procedural and conceptual knowledge in mathematics if the questions in use are well considered and structured. In this paper, we propose a new feature for the ordering question type, that allows some items to be fixed as clues (static/non-draggable items). This feature helps the students to understand how to solve the problem and force the students to think deeply the hidden relationship among the given items. We also show our actual ordering questions with clues in use, that are pre-class learning activities as one of self-assessment tasks in linear algebra lectures.

1 Introduction

Online education such that distance learning, e-learning, blended learning and so on, have been becoming popular even in mathematics education ([3, 4, 7] and references therein), and their demands have been accelerated due to the COVID-19 pandemic (see [15] for example). For those learning environments, computer-based assessment tools and drills are very important since the delivery of lectures at any time or place requires learning activities that are basically scored automatically (e.g. multiple choice questions, fill in the blank questions, and so on).

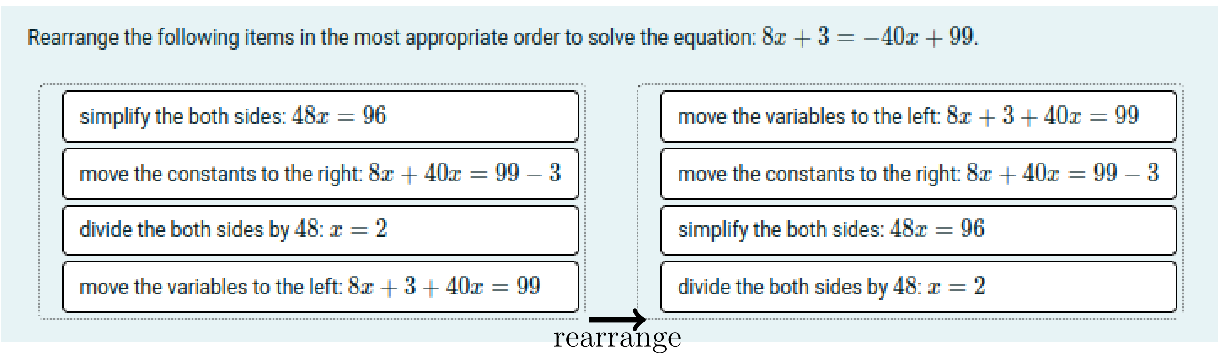

Figure 1: Typical ordering question (left: an initial order, right: the correct order)

In this paper, we are interested in the ordering question type (a Moodle plugin developed by G. Bateson[2]) that is one of the third-party question types for the quiz activity in Moodle. It displays several draggable items in a random order, that may include a couple of mathematical expressions rendered by MathJax or KaTeX (see [12] if you want to use the KaTeX filter instead of the MathJax filter in Moodle). The students are required to rearrange/drag them in the correct order specified in the question text. The figure 1 illustrates a sample ordering question where the given items in some initial positions on the left, and the items in the correct positions on the right (in the actual question, the left part is only shown).

Although the ordering question in mathematics education is not so special (for example, we found some ordering questions in mathematics at Guathmath, Chegg, Vedantu and Course Hero websites[1], and see also [18, 6] for ordering questions outside of mathematics education), and is listed in the computer-based assessment possibilities by Hoogland and Tout[5], unfortunately there are no recent studies in the literature. The one exception we found in the early stage of computer assisted assessment is the study by McCabe et al. [11] that reported several question types (including the ordering question) to assess mainly mathematical proofs (but not limited to) and a higher learning level (in the Bloom cognitive learning levels, see also [1, 9]). They treated the ordering question as the question type: "place a set of statements or groups of statements in the correct order, where typically between 6 and 8 items are used..." with an example to prove that $(AB)^{-1} = B^{-1}A^{-1}$ for matrices A and B. They also argued that "question answering might be made more direct if graphical drag-and-drop components were used to implement ordering...". The ordering question type in Moodle and the following softwares below have this graphical feature.

Additionally, there are some related works in learning mathematical proofs ([10, 17, 16] and references therein). Those softwares basically break and scramble a correct proof into small pieces that are single mathematical expressions, single reasoning sentences and so on. The students are required to rearrange/drag/unscramble them to be one of valid proofs. For example, the application by Kurayama[10] represents the draggable items as cards that will be dropped in the given blanks, hence it can be considered as the drag&drop question type and may be replaced by the ordering question in Moodle. The example of Proof Blocks in [17, 16] seems an ordering question but it uses the directed acyclic graph-based grading that accepts more complex proofs (e.g. multiple valid proofs formed by the same pieces) hence it can not be replaced by any simple ordering question if we really want to allow some non-unique ordering correct answers. In such cases, we think that Proof Blocks is a good choice to assess the reasoning ability, however, in this paper we are interested in the simple ordering question on the wide range of mathematical topics.

In this paper, we briefly show some fundamental expectations and properties of the ordering question type in the following subsections 1.1 and 1.2. In the section 2, we show our actual sets of ordering questions in linear algebra courses, that are early attempts without any clue (static/non-draggable) item in the 1st semester of the 2021 academic year, and recent attempts with clues in the 1st semester of the 2022 academic year. The main contribution is shown in the section 3 where we propose a new feature for the ordering question type in Moodle, that allows some items to be fixed as clues (static/non-draggable items) with examples in linear algebra.

[1] We retrieved them in May 2022.

1.1 Mathematical thinking skills and ordering questions

The aim of our research is basically to develop a computer-based assessment method in mathematics for the students who can use several calculators including computer algebra systems. We note that we want to assess mathematics achievement not computer skills. For example, the figure 2 illustrates this concept. In this question, the students are expected to know how to factorize a quadratic polynomial with integer coefficients and forced to think deeply the hidden relationship among the given items at least at first sight (note: our students reported that with repeated trials, patterns can be discovered and potentially solved without thinking deeply. see also the section 2). This means that the students are forced to guess what approach the teacher expects, even if the students prefer to use another way to factorize the polynomial. In fact, we note that there are several ways to factor a quadratic polynomial with integer coefficients (for example, factoring, linear, average and difference of squares methods are discussed in [8]). For example, if the students can know the resulting factorization $8(x-7)(x+2)$ by some computer algebra system, it means only that the item $8(x-7)(x+2)$ should be the last item. Therefore, the students are required to understand/guess the following intention of each step.

