Research on Development of Teaching Material Put Emphasize on Retroductive Inference to Develop Statistical Thinking in Mathematics Education

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Abstract: *The purpose of this study was to develop teaching materials for 6th-grade elementary school students in Japan, focusing on retroductive inference, to develop their statistical thinking and explain its use. Recently, MEXT (Ministry of Education, Culture, Sports, Science and Technology) has made "Programming Education" compulsory at elementary schools in Japan. The purpose of "Programming Education" in Japan is to develop the programming thinking of students based on their computational thinking and enhance their academic ability in teaching and learning a particular subject, rather than learn coding. However, few studies have been conducted on the development of programming thinking based on computational thinking, for enhancing the academic ability of students. "Retroduction" is effective in promoting computational thinking. Also, the development of mathematical and computational thinking is interacted by one another. Therefore, we developed a teaching material to enable the students to engage in retroductive inference. A context and situation setting were provided to them to make retroductive inferences. The results indicated that it seems reasonable to assume that the teaching material promoted their computational thinking and developed their statistical thinking by creating phases for retroductive inferences.*

1. Introduction

The development of student's mathematical thinking through mathematics lessons is essential. To do so, the teachers need to give due consideration, not only to mathematics teaching materials but also to the lessons, keeping in mind the teaching-learning contents and the mathematical processes.

Recently, MEXT (Ministry of Education, Culture, Sports, Science and Technology) has made "Programming Education" compulsory at elementary schools in Japan. The purpose of "Programming Education" is to develop the "programming thinking" of students, based on computational thinking, and enhance their academic ability in learning a particular subject, not learn to code [1]. It is necessary to develop their "programming thinking" based on their computational thinking and enhance their academic ability in mathematics education. However, surprisingly few studies (for example, [2] and [3]) have been conducted to develop the "programming thinking" of elementary school students based on their computational thinking and enhance their academic ability.

It suggests that the development of mathematical thinking and computational thinking are interacted by one another [3]. Therefore, it is necessary to discuss whether the use of computational thinking in finding solutions to problems has any role in the development of mathematical thinking [3]. Therefore, we developed teaching materials that could aid in the development of mathematical thinking, particularly statistical thinking, by promoting their computational thinking.

"Retroduction" is effective in promoting computational thinking [4]. Therefore, in this paper, we adopt "Retroduction" advocated by Charles Sanders Peirce.

The term “Retroduction” can be defined as “heuristic reasoning” and studying of facts for devising a theory, to explain the surprising fact observed [5, 6]. For example, students guess a relationship between times and water level is liner function while water is drained out from the bottom of a bottle full of water. However, they realize a relationship between these is a quadratic function if they carry out an experiment in which water is drained out from the bottom of a bottle full of water. In this experiment, the surprising fact observed by them is that the relationship between these is a quadratic function. If hypothesis the cause of what relationship between these is quadratic function is water pressure were true, what relationship between these is quadratic function would be a matter of course, hence, there is reason to suspect that hypothesis the cause of what relationship between these is quadratic function is water pressure is true.

We focused on “Retroduction” advocated by Charles Sanders Peirce and developed teaching materials to promote the computational thinking of the students by creating phases on which the students were asked to draw retroductive inference, to develop their statistical thinking.

The purpose of this study was to develop teaching materials and emphasize retroductive inference to develop the statistical thinking of students and discuss it.

2. Theory

It suggests that the development of mathematical and computational thinking is interacted by one another [3]. Therefore, it is necessary to discuss whether one should make full use of computational thinking in locating and finding the solution to problems to develop mathematical thinking [3]. In pursuance of the aim, we developed teaching materials that could enable the development of mathematical thinking, particularly statistical thinking, by promoting computational thinking of elementary school students.

According to Wing, computational thinking is analytical thinking and its essence lies in abstraction and automation [7, 8].

For example, students created programs by using Scratch developed by MIT (Massachusetts Institute of Technology), to construct an equilateral triangle (see Figure 1) by applying the facts that the lengths of the three sides of the triangle are equal and all interior angles are equal to 60° [4].