1 → 2
: factor out the content part (the greatest common divisor of the coefficients),

2 → 3
: factor the constant term that is considered as the product of constants of factors,

3 → 4
: check the possibilities of second leading coefficients that is the sum of constants of factors,

4 → 5
: form the factorization of the primitive part, and

5 → 6
: combine the factorization with the content part.

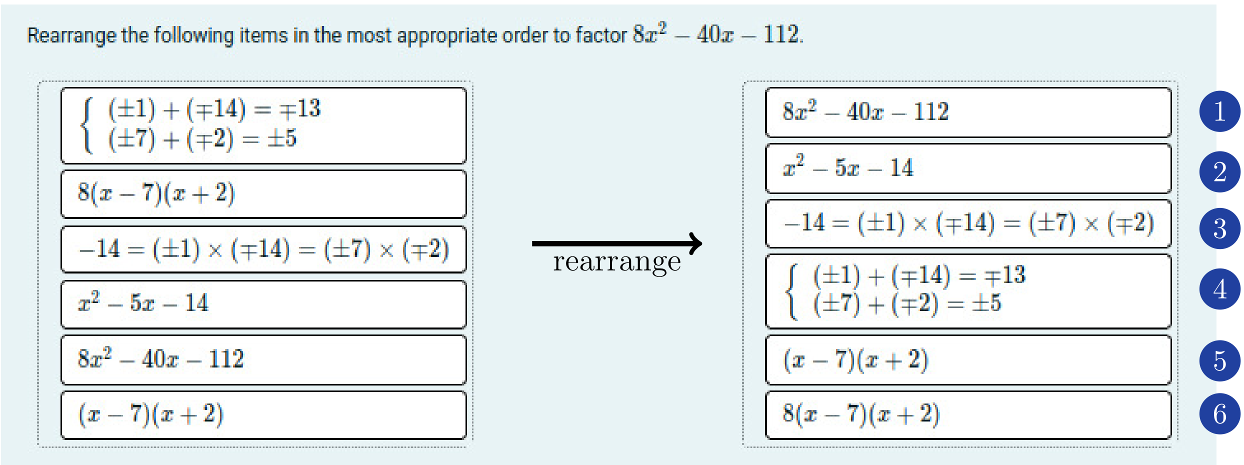

Figure 2: Our concept of ordering question (left: an initial order, right: the correct order)

Simple	Use a simple and straightforward question text and answer w/o misleading.
Unique	Ensure the uniqueness of the correct answer (the correct order of items).
Homogeneous	Homogenize the items in logical/mathematical size and presentation.
Independent	Do not use/repeat any fragment of text/expression in multiple items.
Honest	Avoid any confusing item and refrain from using multiple wildcard items.

Table 1: Writing good ordering questions in mathematics in [14]

The first author expects that this experience will enrich the students' mathematical thinking skills including the procedural and conceptual knowledge. For this purpose, we have developed an automatic generation framework of ordering questions[14] that based on the automatic generation framework of multiple choice questions[13].

1.2 Guideline for ordering questions in mathematics

We have found no guideline for writing good ordering questions (especially in mathematics) while there are many general guidelines for writing/preparing good multiple choice questions ([19] for example). Therefore, in this subsection, we will briefly review the guideline by the first author[14].

The table 1 shows the guideline that consists of the five key rules: **simple**, **unique**, **homogeneous**, **independent** and **honest** where "w/o" stands for "without". The **simple rule** is obvious since we want to assess some appropriate achievement in mathematics and not any reading comprehension of complex texts. We should focus on what we actually want to ask the students. The **unique rule** is also obvious and important. However, we note that the unique and independent rules are conflict in a sense since it is the easiest way to avoid any other possible order (i.e. following the unique rule) that we duplicate some fragment of expressions in each adjacent items (violating the independent rule), and vice versa. Additionally, Poulsen et al.[16] also argue that an erroneous question is because the instructor failed to recognize a possible rearrangement of the proof and it is easy to make such mistakes. Therefore, we must be careful with this rule.

The **homogeneous rule** corresponds to the granularity of the items. If the granularity is not uniform, the students have to translate and disassemble/assemble the items when they understand/guess the intention of each step. This (not uniform granularity) may not be a good situation since again we want to assess some appropriate achievement (procedural and conceptual knowledge) in mathematics and not any reading comprehension of complex items. The **independent rule** is to prevent the question from becoming an easy pattern matching problem without thinking. The **honest rule** is because we want to assess some appropriate achievement in mathematics and not any reading comprehension of tricky items. Especially, an item with wildcards (which hide parts of text/expression) should be used only when it is unavoidable. For example, the item under consideration cannot be changed due to the unique rule, but the content of the item appears in another item and breaks the independent rule.

2 Ordering Questions in Linear Algebra

In this section, we show several examples of ordering questions used in linear algebra courses.

2.1 Early attempts without clues

In the 1st semester of the 2021 academic year, the first author had two courses with our format below: linear algebra 1 (1 credit, 44 students) and linear algebra 2 (1 credit, 41 students).

2021 academic year's class format:

Our class format is a kind of the flipped learning (full online due to COVID-19) where the activity outside the classroom consists of online materials on Moodle (on-demand PDF slides and videos) with our ordering and multiple-choice questions as "weekly self-assessment quiz" on Moodle, and the in-class activity consists of paper-based assignments (submitting their photos to Moodle and responding by image files with marks and comments by hands on tablets) and online video meetings. We note that the grades are based on the weekly ordering questions (10%, the highest scores of multiple attempts during the quarter), the weekly multiple-choice questions (10%, the highest scores of multiple attempts during the quarter) and the weekly in-class paper-based assignments (80%).