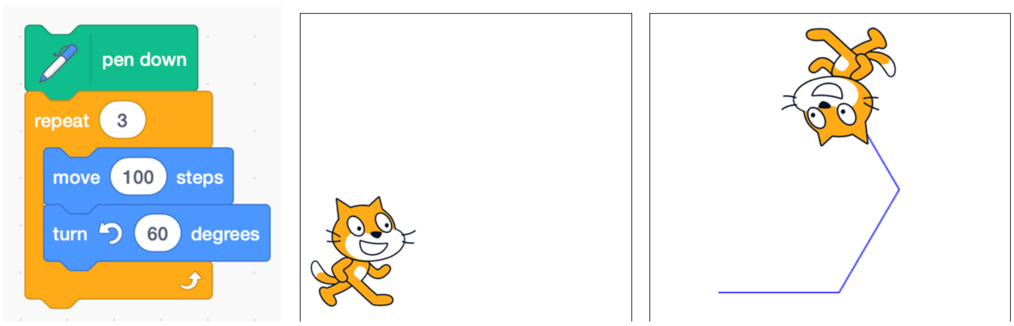

Figure 1 Program to Move A Character and A Movement of A Character

At this juncture, students extracted specific elements (i.e., sides and angles of equilateral triangles) from a variety of elements (i.e., vertices, areas of equilateral triangles, etc.), and created a program to move the character. This is a process of abstraction and automation.

However, if we look at this program (see Figure 1) which students created, we can see that it is not correct. Therefore, students considered improving the program, so that the character could be moved properly. Students repeated and refined the abstraction and automation by inferring why the character doesn’t move as expected in such a case (see Figure 1). This concerned drawing inferences as to why things did or did not go as expected by observing the results, which promotes student's computational thinking.

Therefore, in this paper, we adopt "Retroduction" advocated by Charles Sanders Peirce to promote their computational thinking. The term "Retroduction" can be defined as heuristic reasoning and studying facts and devising a theory to explain the surprising fact observed [5, 6]. Drawing on the concept of Retroduction proposed by Pierce, the surprising fact observed by the students was that an equilateral triangle cannot be constructed using Scratch. Students formulated the following hypotheses: there is an error in setting the lengths of three sides; there is an error in setting interior angles, etc., to elucidate the reasons why they were unable to construct an equilateral triangle. Students found the right program by verifying the hypotheses they formulated, as "Retroduction" is effective in promoting computational thinking [4].

Based on the above example, it can be said that "Retroduction" works appropriately in promoting computational thinking. Therefore, we argue that "Retroduction" is a viewpoint to develop teaching material. Specifically, we created several phases which they inferred upon using retroduction.

3. Development of Teaching-Material

We developed teaching material for learning and teaching statistics in mathematics for 6th-grade elementary school students in Japan.

As mentioned in the preceding section, the development of a viewpoint on the development of teaching material in the creation of several phases of retroductive inference by elementary school students involves computational thinking. Therefore, it is necessary to provide a context and situation so that students draw retroductive inferences, for example, why the automatic vending machine in Japan rejects coins put in the coin slot.

The automatic vending machine in Japan accepts coins based on their weight, diameter, thickness, and quality of the material. The weight, diameter, and thickness of coins may be altered because of any flaw, soil, etc. Therefore, the automatic vending machine is expected to rarely reject coins dropped into the coin slot.

To start the lesson, the teacher presents a picture to show how the automatic vending machine rejects coins put in the coin slot. The teacher asks the students, "why the automatic vending machine would reject coins put in the coin slot?" In an attempt to investigate why the automatic vending machine rejects coins, the students will draw a flow chart (see Figure 2) to elucidate its structure.