For these courses, we made 40 ordering questions and generated 200 instances (same question but with different numeric values) for each question (note that all the examples in this paper have been translated from Japanese to English). The figure 3 is a typical ordering question we used for matrix-vector and matrix-matrix computations (6 questions in total). The figure 4 is a typical ordering question we used for several fundamental computations w.r.t. the elementary row/column operations (including solving a linear equation). We note that the actual quiz in Moodle will not show the numbered circles in the figure 4. Moreover, we note that these questions were generated before the guideline in the table 1 hence the latter question is not following the independent rule.

For the ordering questions used in these courses, we have had the following notable comments from some students and colleagues, that have led our improvement on the ordering question (as shown in the next subsection 2.2 and section 3).

- They may not be appropriate as drills because once we find some patterns, regardless of any duplication between items, we can quickly solve them without mathematics.
- It may not be a good way to force an approach/method to solve a given question.

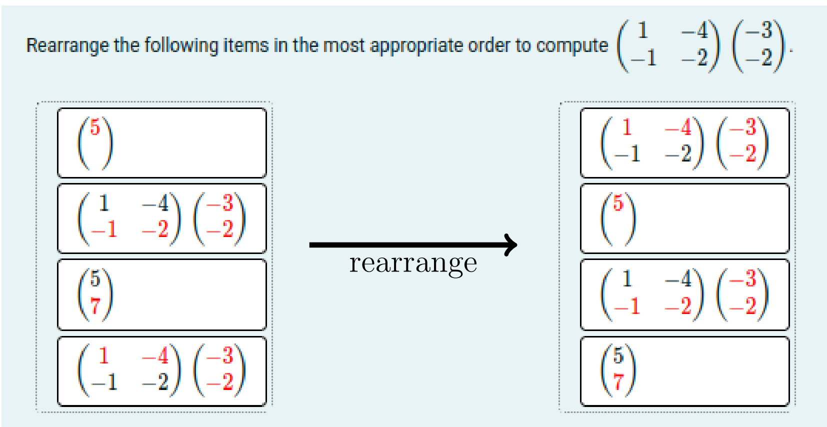

Figure 3: Matrix-vector computation (left: an initial order, right: the correct order)

2.2 Recent attempts with clues

In the 1st semester of the 2022 academic year, the first author had two courses with our format below: linear algebra 1 (1 credit, 45 students) and linear algebra 2 (1 credit, 46 students).

2022 academic year's class format:

Our class format is a kind of the flipped learning where the activity outside the classroom consists of online materials on Moodle (on-demand PDF slides and videos) with our ordering questions as "weekly **pre-class** quiz" on Moodle and multiple-choice questions as "weekly self-assessment quiz" on Moodle, and the in-class (in-person) activity consists of paper-based exercises. We note that the grades are based on the weekly ordering questions (10%, the highest scores of multiple attempts during the quarter), the weekly multiple-choice questions (10%, the highest scores of multiple attempts during the quarter) and the final paper-based examination (80%).

For these courses, we modified the ordering questions used in 2021 and re-generated 200 instances for each question, to make the questions more appropriate as weekly pre-class learning materials. We grouped the items into two categories: 1) the gray background items that are already in the correct order and non-draggable, and 2) the white background items that are the actual rearrangement targets. The gray background items are intended to be a clue. The figures 5 and 6 show some actual questions. Moreover, we note that some questions we used do not follow the independent rule of the guideline in the table 1. This is because they are pre-class materials and some duplications may help the students to understand the process.

Moreover, after the semester, 81.5% of respondents (48.4% of students) answered that the ordering questions with clues had been helpful or moderately helpful to understand the content of lectures, while 18.5% of respondents (11.0% of students) answered that they had been moderately non-helpful or neutral. There is no respondent who thinks that they are non-helpful.

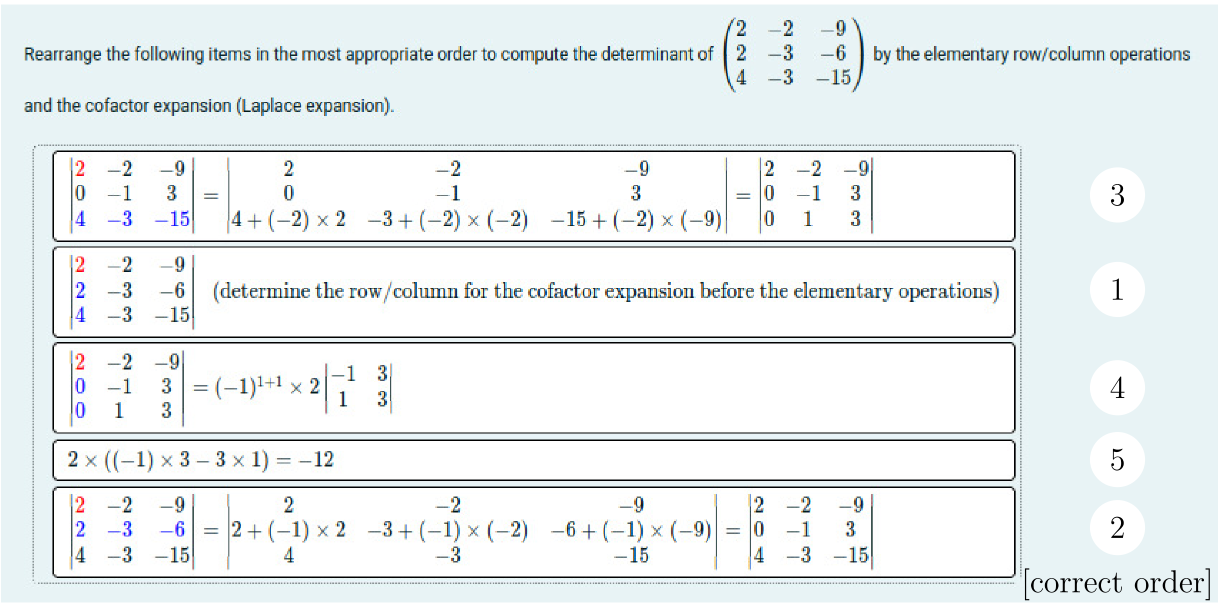

Figure 4: Determinant by cofactor expansion with elementary operations (an initial order)