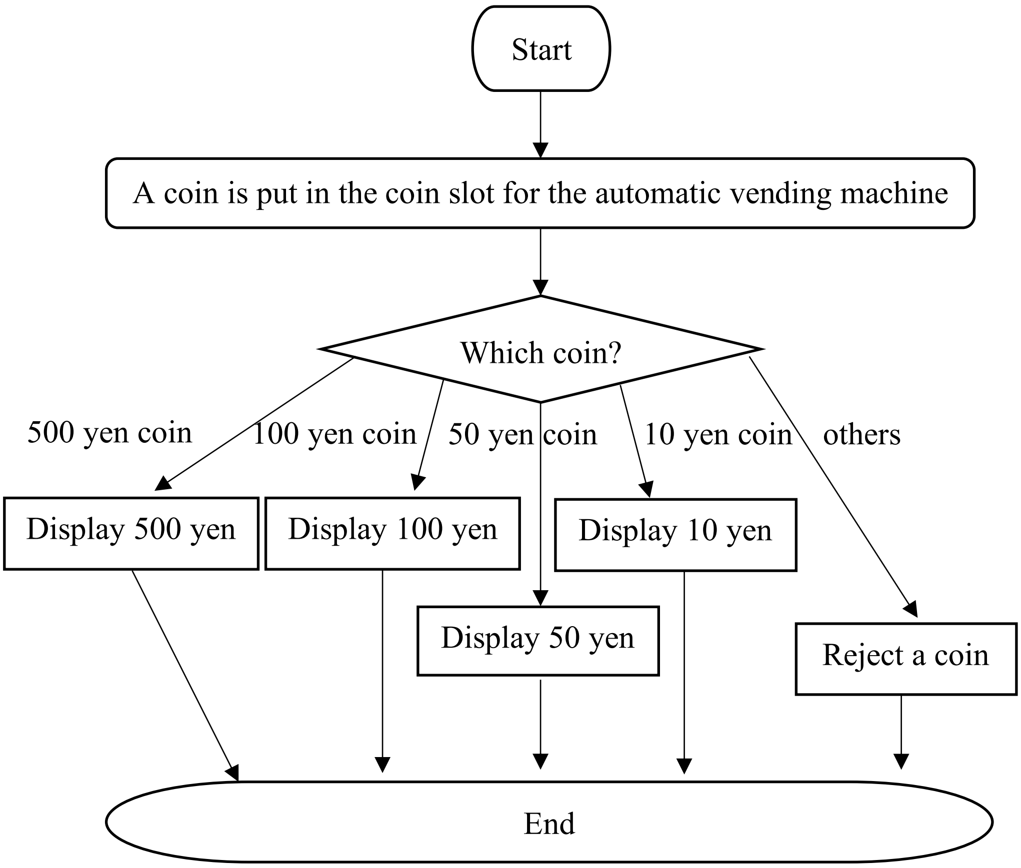

Figure 2 The processes of how the automatic vending machine recognizes coins

Students will consider how to judge coins in "Which coin?" (see Figure 2). With regard to this teaching material, we assumed that students would solve the problem by laying particular emphasis on the weight of the coins. The teacher prepares Scout^{TM} Series Balanced-STX222JP of OHAUS (see Figure 3) if the students emphasized the weight of the coin for solving the problem.

Figure 3 Scout^{TM} Series Balanced-STX222JP of OHAUS

Scout™ Series Balanced-STX222JP by OHAUS (see Figure 3) compares the weight of a sample against target limits and displays results as "Under", "Accept" or "Over".

In Japan, the coins have fixed weights, for example, a 10 yen coin weighs 4.5 grams, a 100 yen coin weighs 4.8 grams, and so on. Therefore, students are likely to consider that "Accept" will be displayed on the screen if the balance is set at 4.50 grams as "Over Limit" and a 10 yen coin can be put on the pan. However, this is not necessary as the weight of the coins might undergo changes depending on the flaw, soil, and so on (see Figure 4).

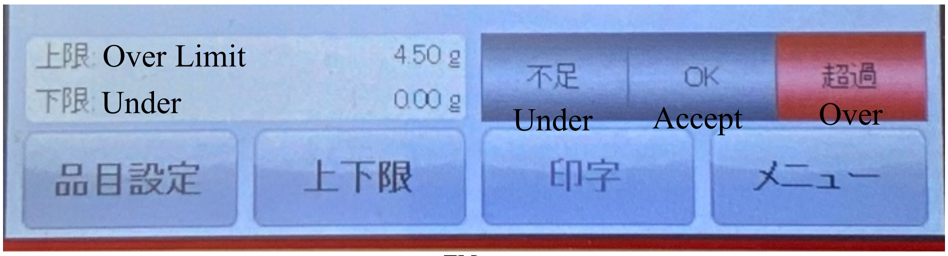

Figure 4 The Screen Display of Scout™ Series Balanced-STX222JP of OHAUS

At this time, students might think why "Over" is displayed on the screen of the balance when they set "Over Limit" as 4.50 grams and put a 10 yen coin on the pan. Students will hypothesize that the weight of a coin put in the coin slot is different from the original one and attempt to solve this problem by using a frequency table, a histogram, a dot plot, and the representative values.