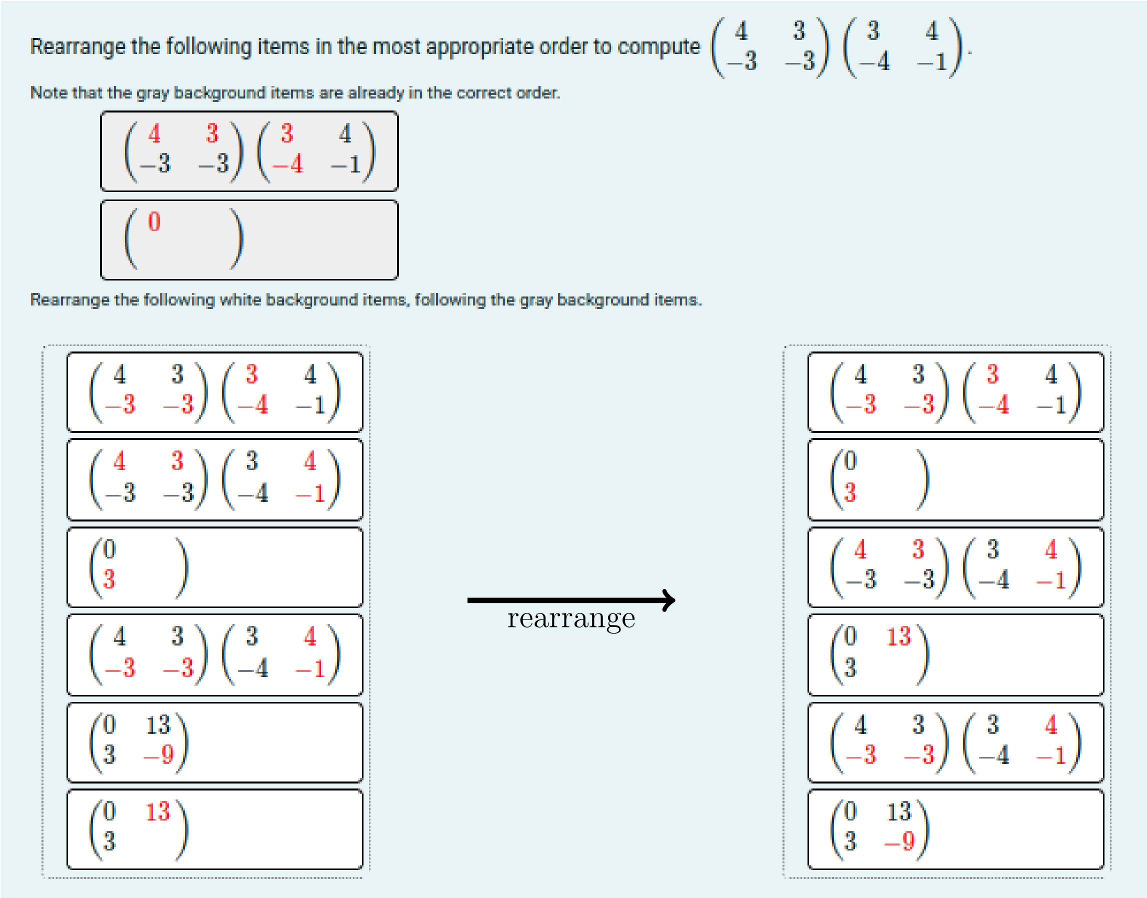

Figure 5: Matrix-matrix computation (left: an initial order, right: the correct order)

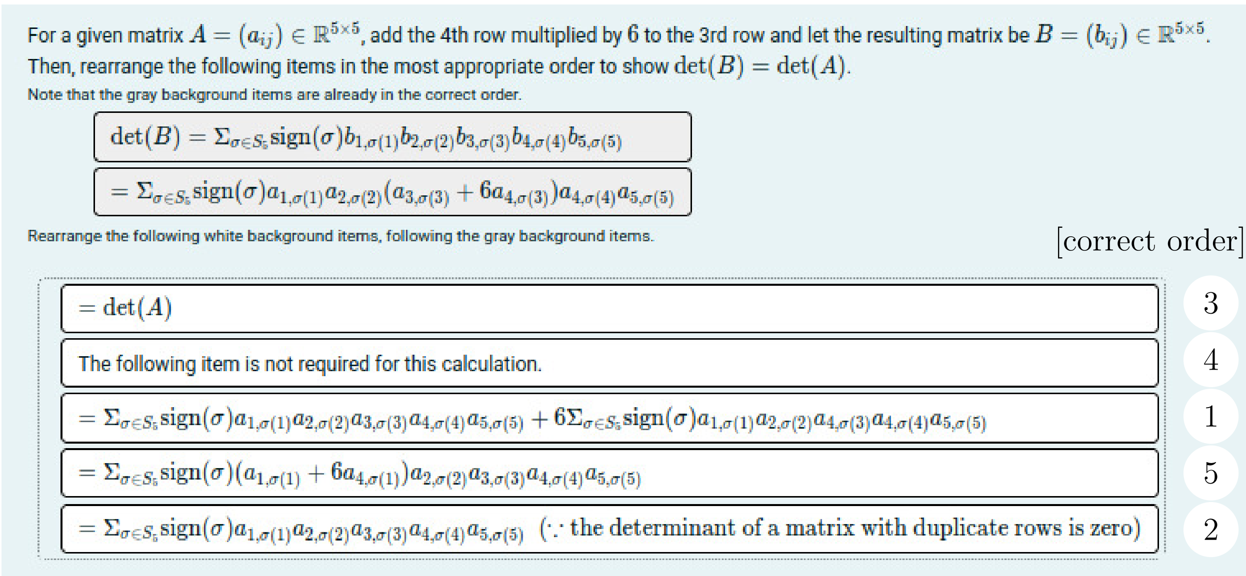

Figure 6: Determinant invariant by row-addition operation (an initial order)