4. Discussion

We discussed the teaching material developed in chapter 3 to promote computational thinking, by creating phases for retroductive inference by elementary school students and the development of statistical thinking by promoting computational thinking.

4.1 Promoting Computational Thinking

The essence of computational thinking is abstraction and automation [7, 8]. Therefore, it needs to be discussed whether repetition and refinement of abstraction and automation are promoted.

By using the teaching material, elementary school students were able to observe that an automatic vending machine in Japan would reject coins. Students hypothesized by doing trial and error to elucidate why this would occur. For example, something is wrong with the automatic vending machine, the coin put in the coin slot is counterfeit money, the weight of a coin put in the coin slot is different from the original one, and so on. At this time, students created a flow chart of the processes by which the automatic vending machine recognized the coins (see Figure 2), repetition and refinement of abstraction and automation through inferring why the automatic vending machine would reject coins to verify if their hypotheses were correct. This was a process of abstraction and automation.

For example, students extract a specific element (the weight of coins) from a variety (color, diameter, thickness of coins, etc.) and hypothesize that the weight of the coin that is put in the coin slot is different from the original one. This shows that students engage in abstract thought with regard to the rejection of the coins by the automatic vending machine.

Creation of the flow chart to depict the processes involved in recognition of the coins by the automatic vending machine (see Figure 2) was done to elucidate how the automatic vending machine judges coins to automate its programs.

From the above example, it can be assumed that setting a context and situation for retroductive inference promotes computational thinking in students.

4.2 Development of Statistical Thinking

To develop statistical thinking, it is important to solve problems and find new ones by employing statistical thinking in PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle [9]. Therefore, setting processes involving solving problems with statistical thinking through PPDAC cycle in mathematics lessons was to be explored.

The students observed the surprising fact that “Over” was displayed on the screen when they set “Over Limit” as 4.50 grams and they put a 10 yen coin on a balance (see Figure 4). Students hypothesized after trial and error that the weight of a 10 yen coin that was put in the automatic vending machine is not 4.50 grams to elucidate why “Over” is displayed on the screen.

The students investigated whether the automatic vending machine judges the 10 yen coins by their weight. This is the process of “Problem” step of PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle.

We assumed that the students solved this problem by following the steps described below.

Firstly, the students considered the data regarding weight (i.e., how many 10 yen coins should be collected ?; how to collect the weight data pertaining to the 10 yen coins? etc.) This comprised the process of “Plan” step of PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle.

Secondly, students collected the weight data pertaining to 10 yen coins and organized them in a table (see Table 1). The weight data of hundred 10 yen coins collected by the author is depicted below. This was the process of “Data” step of PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle.

Table 1 The Weight Data on A Hundred 10 yen Coins

4.42	4.43	4.45	4.45	4.45	4.45	4.45	4.45	4.45	4.46
4.47	4.47	4.47	4.47	4.47	4.47	4.47	4.48	4.48	4.48
4.48	4.48	4.48	4.48	4.48	4.48	4.49	4.49	4.49	4.49
4.49	4.49	4.49	4.49	4.49	4.49	4.49	4.50	4.50	4.50
4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.51	4.51	4.51
4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51
4.51	4.51	4.51	4.52	4.52	4.52	4.52	4.52	4.52	4.52
4.52	4.52	4.52	4.52	4.52	4.52	4.52	4.52	4.52	4.52
4.52	4.52	4.52	4.53	4.53	4.53	4.53	4.53	4.53	4.55

Thirdly, students illustrated the frequency distribution of the weight data of the hundred 10 yen coins with the help of a frequency table (Table 2), histogram (Figure. 6) and the representative values (Table 3) for these data were calculated and the variations in these data were illustrated with the help of a dot plot (Figure 5). We assumed that the students used computers to express in a frequency table, a histogram, and a dot plot of the weight data on a hundred 10 yen coins and calculated the representative values for these data.

Table 2 A Frequency Table of The Weight Data on A Hundred 10 yen Coins

Interval			Frequency
or higher		less than	
4.42	~	4.44	2
4.44	~	4.46	7
4.46	~	4.48	8
4.48	~	4.50	20
4.50	~	4.52	36
4.52	~	4.54	26
4.54	~	4.56	1
total			100

Table 3 Representative Values of The Weight Data on A Hundred 10 yen Coins

	(grams)
Mean	4.4975
Median	4.50
Mode	4.51

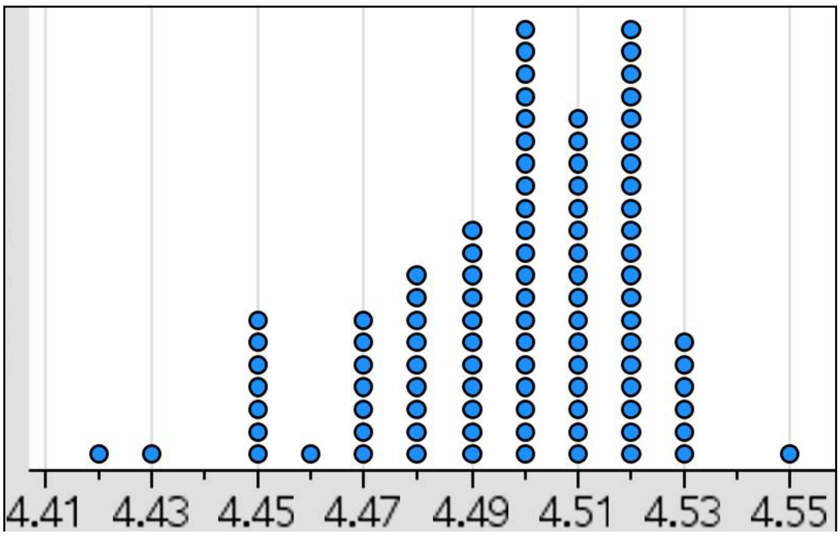

Figure 5 Dot Plot of The Weight Data on A Hundred 10 yen Coins

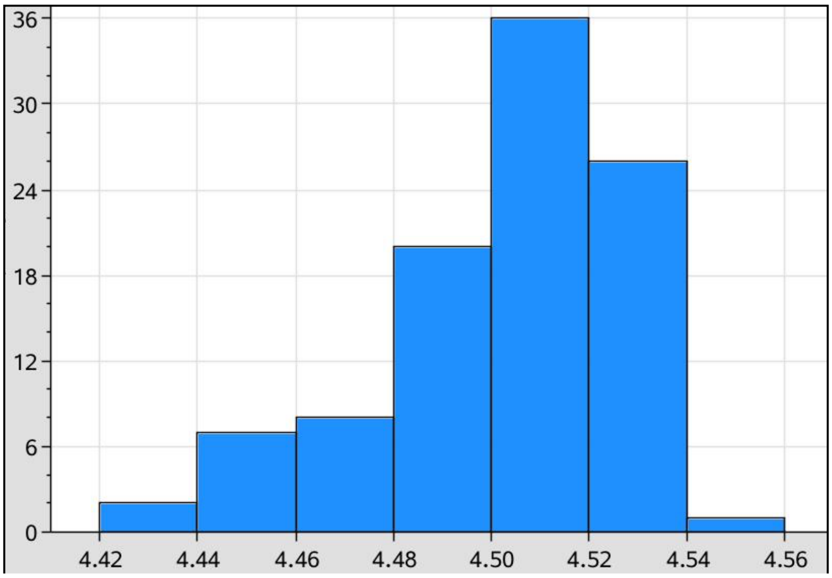

Figure 6 Histogram of The Weight Data on A Hundred 10 yen Coins

The process of "Analysis" step of PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle is described below:

Students found out what the frequency table, histogram, dot plot, and the calculated representative values represented, for example, they found what 10 yen coins weighing 4.53 grams should be ignored with the frequency table because a relative frequency of 10 yen coins weighing 4.54 grams or higher but less than 4.56 grams was 0.001, they found what 10 yen coins weighing 4.58 grams may be existed based on variations of the weight of 4.50 grams or less with the dot plot and so on.