3 Ordering Questions with Static Items

According to our experience of early and recent attempts in the previous section, the ordering question in mathematics is useful and may become more appropriate pre-class learning materials. However, the gray background non-draggable items in the figures 5 and 6 are not really non-draggable items, unfortunately. They are just included in the question text. Therefore, the following configurations cannot be feasible.

- To decrease/increase the number of non-draggable items since they are predetermined.
- To place the non-draggable items at the end since the question text is at the beginning.
- To use the "random subset of items" or "contiguous subset of items" selection types since the non-draggable items are predetermined and cannot be changed on demand.
- To show the non-draggable items side by side with the white background items.

To make these configurations feasible, we propose and will commit [2] a new feature for the ordering question type, that allows some items to be fixed as clues (static/non-draggable items). With our patch, the static items are configurable and shown in a natural way. For example, the figure 7 shows a variant of the question in the figure 2.

3.1 New options for static items

In our proposed version, there are new configuration options as shown in the figure 9. The "beginning static items" option specifies the number of items that will be displayed at the beginning as static/non-draggable/gray background items. The "end static items" option specifies

[2]We have not yet committed our patch at the submission date.

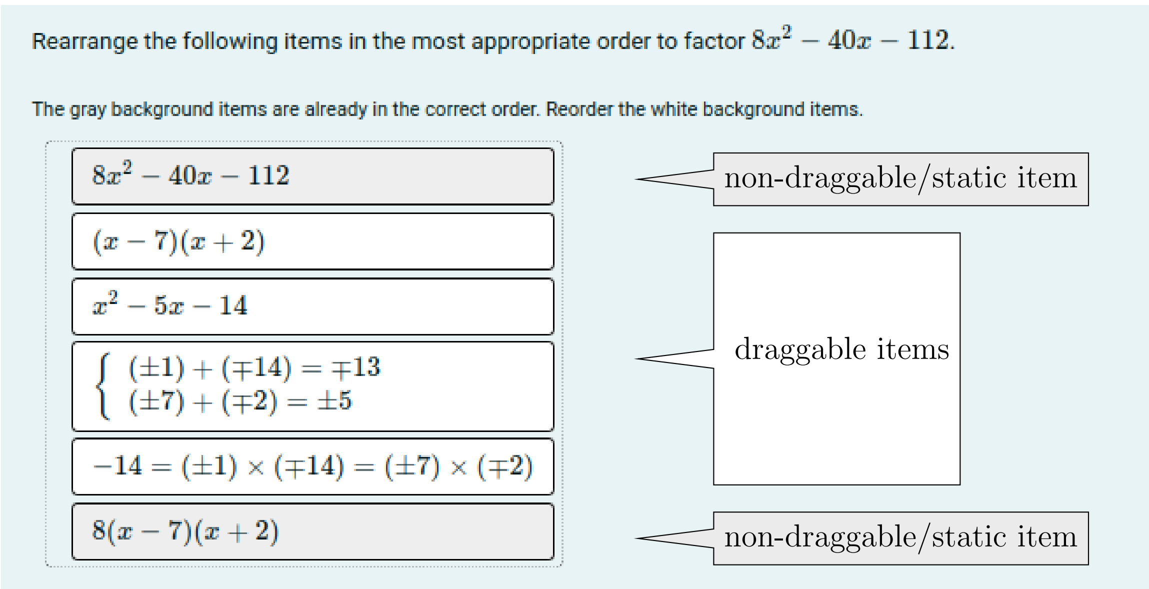

Figure 7: Ordering question with static items (an initial order)

the number of items that will be displayed at the end as static/non-draggable/gray background items. For example, they are both configured to "1" in the question in the figure 7. These options are compatible with all the item selection type (all items, a random subset of items, and a contiguous subset of items) with any size of subset as long as the resulting number of draggable items is not less than two items.

3.2 More examples in linear algebra

With our proposed version, we have been developing new ordering questions in linear algebra. The question in the figure 8 is for a mathematical reasoning (especially a reading comprehension) where the students are required to guess a proof that the matrix A is not the inverse of matrix B under the given assumption. The question in the figure 10 is a sample question of a random subset of items with a wildcard $\star$ (that hides an element of matrix).