Finally, students evaluated the graphs to determine values for their adequacy so that they could set the weight of the coins, in grams, for "Over Limit" and "Under Limit" to find out the number of 10 yen coins which would be adequate. The students concluded that the weight of "Over Limit" should be set as 4.53 grams, the weight of "Over Limit" should be set as 4.58 grams and the weight of "Under Limit" should be set as 4.42 grams, etc. During the processes of setting the weight of "Over Limit" and "Under Limit", students found that setting them with representative values was not adequate.

Students must investigate not only the weight of the 10 yen coin but also the weight of 50, 100, and 500 yen coins. They should not only determine the weight of the coins but also their diameter, thickness, and quality of the material of coins. For example, if a 100 yen coin weighing 4.69 grams is put on balance, it could have been included in the "Accept" category, if they set the "Over Limit" as 4.70 grams. This was because the weight difference between a 10 and 100 yen coin was 0.30 grams (see Figure 7). The data collected by the author has been depicted in the dot plot. The process indicated above comprised the "Conclusion" step of PPDAC (Problem, Plan, Data, Analysis, Conclusion) cycle.

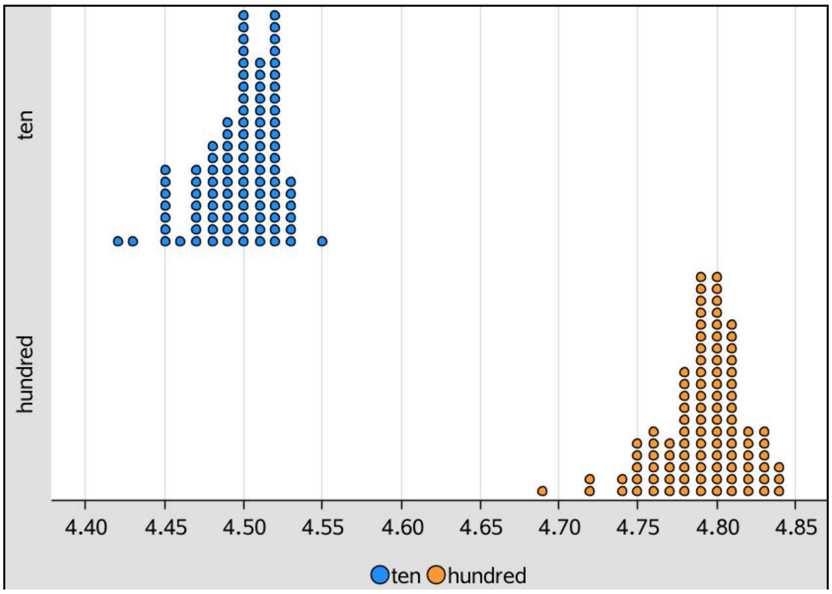

Figure 7 Dot Plot of The Weight Data on Hundred 10 yen Coins and Hundred 100 yen Coins

From the above process, it was expected that students set the problems, solve problems by employing statistical thinking and find new problems in the process during automation of programs, as to how the automatic vending machine judges the coins. Also, it was expected from the students that they validated their own judgment with the help of the frequency table, histogram, dot plot, and the representative values and reflect their essence. Therefore, it seemed reasonable to suppose that it is expected that promoting computational thinking develops statistical thinking.

5. Conclusion

To conclude, we developed teaching material to emphasize retroductive inference for 6th-grade elementary school students in Japan to promote their computational thinking and develop their statistical thinking and discussed it from the view of promoting their computational thinking by creating phases for retroductive inference by students. For example, a context and situation setting so that students inferred upon using retroduction why the automatic vending machine in Japan rejects coins put in the coin slot and development of their statistical thinking by promoting their computational thinking.

The utility of the teaching material in promoting student's computational thinking and development of their statistical thinking became evident.

One of the challenges before us was to use the teaching material for classroom practice and analyze effect of the teaching material on promoting student's computational thinking and development of their statistical thinking.

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