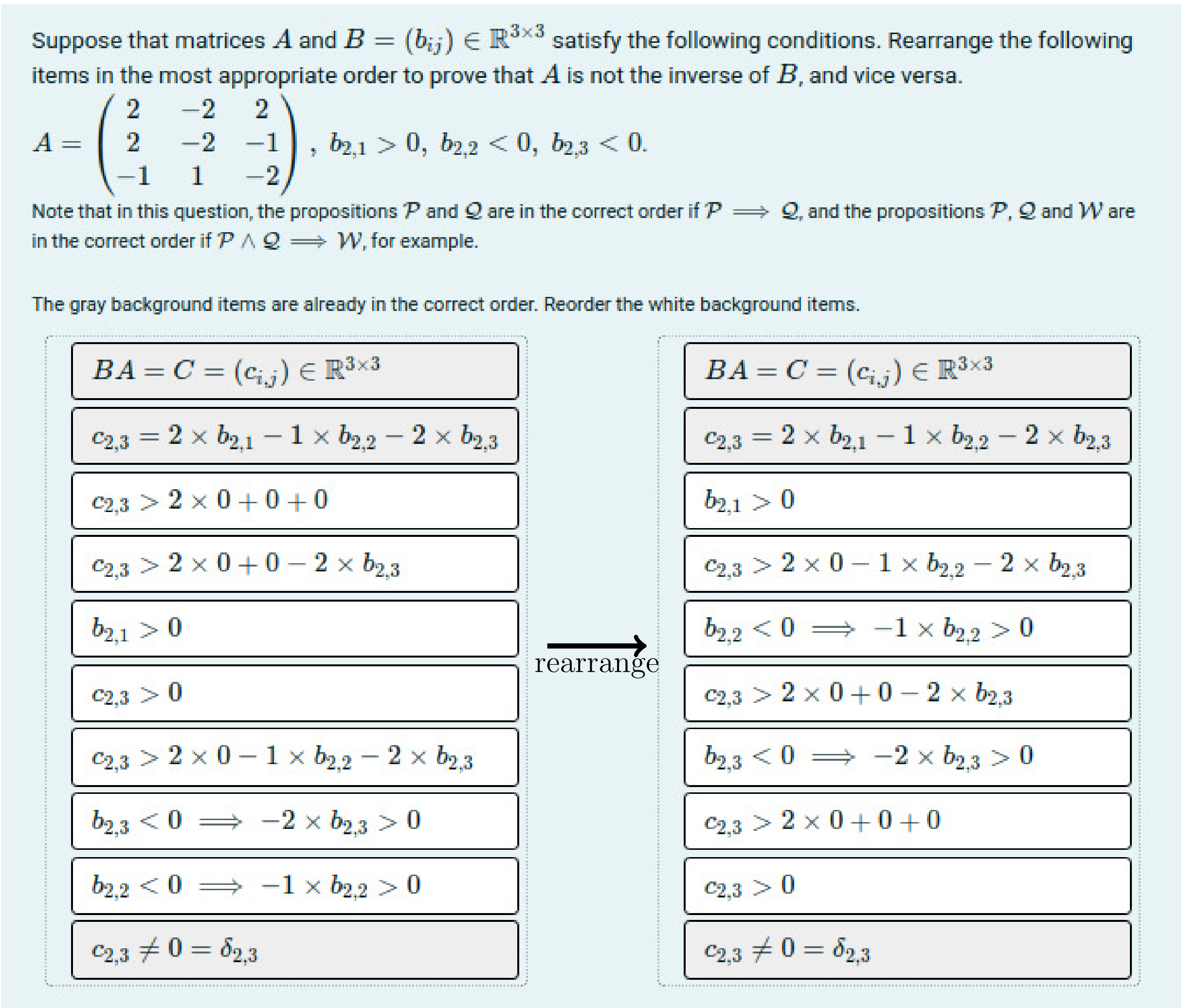

Figure 8: Reasoning question (left: an initial order, right: the correct order)

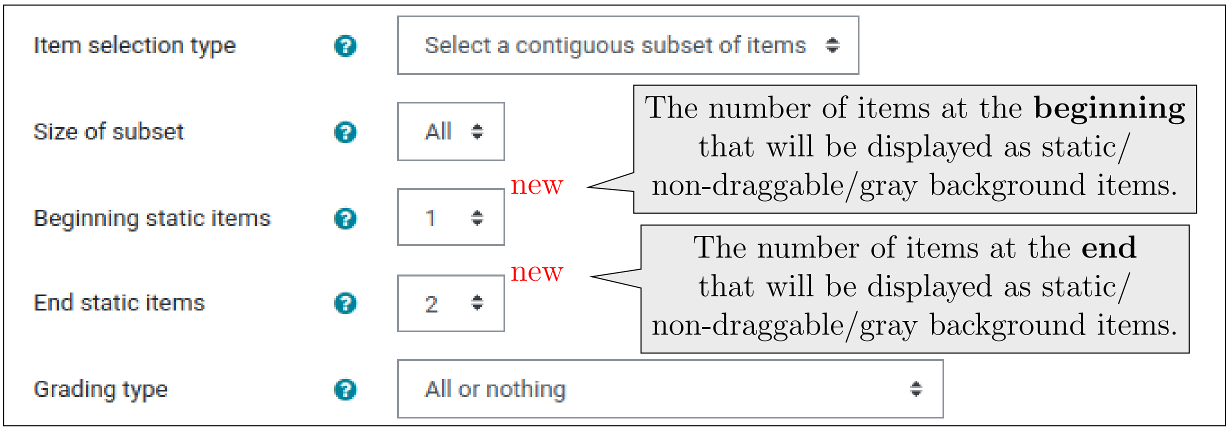

Figure 9: Configuration for static items

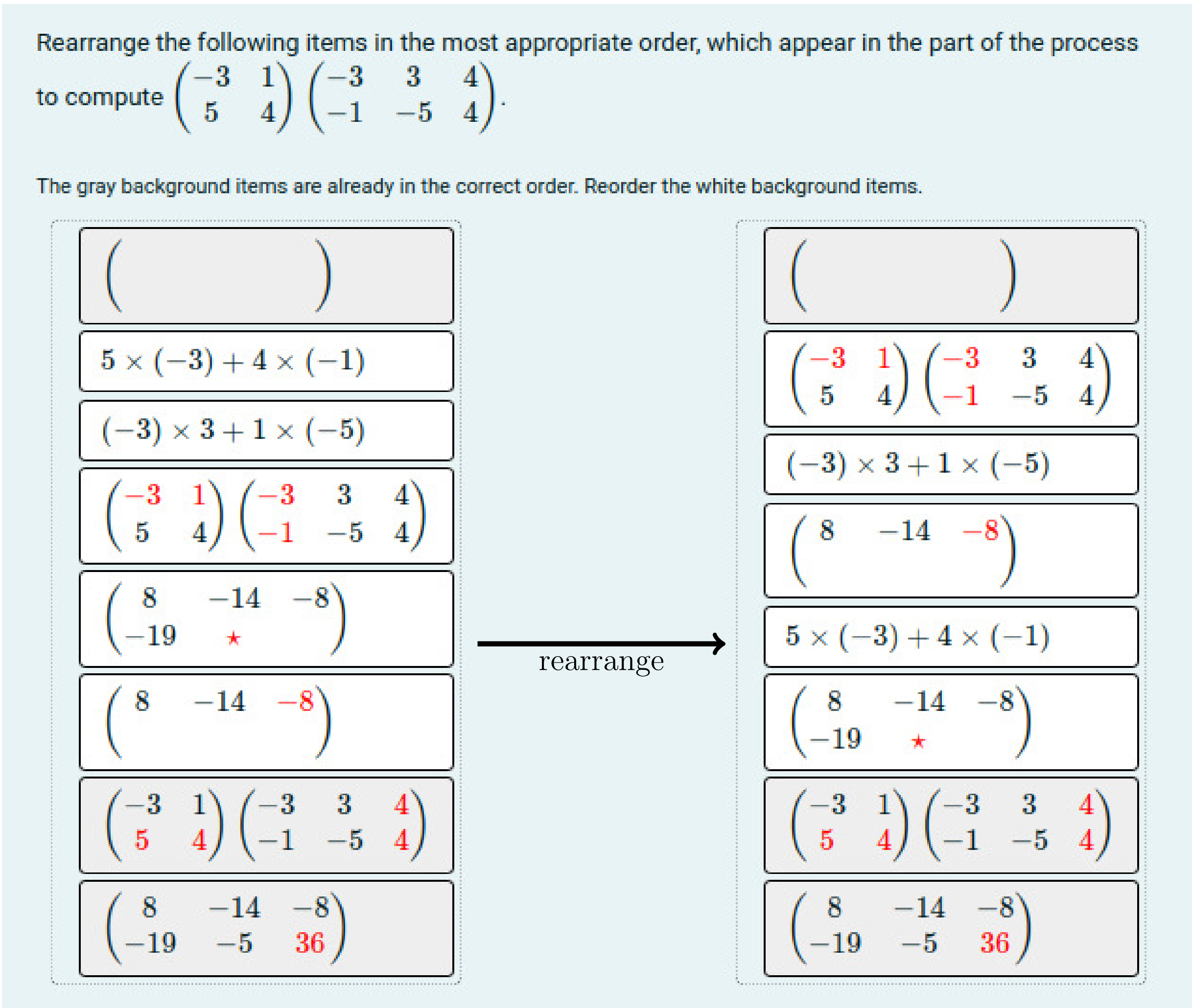

Figure 10: Subset of items with wildcard (left: an initial order, right: the correct order)

4 Conclusion

In this paper, we reviewed and proposed how to use the ordering question with clues in mathematics education, based on our experiments in a couple of linear algebra courses. With these attempts, we also proposed an improved version of ordering question type as a Moodle plugin, that enables us to use the ordering question with clues in a natural way.

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References

[1] Armstrong, P. Bloom's Taxonomy. Vanderbilt University Center for Teaching. `https://cft.vanderbilt.edu/guides-sub-pages/blooms-taxonomy/`, 2010. retrieved 2022-05-20.

[2] Bateson, G. Ordering question type for moodle 2.x and moodle 3.x. `https://moodle.org/plugins/view.php?plugin=qtype_ordering`, 2021. Release 2021-06-13.

[3] Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., and Aguilar, M. S. Blended learning, e-learning and mobile learning in mathematics education. *ZDM Mathematics Education 48* (2016), 589–610.

[4] Engelbrecht, J., Llinares, S., and Borba, M. C. Online mathematics education and e-learning (special issue). *ZDM Mathematics Education 52(5)* (2020).

[5] Hoogland, K., and Tout, D. Computer-based assessment of mathematics into the twenty-first century: pressures and tensions. *ZDM Mathematics Education 50* (2018), 675–686.

[6] Hughes, L. S. Gauging online extensive reading progress via automatically generated comprehension questions. *Journal of Saitama City Educators 2*, 5 (2012), 13–21.

[7] Jason Silverman, V. H. *Distance Learning, E-Learning and Blended Learning in Mathematics Education.* ICME-13 Monographs. Springer Nature, 2018.

[8] Joarder, A. H. Factorizing a quadratic expression with reasoning. *GANIT: Journal of Bangladesh Mathematical Society 41*, 1 (2021), 15–25.

[9] Krathwohl, D. R. A revision of bloom's taxonomy: An overview. *Theory Into Practice 41*, 4 (2002), 212–218.

[10] Kurayama, M. Developing a geometric proof problem-solving support system utilizing card selection. In *Proceedings of the 25 th International Conference on Computers in Education* (2017), pp. 110–112.

[11] McCabe, M., Heal, A., and White, A. Computer assisted assessment of proof = Proof of CAA: New approaches to computer assisted assessment for higher level learning. In *Proceedings of the 5 th International Conference on Technology in Mathematics Teaching* (2001). http://wwwg.uni-klu.ac.at/stochastik.schule/ICTMT_5/ICTMT_5_CD/Strands/CD_Strand_2.htm#b11.

[12] Nagasaka, K. Moodle Katex filter. In *Proceedings of the Study of Mathematical Software and Its Effective Use for Mathematics Education* (2019), no. 2142 in RIMS Kôkyûroku, pp. 106–108. (Japanese).

[13] Nagasaka, K. Multiple-choice questions in mathematics: automatic generation, revisited. In *Proceedings of the 25th Asian Technology Conference in Mathematics* (2020). 21785-1–21785-15.

[14] Nagasaka, K. Mathematical thinking skills and automatic generation of ordering questions. *Bulletin of Japan Society for Symbolic and Algebraic Computation* (in press). 14 pages (Japanese).

[15] Nistor, N., Hofer, S., and Scheibenzuber, C. Online teaching and learning in higher education lessons learned in crisis situations (special issue). *Computers in Human Behavior* (2021).

[16] Poulsen, S., Viswanathan, M., Herman, G. L., and West, M. Proof blocks: Autogradable scaffolding activities for learning to write proofs. Proceedings of the 27th ACM Conference on Innovation and Technology in Computer Science Education Vol 1 (ITiCSE 2022), 2021. arXiv:2106.11032.

[17] Poulsen, S., Viswanathan, M., Herman, G. L., and West, M. Evaluating proof blocks problems as exam questions. *ACM Inroads 13*, 1 (feb 2022), 41–51.

[18] Sayers, D. Computer-assisted language learning in bilingual vocational education. Connecticut State Dept. of Education, Hartford, 1984.

[19] Zimmaro, D. Writing good multiple-choice exams. https://facultyinnovate.utexas.edu/sites/default/files/writing-good-multiple-choice-exams-fic-120116.pdf, 2004. workshop material.

Appendix: Further Examples

There are a couple of approaches for solving a given quadratic equation. Therefore, even if the students can solve the equation by their own methods, rearranging the given items in the most appropriate order may not be an easy task and we can expect that it requires some high order thinking skills. For example, the figure 11 is an ordering question for solving a quadratic equation where the items are not in the correct order, and the figure 12 is a random subset version of the same problem. We recommend the readers to try these questions without looking at the figures 13 and 14 where you can find the correct orders.

Rearrange the following items in the most appropriate order to solve the equation
$5x^2 - 33x - 99 = -3x^2 + 7x + 13$.
Note that some items may be missing, so imagine them if necessary.

The gray background items are already in the correct order. Reorder the white background items.

$5x^2 - 33x - 99 = -3x^2 + 7x + 13$

$\left(x^2 - 2 \times \frac{5}{2}x\right) - 14 = 0$

$\left(x - \frac{5}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = 0$

$\left(x - \frac{5}{2} - \frac{9}{2}\right)\left(x - \frac{5}{2} + \frac{9}{2}\right) = 0$

$(x - \frac{5}{2})^2 - \frac{81}{4} = 0$

$8x^2 - 40x - 112 = 0$

$8\left(x^2 - 5x - 14\right) = 0$

$x^2 - 5x - 14 = 0$

$5x^2 - 33x - 99 + \left(3x^2 - 7x - 13\right) = -3x^2 + 7x + 13 + \left(3x^2 - 7x - 13\right)$

$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7, -2$

Figure 11: Solving a quadratic equation with all the items (an initial order)

Rearrange the following items in the most appropriate order to solve the equation
$5x^2 - 33x - 99 = -3x^2 + 7x + 13$.
Note that some items may be missing, so imagine them if necessary.

The gray background items are already in the correct order. Reorder the white background items.

$5x^2 - 33x - 99 + \left(3x^2 - 7x - 13\right) = -3x^2 + 7x + 13 + \left(3x^2 - 7x - 13\right)$

$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14 = 0$

$8\left(x^2 - 5x - 14\right) = 0$

$8x^2 - 40x - 112 = 0$

$\left(x^2 - 2 \times \frac{5}{2}x\right) - 14 = 0$

$\left(x - \frac{5}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = 0$

$(x - 7)(x + 2) = 0$

Figure 12: Solving a quadratic equation with a random subset (an initial order)

Rearrange the following items in the most appropriate order to solve the equation
$5x^2 - 33x - 99 = -3x^2 + 7x + 13$.
Note that some items may be missing, so imagine them if necessary.

The gray background items are already in the correct order. Reorder the white background items.

$$5x^2 - 33x - 99 = -3x^2 + 7x + 13$$
$$5x^2 - 33x - 99 + \left(3x^2 - 7x - 13\right) = -3x^2 + 7x + 13 + \left(3x^2 - 7x - 13\right)$$
$$8x^2 - 40x - 112 = 0$$
$$8\left(x^2 - 5x - 14\right) = 0$$
$$x^2 - 5x - 14 = 0$$
$$\left(x^2 - 2 \times \frac{5}{2}x\right) - 14 = 0$$
$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14 = 0$$
$$(x - \frac{5}{2})^2 - \frac{81}{4} = 0$$
$$\left(x - \frac{5}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = 0$$
$$\left(x - \frac{5}{2} - \frac{9}{2}\right)\left(x - \frac{5}{2} + \frac{9}{2}\right) = 0$$
$$(x - 7)(x + 2) = 0$$
$$x = 7, -2$$

Figure 13: Solving a quadratic equation with all the items (the correct order)

Rearrange the following items in the most appropriate order to solve the equation
$5x^2 - 33x - 99 = -3x^2 + 7x + 13$.
Note that some items may be missing, so imagine them if necessary.

The gray background items are already in the correct order. Reorder the white background items.

$$5x^2 - 33x - 99 + \left(3x^2 - 7x - 13\right) = -3x^2 + 7x + 13 + \left(3x^2 - 7x - 13\right)$$
$$8x^2 - 40x - 112 = 0$$
$$8\left(x^2 - 5x - 14\right) = 0$$
$$\left(x^2 - 2 \times \frac{5}{2}x\right) - 14 = 0$$
$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14 = 0$$
$$\left(x - \frac{5}{2}\right)^2 - \left(\frac{9}{2}\right)^2 = 0$$
$$(x - 7)(x + 2) = 0$$

Figure 14: Solving a quadratic equation with a random subset (the correct order)